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Almost Contra Pre Generalized b - Continuous Functions in Topological Spaces

S. Sekar[a,*] and R. Brindha[b]

[a] Department of Mathematics, Government Arts College (Autonomous), Salem – 636 007, Tamil Nadu, India.

[b] Department of Mathematics, King College of Technology, Namakkal – 637 020, Tamil Nadu, India.

Abstract

In this paper, the authors introduce a new class of functions called almost contra pre generalized b - continuous function (briefly almost contra pgb-continuous) in topological spaces. Some characterizations and several properties concerning almost contra pgb-continuous functions are obtained.

Keywords: pgb-closed sets, pgb-closed map, pgb-continuous map, contra pgb-continuity.

2010 MSC: 54C05, 54C08, 54C10.

1 Introduction

In 2002, Jafari and Noiri introduced and studied a new form of functions called contra-pre continuous functions. The purpose of this paper is to introduce and study almost contra pgb-continuous functions via the concept of pgb-closed sets. Also, properties of almost contra pgb-continuity are discussed. Moreover, we obtain basic properties and preservation theorems of almost contra pgb-continuous functions and relationships between almost contra pgb-continuity and pgb-regular graphs.

Through out this paper (X,τ) and (Y,σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all pgb-open sets X contained in A is called pgb-interior of A and it is denoted by $pgbint(A)$, the intersection of all pgb-closed sets of X containing A is called pgb-closure of A and it is denoted by $pgbcl(A)$ [9].

2 Preliminaries

Definition 2.1. *Let a subset A of a topological space (X,τ), is called*
1) a pre-open set [8] if $A \subseteq int(cl(A))$.
2) a semi-open set [6] if $A \subseteq cl(int(A))$.
3) a b -open set [3] if $A \subseteq cl(int(A)) \cup int(cl(A))$.
4) a generalized b- closed set (briefly gb- closed) [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
5) a generalized αb- closed set (briefly gαb- closed) [11] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.
6) a regular generalized b- closed set (briefly rgb- closed) [7] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
7) a pre generalized b- closed set (briefly pgb- closed) [9] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in X.

*Corresponding author.
E-mail address: sekar_nitt@rediffmail.com (S. Sekar), brindhaaramasamy@gmail.com (R. Brindha).

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$, is called*
1) almost contra continuous [1] if $f^{-1}(V)$ is closed in (X, τ) for every regular-open set V of (Y, σ).
2) almost contra b-continuous [2] if $f^{-1}(V)$ is b-closed in (X, τ) for every regular-open set V of (Y, σ). 3) almost contra pre-continuous [5] if $f^{-1}(V)$ is pre-closed in (X, τ) for every regular-open set V of (Y, σ).
4) almost contra semi-continuous [4] if $f^{-1}(V)$ is semi-closed in (X, τ) for every regular-open set V of (Y, σ).
5) almost contra rgb-continuous [10] if $f^{-1}(V)$ is rgb-closed in (X, τ) for every regular-open set V of (Y, σ).

3 Almost Contra Pre Generalized b - Continuous Functions

In this section, we introduce almost contra pre generalized b - continuous functions and investigate some of their properties.

Definition 3.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is called almost contra pre generalized b - continuous if $f^{-1}(V)$ is pgb - closed in (X, τ) for every regular open set V in (Y, σ).*

Example 3.1. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is almost contra pgb - continuous.*

Theorem 3.1. *If $f : X \to Y$ is contra pgb - continuous then it is almost contra pgb - continuous.*

Proof. Obvious, because every regular open set is open set. □

Remark 3.1. *Converse of the above theorem need not be true in general as seen from the following example.*

Example 3.2. *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is almost contra pgb - continuous function but not contra pgb - continuous, because for the open set $\{a, c\}$ in Y and $f^{-1}\{a, c\} = \{a, b\}$ is not pgb - closed in X.*

Theorem 3.2. *1) Every almost contra b - continuous function is almost contra pgb - continuous function.*
2) Every almost contra $g\alpha$ - continuous function is almost contra pgb - continuous function.
3) Every almost contra $g\alpha*$ - continuous function is almost contra pgb - continuous function.
4) Every almost contra g - continuous function is almost contra pgb - continuous function.
5) Every almost contra rgb - continuous function is almost contra pgb - continuous function.
6) Every almost contra $g\alpha b$ - continuous function is almost contra pgb - continuous function.

Remark 3.2. *Converse of the above statements is not true as shown in the following example.*

Example 3.3. *i) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$. Clearly f is almost contra pgb - continuous but f is not almost contra b - continuous. Because $f^{-1}(\{b\}) = \{c\}$ is not b - closed in (X, τ) where $\{b\}$ is regular - open in (Y, σ).*
ii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is almost contra pgb - continuous but f is not almost contra $g\alpha$ - continuous. Because $f^{-1}(\{b\}) = \{a\}$ is not $g\alpha$ - closed in (X, τ) where $\{a\}$ is regular - open in (Y, σ).
iii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly f is almost contra pgb - continuous but f is not almost contra $g\alpha*$ - continuous. Because $f^{-1}(\{b\}) = \{b\}$ is not $g\alpha*$ - closed in (X, τ) where $\{b\}$ is regular - open in (Y, σ).
iv) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is almost contra pgb - continuous but f is not almost contra g - continuous. Because $f^{-1}(\{b\}) = \{a\}$ is not g - closed in (X, τ) where $\{b\}$ is regular - open in (Y, σ).
v) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is almost contra pgb - continuous but f is not almost contra rgb - continuous. Because $f^{-1}(\{c\}) = \{a\}$ is not rgb - closed in (X, τ) where $\{c\}$ is regular - open in (Y, σ).
vi) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly f is almost contra pgb - continuous but f is not almost contra $g\alpha b$ - continuous. Because $f^{-1}(\{a\}) = \{a\}$ is not $g\alpha b$ - closed in (X, τ) where $\{b\}$ is regular - open in (Y, σ).

Theorem 3.3. *The following are equivalent for a function $f : X \to Y$,*
(1) f is almost contra pgb - continuous.
(2) for every regular closed set F of Y, $f^{-1}(F)$ is pgb - open set of X.
(3) for each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists pgb - open U containing x such that $f(U) \subset F$.
(4) for each $x \in X$ and each regular open set V of Y not containing $f(x)$, there exists pgb - closed set K not containing x such that $f^{-1}(V) \subset K$.

Proof. (1) ⇒ (2) : Let F be a regular closed set in Y, then $Y - F$ is a regular open set in Y. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is pgb - closed set in X. This implies $f^{-1}(F)$ is pgb - open set in X. Therefore, (2) holds.
(2) ⇒ (1) : Let G be a regular open set of Y. Then $Y - G$ is a regular closed set in Y. By (2), $f^{-1}(Y - G)$ is pgb - open set in X. This implies $X - f^{-1}(G)$ is pgb - open set in X, which implies $f^{-1}(G)$ is pgb - closed set in X. Therefore, (1) hold.
(2) ⇒ (3) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. By (2), $f^{-1}(F)$ is pgb - open in X containing x. Set $U = f^{-1}(F)$, which implies U is pgb - open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$. Therefore (3) holds.
(3) ⇒ (2) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. From (3), there exists pgb - open U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$. Thus $f^{-1}(F) = \{\cup U_x : x \in f^{-1}(F)$, which is union of pgb - open sets. Therefore, $f^{-1}(F)$ is pgb - open set of X.
(3) ⇒ (4) : Let V be a regular open set in Y not containing $f(x)$. Then $Y - V$ is a regular closed set in Y containing $f(x)$. From (3), there exists a pgb - open set U in X containing x such that $f(U) \subset Y - V$ .This implies $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X - U$. Set $K = X - V$, then K is pgb - closed set not containing x in X such that $f^{-1}(V) \subset K$.
(4) ⇒ (3) : Let F be a regular closed set in Y containing $f(x)$. Then $Y - F$ is a regular open set in Y not containing $f(x)$. From (4), there exists pgb - closed set K in X not containing x such that $f^{-1}(Y - F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X - K) \subset F$. Set $U = X - K$, then U is pgb - open set containing x in X such that $f(U) \subset F$. □

Theorem 3.4. *The following are equivalent for a function $f : X \to Y$,*
(1) f is almost contra pgb - continuous.
(2) $f^{-1}(Int(Cl(G)))$ is pgb - closed set in X for every open subset G of Y.
(3) $f^{-1}(Cl(Int(F)))$ is pgb - open set in X for every closed subset F of Y.

Proof. (1) ⇒ (2) : Let G be an open set in Y. Then $Int(Cl(G))$ is regular open set in Y. By (1), $f^{-1}(Int(Cl(G)) \in pgb - C(X)$.
(2) ⇒ (1) : Proof is obvious.
(1) ⇒ (3) : Let F be a closed set in Y. Then $Cl(Int(G))$ is regular closed set in Y. By (1), $f^{-1}(Cl(Int(G)) \in pgb - O(X)$.
(3) ⇒ (1) : Proof is obvious. □

Definition 3.4. *A function $f : X \to Y$ is said to be R - map if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.*

Definition 3.5. *A function $f : X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for each open set V of Y.*

Theorem 3.5. *For two functions $f : X \to Y$ and $g : Y \to Z$, let $g \circ f : X \to Z$ be a composition function. Then, the following properties hold.*
(1) If f is almost contra pgb - continuous and g is an R - map, then $g \circ f$ is almost contra pgb - continuous.
(2) If f is almost contra pgb - continuous and g is perfectly continuous, then $g \circ f$ is contra pgb - continuous.
(3) If f is contra pgb - continuous and g is almost continuous, then $g \circ f$ is almost contra pgb - continuous.

Proof. (1) Let V be any regular open set in Z. Since g is an R - map, $g^{-1}(V)$ is regular open in Y. Since f is almost contra pgb - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is pgb - closed set in X. Therefore $g \circ f$ is almost contra pgb - continuous.
(2) Let V be any regular open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is almost contra pgb - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is pgb - open and pgb - closed set in X. Therefore $g \circ f$

is pgb continuous and contra pgb - continuous.
(3) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is open in Y. Since f is almost contra pgb - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is pgb - closed set in X. Therefore $g \circ f$ is almost contra pgb - continuous. □

Theorem 3.6. *Let $f : X \to Y$ be a contra pgb - continuous and $g : Y \to Z$ be pgb - continuous. If Y is $Tpgb$ - space, then $g \circ f : X \to Z$ is an almost contra pgb - continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is pgb - continuous $g^{-1}(V)$ is pgb - open in Y and Y is $Tpgb$ - space implies $g^{-1}(V)$ open in Y. Since f is contra pgb - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is pgb - closed set in X. Therefore, $g \circ f$ is an almost contra pgb - continuous. □

Theorem 3.7. *If $f : X \to Y$ is surjective strongly pgb - open (or strongly pgb - closed) and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is an almost contra pgb - continuous, then g is an almost contra pgb - continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g \circ f$ is an almost contra pgb - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is pgb - open (resp. pgb - closed) in X. Since f is surjective and strongly pgb - open (or strongly pgb - closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is pgb - open(or pgb - closed). Therefore g is an almost contra pgb - continuous. □

Definition 3.6. *A function $f : X \to Y$ is called weakly pgb - continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in pgb - O(X; x)$ such that $f(U) \subset cl(V)$.*

Theorem 3.8. *If a function $f : X \to Y$ is an almost contra pgb - continuous, then f is weakly pgb - continuous function.*

Proof. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $cl(V)$ is regular closed in Y containing $f(x)$. Since f is an almost contra pgb - continuous function by Theorem 3.4 (2), $f^{-1}(cl(V))$ is pgb - open set in X containing x. Set $U = f^{-1}(cl(V))$, then $f(U) \subset f(f^{-1}(Cl(V))) \subset cl(V)$. This shows that f is weakly pgb - continuous function. □

Definition 3.7. *A space X is called locally pgb - indiscrete if every pgb - open set is closed in X.*

Theorem 3.9. *If a function $f : X \to Y$ is almost contra pgb - continuous and X is locally pgb - indiscrete space, then f is almost continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra pgb - continuous $f^{-1}(U)$ is pgb - closed set in X and X is locally pgb - indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous. □

Lemma 3.1. *Let A and X_0 be subsets of a space X. If $A \in pgb - O(X)$ and $X_0 \in \tau^\alpha$, then $A \cap X_0 \in pgb - O(X_0)$.*

Theorem 3.10. *If $f : X \to Y$ is almost contra pgb - continuous and $X_0 \in \tau^\alpha$ then the restriction $f/X_0 : X_0 \to Y$ is almost contra pgb - continuous.*

Proof. Let V be any regular open set of Y. By Theorem, we have $f^{-1}(V) \in pgb - O(X)$ and hence $(f/X_0)^{-1}(V) = f^{-1}(V) \cap X_0 \in pgb - O(X_0)$. By Lemma 3.1, it follows that f/X_0 is almost contra pgb - continuous. □

Theorem 3.11. *If $f : X \to \prod Y_\lambda$ is almost contra pgb - continuous, then $P_\lambda \circ f : X \to Y_\lambda$ is almost contra pgb - continuous for each $\lambda \in \nabla$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

Proof. Let Y_λ be any regular open set of Y. Since P_λ is continuous open, it is an R - map and hence $(P_\lambda)^{-1} \in RO(\prod Y_\lambda)$.
By theorem, $f^{-1}(P_\lambda^{-1}(V)) = (P_\lambda \circ f)^{-1} \in pgb - O(X)$. Hence $P_\lambda \circ f$ is almost contra pgb - continuous. □

4 Pre Generalized b - Regular Graphs and Strongly Contra Pre Generalized b - Closed Graphs

Definition 4.8. *A graph G_f of a function $f : X \to Y$ is said to be pgb - regular (strongly contra pgb - closed) if for each $(x,y) \in (X \times Y)\backslash G_f$, there exist a pgb - closed set U in X containing x and $V \in R - O(Y)$ such that $(U \times V) \cap G_f = \varphi$.*

Theorem 4.12. *If $f : X \to Y$ is almost contra pgb - continuous and Y is T_2, then G_f is pgb - regular in $X \times Y$.*

Proof. Let $(x,y) \in (X \times Y)\backslash G_f)$. It is obvious that $f(x) \neq y$. Since Y is T_2, there exists $V, W \in RO(Y)$ such that $f(x) \in V$, $y \in W$ and $V \cap W = \varphi$. Since f is almost contra pgb - continuous, $f^{-1}(V)$ is a pgb - closed set in X containing x. If we take $U = f^{-1}(V)$, we have $f(U) \subset V$. Hence, $f(U) \cap W = \varphi$ and G_f is pgb - regular. □

Theorem 4.13. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function and $g : (X,\tau) \to (X \times Y, \tau \times \sigma)$ the graph function defined by $g(x) = (x, f(x))$ for every $x \in X$. Then f is almost pgb - continuous if and only if g is almost pgb - continuous.*

Proof. **Necessary :** Let $x \in X$ and $V \in pgb - O(Y)$ containing $f(x)$. Then, we have $g(x) = (x, f(x)) \in R - O(X \times Y)$. Since f is almost pgb - continuous, there exists a pgb - open set U of X containing x such that $g(U) \subset X \times Y$. Therefore, we obtain $f(U) \subset V$. Hence f is almost pgb continuous.
Sufficiency : Let $x \in X$ and w be a regular open set of $X \times Y$ containing $g(x)$. There exists $U_1 \in RO(X,\tau)$ and $V \in RO(Y,\sigma)$ such that $(x, f(x)) \in (U_1 \times V) \subset W$. Since f is almost pgb - continuous, there exists $U_2 \in pgb - O(X,\tau)$ such that $x \in U_2$ and $f(U_2) \subset V$. Set $U = U_1 \cap U_2$. We have $x \in U_x \in pgb - O(X,\tau)$ and $g(U) \subset (U_1 \times V) \subset W$. This shows that g is almost pgb - continuous. □

Theorem 4.14. *If a function $f : X \to Y$ be a almost contra pgb - continuous and almost continuous, then f is regular set - connected.*

Proof. Let $V \in RO(Y)$. Since f is almost contra pgb - continuous and almost continuous, $f^{-1}(V)$ is pgb - closed and open. So $f^{-1}(V)$ is clopen. It turns out that f is regular set - connected. □

5 Connectedness

Definition 5.9. *A space X is called pgb - connected if X cannot be written as a disjoint union of two non - empty pgb - open sets.*

Theorem 5.15. *If $f : X \to Y$ is an almost contra pgb - continuous surjection and X is pgb - connected, then Y is connected.*

Proof. Suppose that Y is not a connected space. Then Y can be written as $Y = U_0 \cup V_0$ such that U_0 and V_0 are disjoint non - empty open sets. Let $U = int(cl(U_0))$ and $V = int(cl(V_0))$. Then U and V are disjoint nonempty regular open sets such that $Y = U \cup V$. Since f is almost contra pgb - continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are pgb - open sets of X. We have $X = f^{-1}(U) \cup f^{-1}(V)$ such that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint. Since f is surjective, this shows that X is not pgb - connected. Hence Y is connected. □

Theorem 5.16. *The almost contra pgb - continuous image of pgb - connected space is connected.*

Proof. Let $f : X \to Y$ be an almost contra pgb - continuous function of a pgb - connected space X onto a topological space Y. Suppose that Y is not a connected space. There exist non - empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra pgb - continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are pgb - open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non - empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not pgb - connected. This is a contradiction and hence Y is connected. □

Definition 5.10. *A topological space X is said to be pgb - ultra connected if every two non - empty pgb - closed subsets of X intersect.*

A topological space X is said to be hyper connected if every open set is dense.

Theorem 5.17. *If X is pgb - ultra connected and $f : X \to Y$ is an almost contra pgb - continuous surjection, then Y is hyper connected.*

Proof. Suppose that Y is not hyperconnected. Then, there exists an open set V such that V is not dense in Y. So, there exist non - empty regular open subsets $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. Since f is almost contra pgb - continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint pgb - closed. This is contrary to the pgb - ultra - connectedness of X. Therefore, Y is hyperconnected. □

6 Separation axioms

Definition 6.11. *A topological space X is said to be $pgb - T_1$ space if for any pair of distinct points x and y, there exist a pgb - open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.*

Theorem 6.18. *If $f : X \to Y$ is an almost contra pgb - continuous injection and Y is weakly Hausdorff, then X is $pgb - T_1$.*

Proof. Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra pgb - continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are pgb - open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $pgb - T_1$. □

Corollary 6.1. *If $f : X \to Y$ is a contra pgb - continuous injection and Y is weakly Hausdorff, then X is $pgb - T_1$.*

Definition 6.12. *A topological space X is called Ultra Hausdorff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X containing x and y, respectively.*

Definition 6.13. *A topological space X is said to be $pgb - T_2$ space if for any pair of distinct points x and y, there exist disjoint pgb - open sets G and H such that $x \in G$ and $y \in H$.*

Theorem 6.19. *If $f : X \to Y$ is an almost contra pgb - continuous injective function from space X into a Ultra Hausdorff space Y, then X is $pgb - T_2$.*

Proof. Let x and y be any two distinct points in X. Since f is an injective $f(x) \neq f(y)$ and Y is Ultra Hausdorff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint pgb - open sets in X. Therefore X is $pgb - T_2$. □

Definition 6.14. *A topological space X is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.*

Definition 6.15. *A topological space X is said to be pgb - normal if each pair of disjoint closed sets can be separated by disjoint pgb - open sets.*

Theorem 6.20. *If $f : X \to Y$ is an almost contra pgb - continuous closed injection and Y is ultra normal, then X is pgb - normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y . Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra pgb - continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint pgb - open sets in X. This shows X is pgb - normal. □

Theorem 6.21. *If $f : X \to Y$ is an almost contra pgb - continuous and Y is semi - regular, then f is pgb - continuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. By definition of semi - regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost contra pgb - continuous, there exists $U \in pgb - O(X, x)$ such that $f(U) \subset G$. Hence we have $f(U) \subset G \subset V$. This shows that f is pgb - continuous function. □

7 Compactness

Definition 7.16. *A space X is said to be:*
(1) pgb - compact if every pgb - open cover of X has a finite subcover.
(2) pgb - closed compact if every pgb - closed cover of X has a finite subcover.
(3) Nearly compact if every regular open cover of X has a finite subcover.
(4) Countably pgb - compact if every countable cover of X by pgb - open sets has a finite subcover.
(5) Countably pgb - closed compact if every countable cover of X by pgb - closed sets has a finite sub cover.
(6) Nearly countably compact if every countable cover of X by regular open sets has a finite sub cover.
(7) pgb - Lindelof if every pgb - open cover of X has a countable sub cover.
(8) pgb - Lindelof if every pgb - closed cover of X has a countable sub cover.
(9) Nearly Lindelof if every regular open cover of X has a countable sub cover.
(10) S - Lindelof if every cover of X by regular closed sets has a countable sub cover.
(11) Countably S - closed if every countable cover of X by regular closed sets has a finite sub - cover.
(12) S - closed if every regular closed cover of x has a finite sub cover.

Theorem 7.22. *Let $f : X \to Y$ be an almost contra pgb - continuous surjection. Then, the following properties hold:*
(1) If X is pgb - closed compact, then Y is nearly compact.
(2) If X is countably pgb - closed compact, then Y is nearly countably compact.
(3) If X is pgb - Lindelof, then Y is nearly Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is pgb - closed cover of X. Since X is pgb - closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ which is finite sub cover of Y, therefore Y is nearly compact.
(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y . Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable pgb - closed cover of X. Since X is countably pgb - closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ is finite subcover for Y . Hence Y is nearly countably compact.
(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is pgb - closed cover of X. Since X is pgb - Lindelof, there exists a countable subset I_0 of I such that $X = \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ is finite sub cover for Y . Therefore, Y is nearly Lindelof. □

Theorem 7.23. *Let $f : X \to Y$ be an almost contra pgb - continuous surjection. Then, the following properties hold:*
(1) If X is pgb - compact, then Y is S - closed.
(2) If X is countably pgb - closed, then Y is is countably S - closed.
(3) If X is pgb - Lindelof, then Y is S - Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is pgb - open cover of X. Since X is pgb - compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Therefore, Y is S - closed.
(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular closed cover of Y. Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable pgb - open cover of X. Since X is countably pgb - compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is countably S - closed.
(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra pgb - continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is pgb - open cover of X. Since X is pgb - Lindelof, there exists a countable sub - set I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is S - Lindelof. □

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