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Ulam-Hyers Stability of Quadratic Reciprocal Functional Equation in Intuitionistic Random Normed spaces: Various Methods

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Abstract

In this paper, the authors investigated the intuitionistic random stability of a quadratic reciprocal functional equation

$$f(x+2y)+f(2x+y)=\frac{f(x)f(y)\left[5f(x)+5f(y)+8\sqrt{f(x)f(y)}\right]}{\left[2f(x)+2f(y)+5\sqrt{f(x)+f(y)}\right]^2}$$

using direct and fixed point methods.

Keywords: Quadratic reciprocal functional equation, generalized Ulam-Hyers stability, intuitionistic random normed space, fixed point.

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1 Introduction

The study of stability problem for functional equations goes back to a question raised by Ulam [44] concerning the stability of group homomorphisms that affirmatively answered for Banach spaces by Hyers [24]. Hyers Theorem was generalized by Aoki [3] for additive mappings and by Th.M. Rassias [37] for linear mappings by considering an unbounded Cauchy difference. The paper by Rassias has provided a lot of influences in the development of what we now call the generalized Hyers-Ulam stability or Hyers- Ulam-Rassias stability of functional equations. J.M. Rassias [35] considered the Cauchy difference controlled by a product of different powers of norm. Afterwards, Găvruţa [21] generalized the Rassas's theorem by using a general control function. In 2008, a special case of Găvruţa's theorem for the unbounded Cauchy difference was obtained by Ravi et al. [38] by considering the summation of both the sum and the product of two p-norms in the sprit of Rassias approach. A large part of proofs in this topic used the direct method (of Hyers): the exact solution of the functional equation is explicitly constructed as a limit of a sequence, starting from the given approximate solution.

In 2003, V. Radu [11] proposed a new method, successively developed in [12–14], to obtaining the existence of the exact solutions and the error estimations, based on the fixed point alternative.

The theory of random normed spaces (RN-spaces) is important as a generalization of the deterministic result of linear normed spaces and also in the study of random operator equations. The RN-spaces may also

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provide us with the appropriate tools to study the geometry of nuclear physics and have important application in quantum particle physics. Recently, J.M. Rassias et al. [36] investigated the intuitionistic random stability of the quartic functional equation and C. Park et al. [33] presented the Hyers-Ulam stability of the additive-quadratic functional equation in intuitionistic random normed space.

In 2014, M. Arunkumar and S. Karthikeyan [5] introduced and investigated Hyers-Ulam stability of n-dimensional reciprocal functional equation

$$f\left(\frac{2x}{n}\right)=\sum_{\ell=1}^{n}\left(\frac{f\left(x+\ell y_{\ell}\right)f\left(x-\ell y_{\ell}\right)}{f\left(x+\ell y_{\ell}\right)+f\left(x-\ell y_{\ell}\right)}\right) \tag{1.1}$$

which originates from n-consecutive terms of a harmonic progression in RN-space using direct and fixed point methods.

Recently, Abasalt Bodaghi and Sang Og Kim [1] introduced new 2-dimensional quadratic reciprocal functional equation

$$f(x+2y)+f(2x+y)=\frac{f(x)f(y)\left[5f(x)+5f(y)+8\sqrt{f(x)f(y)}\right]}{\left[2f(x)+2f(y)+5\sqrt{f(x)+f(y)}\right]^{2}}. \tag{1.2}$$

It is easily verified that the quadratic reciprocal function $f(x)=\frac{1}{x^2}$ is a solution of the functional equation (1.2).

In this paper, the authors establish intuitionistic random norm stability of a quadratic reciprocal functional equation (1.2) using direct and fixed point methods.

2 Preliminaries of Intuitionistic Random Normed Spaces

In this section, using the idea of intuitionistic random normed spaces introduced by Chang et al. [16], we define the notion of intuitionistic random normed spaces as in [15, 22, 29, 31, 40–42].

Definition 2.1. *A measure distribution function is a function $\mu:\mathbb{R}\to[0,1]$ which is left continuous, non-decreasing on $\mathbb{R}$, $inf_{t\in\mathbb{R}}\mu(t)=0$ and $sup_{t\in\mathbb{R}}\mu(t)=1$.*

We will denote by D the family of all measure distribution functions and by H a special element of D defined by

$$H(t)=\begin{cases}0, & if\ t\le 0,\\ 1, & if\ t>0.\end{cases} \tag{2.1}$$

If X is a nonempty set, then $\mu:X\to D$ is called a probabilistic measure on X and $\mu(x)$ is denoted by μ_x.

Definition 2.2. *A non-measure distribution function is a function $\nu:\mathbb{R}\to[0,1]$ which is right continuous, non-decreasing on $\mathbb{R}$, $inf_{t\in\mathbb{R}}\nu(t)=0$ and $sup_{t\in\mathbb{R}}\nu(t)=1$.*

We will denote by B the family of all non-measure distribution functions and by G a special element of B defined by

$$G(t)=\begin{cases}1, & if\ t\le 0,\\ 0, & if\ t>0.\end{cases} \tag{2.2}$$

If X is a nonempty set, then $\nu:X\to B$ is called a probabilistic non-measure on X and $\nu(x)$ is denoted by ν_x.

Lemma 2.1. *[8, 20] Consider the set L^* and operation $\le_{L^*}$ defined by:*

$$L^*=\left\{(x_1,x_2):(x_1,x_2)\in[0,1]^2\ \ and\ \ x_1+x_2\le 1\right\},$$

$$(x_1,x_2)\le_{L^*}(y_1,y_2)\Leftrightarrow x_1\le y_1, x_2\ge y_2,\quad\forall\,(x_1,x_2),\,(y_1,y_2)\in L^*.$$

Then $(L^*,\le_{L^*})$ is a complete lattice.

Definition 2.3. *[8] An intuitionistic fuzzy set $A_{\zeta,\eta}$ in a universal set U is an object*

$$A_{\zeta,\eta} = \{(\zeta_A(u), \eta_A(u)) \,|\, u \in U\}$$

for all $u \in U$, $\zeta_A(u) \in [0,1]$ and $\eta_A(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of u in $A_{\zeta,\eta}$ and, furthermore, they satisfy $\zeta_A(u) + \eta_A(u) \leq 1$.

We denote its units by $0_{L^*} = (0,1)$ and $1_{L^*} = (1,0)$. Classically, a *triangular norm* $* = T$ on $[0,1]$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1,x) = 1 * x = x$ for all $x \in [0,1]$. A *triangular conorm* $S = \diamondsuit$ is defined as an increasing, commutative, associative mapping $S : [0,1]^2 \to [0,1]$ satisfying $S(0,x) = 0 \diamondsuit x = x$ for all $x \in [0,1]$.

Using the lattice $(L^*, \leq_{L^*})$, these definitions can be straightforwardly extended.

Definition 2.4. *[8] A triangular norm ($t-$norm) on L^* is a mapping $T : (L^*)^2 \to L^*$ satisfying the following conditions:*

(i) $(\forall \in L^*)\,(T(x, 1_{L^*}) = x)$ *(boundary condition);*

(ii) $\left(\forall (x,y) \in (L^*)^2\right)(T(x,y) = T(y,x))$ *(commutativity);*

(iii) $\left(\forall (x,y,z) \in (L^*)^3\right)(T(x, T(y,z)) = T(T(x,y), z))$ *(associativity);*

(iv) $\left(\forall (x,x',y,y') \in (L^*)^4\right)(x \leq_{L^*} x' \text{ and } y \leq_{L^*} y' \Rightarrow T(x,y) \leq_{L^*} T(x',y'))$ *(monotonicity).*

If $(L^*, \leq_{L^*}, T)$ is an Abelian topological monoid with unit 1_{L^*}, then T is said to be a *continuous $t-$norm*.

Definition 2.5. *[8] A continuous $t-$norms T on L^* is said to be continuous $t-$representable if there exist a continuous $t-$norm $*$ and a continuous $t-$conorm $\diamondsuit$ on $[0,1]$ such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,*

$$T(x,y) = (x_1 * y_1, x_2 \diamondsuit y_2).$$

For example,

$$T(a,b) = (a_1 b_1,\ \min\{a_2 + b_2, 1\})$$

and

$$M(a,b) = (\min\{a_1, b_1\}, \max\{a_2, b_2\})$$

for all $a = (a_1, a_2),\ b = (b_1, b_2) \in L^*$ are continuous $t-$representable.

Now, we define a sequence T^n recursively by $T^1 = T$ and

$$T^n\left(x^{(1)}, \ldots, x^{(n+1)}\right) = T\left(T^{n-1}\left(x^{(1)}, \ldots, x^{(n)}\right), x^{(n+1)}\right), \quad \forall n \geq 2,\ x^{(i)} \in L^*.$$

Definition 2.6. *[43] A negator on L^* is any decreasing mapping $N : L^* \to L^*$ satisfying $N : (0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$. If $N(N(x)) = x$ for all $x \in L^*$, then N is called an involutive negator. A negator on $[0,1]$ is a decreasing mapping $N : [0,1] \to [0,1]$ satisfying $P_{\mu,\nu}(0) = 1$ and $P_{\mu,\nu}(1) = 0$. N_s denotes the standard negator on $[0,1]$ defined by*

$$N_s(x) = 1 - x, \quad \forall x \in [0,1].$$

Definition 2.7. *[43] Let μ and ν be measure and non-measure distribution functions from $X \times (0, +\infty)$ to $[0,1]$ such that $\mu_x(t) + \nu_x(t) \leq 1$ for all $x \in X$ and all $t > 0$. The triple $\left(X, P_{\mu,\nu}, T\right)$ is said to be an intuitionistic random normed space (briefly IRN-space) if X is a vector space, T is a continuous $t-$representable and $P_{\mu,\nu}$ is a mapping $X \times (0, +\infty) \to L^*$ satisfying the following conditions: for all $x, y \in X$ and $t, s > 0$,*

(IRN1) $P_{\mu,\nu}(x, 0) = 0_{L^*}$;

(IRN2) $P_{\mu,\nu}(x, t) = 1_{L^*}$ if and only if $x = 0$;

(IRN3) $P_{\mu,\nu}(\alpha x, t) = P_{\mu,\nu}\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;

(IRN4) $P_{\mu,\nu}(x+y,t+s) \geq_{L^*} T\left(P_{\mu,\nu}(x,t), P_{\mu,\nu}(y,s)\right).$

In this case, $P_{\mu,\nu}$ *is called an intuitionistic random norm. Here,* $P_{\mu,\nu}(x,t) = (\mu_x(t), \nu_x(t))$.

Example 2.1. *[43] Let* $(X, \|\cdot\|)$ *be a normed space. Let* $T(a,b) = (a_1 b_1, \min(a_2+b_2, 1))$ *for all* $a = (a_1,a_2), b = (b_1,b_2) \in L^*$ *and* μ, ν *be measure and non-measure distribution functions defined by*

$$P_{\mu,v}(x,t) = (\mu_x(t), v_x(t)) = \left(\frac{t}{t+\|x\|}, \frac{\|x\|}{t+\|x\|}\right), \ \forall t \in \mathbb{R}^+.$$

Then $\left(X, P_{\mu,\nu}, T\right)$ *is an IRN-sapce.*

Definition 2.8. *[43] A sequence* $\{x_n\}$ *in an IRN-space* $\left(X, P_{\mu,\nu}, T\right)$ *is called a Cauchy sequence if, for any* $\varepsilon > 0$ *and* $t > 0$*, there exists* $n_0 \in \mathbb{N}$ *such that*

$$P_{\mu,\nu}(x_n - x_m, t) > L^* \ (N_s(\varepsilon), \varepsilon), \quad \forall n, \ m \geq n_0,$$

where N_s *is the standard negator.*

Definition 2.9. *[43] The sequence* $\{x_n\}$ *is said to be convergent to a point* $x \in X$ *(denoted by* $x_n \xrightarrow{P_{\mu,\nu}} x$*) if* $P_{\mu,\nu}(x_n - x, t) \to 1_{L^*}$ *as* $n \to \infty$ *for every* $t > 0$.

Definition 2.10. *[43] An IRN-space* $\left(X, P_{\mu,\nu}, T\right)$ *is said to be complete if every Cauchy sequence in* X *is convergent to a point* $x \in X$.

Now, we use the following notation for a given mapping $\Delta : X \to Y$

$$\Delta(x,y) = f(x+2y) + f(2x+y) - \frac{f(x)f(y)\left[5f(x)+5f(y)+8\sqrt{f(x)f(y)}\right]}{\left[2f(x)+2f(y)+5\sqrt{f(x)+f(y)}\right]^2}.$$

3 Stability Results: Direct Method

In this section, the authors presented the generalized Ulam-Hyers stability of the functional equation (1.2) in intuitionistic random normed spaces using direct method.

Theorem 3.1. *Let* X *be a linear space and* $\left(Y, P_{\mu,\nu}, T\right)$ *be a complete IRN-space. Let* $f : X \to Y$ *be a mapping with* $f(0) = 0$ *for which there are* $\xi, \zeta : X^2 \to D^+$*,* $\xi(x,y)$ *is denoted by* $\xi_{x,y}$ *and* $\zeta(x,y)$ *is denoted by* $\zeta_{x,y}$*, furthur,*$\left(\xi_{x,y}(t), \zeta_{x,y}(t)\right)$ *is denoted by* $P'_{\xi,\zeta}(x,y,t)$ *with the property:*

$$P_{\mu,\nu}(\Delta(x,y), t) \geq_{L^*} P'_{\xi,\zeta}(x,y,t) \tag{3.1}$$

for all $x, y \in X$ *and all* $t > 0$*. If*

$$T_{i=1}^{\infty} P'_{\xi,\zeta}\left(\frac{x}{3^{i+n}}, \frac{x}{3^{i+n}}, 3^{i-1+2n}t\right) = 1_{L^*} \tag{3.2}$$

and

$$lim_{n\to\infty} P'_{\xi,\zeta}\left(\frac{x}{3^n}, \frac{x}{3^n}, 3^{2n}t\right) = 1_{L^*} \tag{3.3}$$

for all $x \in X$ *and all* $t > 0$*, then there exists a unique quadratic reciprocal mapping* $R : X \to Y$ *satisfies the inequality*

$$P_{\mu,\nu}(f(x) - R(x), t) \geq_{L^*} T_{i=1}^{\infty} P'_{\xi,\zeta}\left(\frac{x}{3^i}, \frac{x}{3^i}, 3^{i-1}t\right) \tag{3.4}$$

for all $x \in X$ *and all* $t > 0$.

Proof. Replacing (x,y) by (x,x) in (3.1), we get

$$P_{\mu,\nu}\left(f(3x) - \frac{f(x)}{3^2}, t\right) \geq_{L^*} P'_{\xi,\zeta}(x,x,t) \tag{3.5}$$

for all $x \in X$ and all $t > 0$. Replacing x by $\frac{x}{3}$ in (3.5), we obtain

$$P_{\mu,\nu}\left(f(x) - \frac{1}{3^2} f\left(\frac{x}{3}\right), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3}, \frac{x}{3}, t\right) \tag{3.6}$$

for all $x \in X$ and all $t > 0$. Replacing x by $\frac{x}{3^n}$ in (3.5)and using (IRN3), we have

$$P_{\mu,\nu}\left(\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right) - \frac{1}{3^{2(n+1)}}f\left(\frac{x}{3^{n+1}}\right), \frac{t}{3^{2n}}\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3^{n+1}}, \frac{x}{3^{n+1}}, t\right) \tag{3.7}$$

for all $x \in X$ and all $t > 0$. Using $(IRN3)$ in (3.7), we arrive

$$P_{\mu,\nu}\left(\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right) - \frac{1}{3^{2(n+1)}}f\left(\frac{x}{3^{n+1}}\right), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3^{n+1}}, \frac{x}{3^{n+1}}, 3^{2n}t\right) \tag{3.8}$$

that is,

$$P_{\mu,\nu}\left(\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right) - \frac{1}{3^{2(n+1)}}f\left(\frac{x}{3^{n+1}}\right), \frac{t}{3^n}\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3^{n+1}}, \frac{x}{3^{n+1}}, 3^n t\right) \tag{3.9}$$

for all $n \in \mathbb{N}$ and all $t > 0$. As $3 > 1/3 + 1/3^2 + \ldots + 1/3^k$, by the triangle inequality it follows

$$\begin{aligned} P_{\mu,\nu}\left(f(x) - \frac{1}{3^{2k}}f\left(\frac{x}{3^k}\right), t\right) &\geq_{L^*} T_{n=0}^{k-1}\left\{P'_{\xi,\zeta}\left(\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right) - \frac{1}{3^{2(n+1)}}f\left(\frac{x}{3^{n+1}}\right), \sum_{n=0}^{k-1}\frac{1}{3^n}t\right)\right\} \\ &\geq_{L^*} T_{i=1}^{k}\left\{P'_{\xi,\zeta}\left(\frac{x}{3^i}, \frac{x}{3^i}, 3^{i-1}t\right)\right\} \end{aligned} \tag{3.10}$$

for all $x \in X$ and all $t > 0$. In order to prove the convergence of the sequence $\left\{\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right)\right\}$, replacing x by $\frac{x}{3^m}$ in (3.10), we obtain

$$P_{\mu,\nu}\left(\frac{1}{3^{2m}}f\left(\frac{x}{3^m}\right) - \frac{1}{3^{2(k+m)}}f\left(\frac{x}{3^{k+m}}\right), t\right) \geq_{L^*} T_{i=1}^{k}\left\{P'_{\xi,\zeta}\left(\frac{x}{3^{i+m}}, \frac{x}{3^{i+m}}, 3^{i-1+2m}t\right)\right\} \tag{3.11}$$

for all $x \in X$ and all $t > 0$ and all $k, m \geq 0$. Since the right hand-side of the inequality tends to 1_{L^*} as m tends to infinity, the sequence $\left\{\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right)\right\}$ is a Cauchy sequence. Therefore, we may define $R(x) = lim_{n\to\infty}\frac{1}{3^{2n}}f\left(\frac{x}{3^n}\right)$ for all $x \in X$.

Now, we prove that R satisfies (1.2). Replacing (x, y) by $\left(\frac{x}{3^n}, \frac{y}{3^n}\right)$ in (3.1), we get

$$P_{\mu,\nu}\left(\frac{1}{3^{2n}}\Delta\left(\frac{x}{3^n}, \frac{y}{3^n}\right), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3^n}, \frac{y}{3^n}, 3^{2n}t\right) \tag{3.12}$$

for all $x, y \in X$ and $t > 0$. Letting $n \to \infty$ in the above inequality and using the definition of $R(x)$, we see that R satisfies (1.2) for all $x, y \in X$.

Finally, to prove the uniqueness of the quadratic reciprocal function R subject to (3.4), let us assume that there exists another quadratic reciprocal function S which satisfies (3.4). Obviously, we have $R\left(\frac{x}{3^n}\right) = 3^{2n}R(x)$ and $S\left(\frac{x}{3^n}\right) = 3^{2n}S(x)$ for all $x \in X$ and $n \in \mathbb{N}$. Hence, it follows from (3.4) that

$$\begin{aligned} P_{\mu,\nu}\left(R(x) - S(x), t\right) &\geq_{L^*} P_{\mu,\nu}\left(R\left(\frac{x}{3^n}\right) - S\left(\frac{x}{3^n}\right), 3^{2n}t\right) \\ &\geq_{L^*} T\left(P_{\mu,\nu}\left(R\left(\frac{x}{3^n}\right) - f\left(\frac{x}{3^n}\right), \frac{3^{2n}t}{2}\right), P_{\mu,\nu}\left(f\left(\frac{x}{3^n}\right) - S\left(\frac{x}{3^n}\right), \frac{3^{2n}t}{2}\right)\right) \\ &\geq_{L^*} T\left(T_{i=1}^{\infty}\left(P'_{\xi,\zeta}\left(\frac{x}{3^{i+m}}, \frac{x}{3^{i+m}}, \frac{3^{i-1+2m}t}{2}\right)\right), T_{i=1}^{\infty}\left(P'_{\xi,\zeta}\left(\frac{x}{3^{i+m}}, \frac{x}{3^{i+m}}, \frac{3^{i-1+2m}t}{2}\right)\right)\right) \end{aligned}$$

for all $x \in X$ and $t > 0$. By letting $n \to \infty$ in (3.4), we prove the uniqueness of R. This completes the proof. □

From Theorem 3.1, we obtain the following corollary concerning the Hyers-Ulam-Rassias and JMRassias stabilities for the functional equation (1.2).

Corollary 3.1. *Suppose that a function $f : X \to Y$ satisfies the inequality*

$$P_{\mu,\nu}\left(\Delta(x, y), t\right) \geq_{L^*} \begin{cases} P'_{\xi,\zeta}\left(\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s + ||y||^s\right), t\right); \\ P'_{\xi,\zeta}\left(\epsilon||x||^s||y||^s, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s||y||^s + ||x||^{2s} + ||y||^{2s}\right), t\right); \end{cases} \tag{3.13}$$

for all $x, y \in X$ and $t > 0$, where ϵ, s are constants with $\epsilon > 0$. Then there exists a unique quadratic reciprocal mapping $R : X \to Y$ such that

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} \begin{cases} P'_{\xi,\zeta}\left(\left|\frac{9}{8}\right| \epsilon, t\right); & \\ P'_{\xi,\zeta}\left(\frac{18\epsilon}{|3^{s+2}-1|}||x||^s, t\right), & s < -2 \quad or \quad s > -2; \\ P'_{\xi,\zeta}\left(\frac{9\epsilon}{|3^{2s+2}-1|}||x||^{2s}, t\right), & s < -1 \quad or \quad s > -1; \\ P'_{\xi,\zeta}\left(\frac{27\epsilon}{|3^{2s+2}-1|}||x||^{2s}, t\right), & s < -1 \quad or \quad s > -1; \end{cases} \tag{3.14}$$

for all $x \in X$ and all $t > 0$.

4 Stability Results: Fixed Point Method

In this section, the authors proved the generalized Ulam-Hyers stability of the functional equation (1.2) in intuitionistic random normed spaces using fixed point method.

Now, we will recall the fundamental results in fixed point theory.

Theorem 4.2. *(Banach's contraction principle) Let (X, d) be a complete metric space and consider a mapping $\Gamma : X \to X$ which is strictly contractive mapping, that is*

(A1) $d(\Gamma x, \Gamma y) \leq Ld(x, y)$ for some (Lipschitz constant) $L < 1$. Then,
(i) The mapping Γ has one and only fixed point $x^* = \Gamma(x^*)$;
(ii)The fixed point for each given element x^* is globally attractive, that is

(A2) $lim_{n\to\infty}\Gamma^n x = x^*$, for any starting point $x \in X$;
(iii) One has the following estimation inequalities:

(A3) $d(\Gamma^n x, x^*) \leq \frac{1}{1-L}\ d(\Gamma^n x, \Gamma^{n+1} x), \forall\ n \geq 0, \forall\ x \in X$;

(A4) $d(x, x^*) \leq \frac{1}{1-L}\ d(x, x^*), \forall\ x \in X$.

Theorem 4.3. *[30](The alternative of fixed point) Suppose that for a complete generalized metric space (X, d) and a strictly contractive mapping $\Gamma : X \to X$ with Lipschitz constant L. Then, for each given element $x \in X$, either*
$(B1)$ $\qquad d(\Gamma^n x, \Gamma^{n+1} x) = \infty \quad \forall\ n \geq 0$,
or
$(B2)$ there exists a natural number n_0 such that:
(i) $d(\Gamma^n x, \Gamma^{n+1} x) < \infty$ for all $n \geq n_0$;
(ii)The sequence $(\Gamma^n x)$ is convergent to a fixed point y^* of Γ
(iii) y^* is the unique fixed point of Γ in the set $Y = \{y \in X : d(\Gamma^{n_0} x, y) < \infty\}$;
(iv) $d(y^*, y) \leq \frac{1}{1-L}\ d(y, \Gamma y)$ for all $y \in Y$.

Using above fixed point theorems to prove the stability results, we define the following:
δ_i is a constant such that

$$\delta_i = \begin{cases} 3 & if \quad i = 0; \\ \frac{1}{3} & if \quad i = 1; \end{cases}$$

and Ω is the set such that

$$\Omega = \{g \mid g : X \to Y, g(0) = 0\}.$$

Theorem 4.4. *Let X be a linear space and $(Y, P_{\mu,\nu}, T)$ be a complete IRN-space. Let $f : X \to Y$ be a mapping for which there exist a function $\xi, \zeta : X^2 \to D^+$ with the condition*

$$T_{i=1}^{\infty} P'_{\xi,\zeta}\left(\frac{x}{3^{i+n}}, \frac{x}{3^{i+n}}, 3^{i-1+2n} t\right) = 1_{L^*} \tag{4.1}$$

and

$$lim_{n\to\infty} P'_{\xi,\zeta}\left(\frac{x}{3^n}, \frac{x}{3^n}, 3^{2n} t\right) = 1_{L^*}, \tag{4.2}$$

and satisfying the functional inequality

$$P_{\mu,\nu}(\Delta(x,y),t) \geq_{L^*} P'_{\xi,\zeta}(x,y,t), \ \forall\, x,y \in X, t > 0. \tag{4.3}$$

If there exists L such that the function

$$x \to \beta(x) = \frac{x}{3}, \frac{x}{3} \tag{4.4}$$

has the property

$$P'_{\xi,\zeta}\left(L\delta_i^2\beta(\delta_i x), r\right) = P'_{\xi,\zeta}(\beta(x),t), \ \forall\, x \in X, t > 0. \tag{4.5}$$

Then there exists a unique quadratic reciprocal function $R : X \to Y$ satisfying the functional equation (1.2) and

$$P_{\mu,\nu}(f(x) - R(x),t) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{L^{1-i}}{1-L}\beta(x),t\right), \ \forall\, x \in X, t > 0. \tag{4.6}$$

Proof. Let d be a general metric on Ω, such that

$$d(g,h) = inf\left\{K \in (0,\infty) | P_{\mu,\nu}(g(x) - h(x),r) \geq_{L^*} P'_{\xi,\zeta}(K\beta(x),t), x \in X, t > 0\right\}.$$

It is easy to see that (Ω,d) is complete. Define $\Gamma : \Omega \to \Omega$ by $\Gamma g(x) = \delta_i^2 g(\delta_i x)$, for all $x \in X$. For $g,h \in \Omega$, we have $d(g,h) \leq K$

$$\begin{aligned}
\Rightarrow &\quad P_{\mu,\nu}(g(x) - h(x),t) \geq_{L^*} P'_{\xi,\zeta}(K\beta(x),t)\\
\Rightarrow &\quad P_{\mu,\nu}\left(\delta_i^2 g(\delta_i x) - \delta_i^2 h(\delta_i x),t\right) \geq_{L^*} P'_{\xi,\zeta}\left(K\beta(\delta_i x), \frac{t}{\delta_i^2}\right)\\
\Rightarrow &\quad P_{\mu,\nu}(\Gamma g(x) - \Gamma h(x),t) \geq_{L^*} P'_{\xi,\zeta}(KL\beta(x),t)\\
\Rightarrow &\quad d(\Gamma g(x), \Gamma h(x)) \leq KL\\
\Rightarrow &\quad d(\Gamma g, \Gamma h) \leq Ld(g,h)
\end{aligned} \tag{4.7}$$

for all $g,h \in \Omega$. Therefore, Γ is strictly contractive mapping on Ω with Lipschitz constant L. Replacing (x,y) by (x,x) in (4.3), we get

$$P_{\mu,\nu}\left(f(3x) - \frac{f(x)}{9},t\right) \geq_{L^*} P'_{\xi,\zeta}(x,x,t) \tag{4.8}$$

for all $x \in X, t > 0$. Using (IRN3) in (4.8), we arrive

$$P_{\mu,\nu}(9f(3x) - f(x),t) \geq_{L^*} P'_{\xi,\zeta}\left(x,x,\frac{t}{9}\right) \tag{4.9}$$

for all $x \in X, t > 0$, with the help of (4.5) when $i = 0$, it follows from (4.8), we get

$$\begin{aligned}
\Rightarrow &\quad P_{\mu,\nu}(9f(3x) - f(x),t) \geq_{L^*} P'_{\xi,\zeta}(L\beta(x),t)\\
\Rightarrow &\quad d(\Gamma f, f) \leq L = L^1 = L^{1-i}.
\end{aligned} \tag{4.10}$$

Replacing x by $\frac{x}{3}$ in (4.8) and using (IRN3), we obtain

$$P_{\mu,\nu}\left(f(x) - \frac{1}{9}f\left(\frac{x}{3}\right),t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{x}{3},\frac{x}{3},t\right) \tag{4.11}$$

for all $x \in X, t > 0$, with the help of (4.5) when $i = 1$, it follows from (4.11) we get

$$\begin{aligned}
&\quad P_{\mu,\nu}\left(f(x) - \frac{1}{9}f\left(\frac{x}{3}\right),t\right) \geq_{L^*} P'_{\xi,\zeta}(\beta(x),t)\\
\Rightarrow &\quad d(f,\Gamma f) \leq 1 = L^0 = L^{1-i}
\end{aligned} \tag{4.12}$$

for all $x \in X, t > 0$. Then, from (4.10) and (4.12) we can conclude,

$$d(f,\Gamma f) \leq L^{1-i} < \infty.$$

Now, from the fixed point alternative in both cases, it follows that there exists a fixed point R of Γ in Ω such that

$$\lim_{n\to\infty} P_{\mu,\nu}\left(\delta_i^{2n} f\left(\delta_i^n x\right) - R(x), t\right) \to 1_{L^*}, \qquad \forall x \in X, t > 0. \tag{4.13}$$

Replacing (x, y) by $(\delta_i x, \delta_i y)$ in (4.3), we arrive

$$P_{\mu,\nu}\left(\delta_i^{2n}\Delta(\delta_i x, \delta_i y), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\delta_i x, \delta_i y, \frac{t}{\delta_i^{2n}}\right) \tag{4.14}$$

for all $x, y \in X$ and $t > 0$.

By proceeding the same procedure as in the Theorem 3.1, we can prove the function, $R : X \to Y$ satisfies the functional equation (1.2).

By fixed point alternative, since R is unique fixed point of Γ in the set

$$\nabla = \{f \in \Omega | d(f, Q) < \infty\},$$

therefore, R is a uniqe function such that

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(K\beta(x), t\right) \tag{4.15}$$

for all $x \in X, t > 0$ and $K > 0$. Again using the fixed point alternative, we obtain

$$\begin{aligned} & d(f,R) \leq \frac{1}{1-L} d(f,\Gamma f) \\ \Rightarrow \quad & d(f,R) \leq \frac{L^{1-i}}{1-L} \\ \Rightarrow \quad & P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{L^{1-i}}{1-L}\beta(x), t\right) \end{aligned} \tag{4.16}$$

for all $x \in X$ and $t > 0$. This completes the proof. □

From Theorem 4.4, we obtain the following corollary concerning the stability for the functional equation (1.2).

Corollary 4.2. *Suppose that a function $f : X \to Y$ satisfies the inequality*

$$P_{\mu,\nu}\left(\Delta(x,y), t\right) \geq_{L^*} \begin{cases} P'_{\xi,\zeta}\left(\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s + ||y||^s\right), t\right); \\ P'_{\xi,\zeta}\left(\epsilon ||x||^s ||y||^s, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s||y||^s + ||x||^{2s} + ||y||^{2s}\right), t\right); \end{cases} \tag{4.17}$$

for all $x, y \in X$ and $t > 0$, where ϵ, s are constants with $\epsilon > 0$. Then there exists a unique quadratic reciprocal mapping $R : X \to Y$ such that

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} \begin{cases} P'_{\xi,\zeta}\left(\left|\frac{9}{8}\right|\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\frac{18\epsilon}{|3^{s+2}-1|}||x||^s, t\right), & s < -2 \;\; or \;\; s > -2; \\ P'_{\xi,\zeta}\left(\frac{9\epsilon}{|3^{2s+2}-1|}||x||^{2s}, t\right), & s < -1 \;\; or \;\; s > -1; \\ P'_{\xi,\zeta}\left(\frac{27\epsilon}{|3^{2s+2}-1|}||x||^{2s}, t\right), & s < -1 \;\; or \;\; s > -1; \end{cases} \tag{4.18}$$

for all $x \in X$ and all $t > 0$.

Proof. Setting

$$P'_{\xi,\zeta}(x,y,t) = \begin{cases} P'_{\xi,\zeta}\left(\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s + ||y||^s\right), t\right); \\ P'_{\xi,\zeta}\left(\epsilon ||x||^s ||y||^s, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s||y||^s + ||x||^{2s} + ||y||^{2s}\right), t\right); \end{cases}$$

for all $x, y \in X$ and $t > 0$. Then,

$$P'_{\xi,\zeta}\left(\delta_i^k x, \delta_i^k y, \frac{t}{\delta_i^{2k}}\right) = \begin{cases} P'_{\xi,\zeta}\left(\epsilon, \frac{t}{\delta_i^{2k}}\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||\delta_i^k x||^s + ||\delta_i^k y||^s\right), \frac{t}{\delta_i^{2k}}\right); \\ P'_{\xi,\zeta}\left(\epsilon||\delta_i^k x||^s||\delta_i^k y||^s, \frac{t}{\delta_i^{2k}}\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||\delta_i^k x||^s||\delta_i^k y||^s + ||\delta_i^k x||^{2s} + ||\delta_i^k y||^{2s}\right), \frac{t}{\delta_i^{2k}}\right); \end{cases}$$

$$= \begin{cases} P'_{\xi,\zeta}\left(\epsilon, \delta_i^{-2k} t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s + ||y||^s\right), \delta_i^{-(2+s)k} t\right); \\ P'_{\xi,\zeta}\left(\epsilon||x||^s||y||^s, \delta_i^{-(2+2s)k} t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(||x||^s||y||^s + ||x||^{2s} + ||y||^{2s}\right), \delta_i^{-(2+2s)k} t\right); \end{cases}$$

$$= \begin{cases} \to 1_{L^*} \text{ as } k \to \infty; \\ \to 1_{L^*} \text{ as } k \to \infty; \\ \to 1_{L^*} \text{ as } k \to \infty; \\ \to 1_{L^*} \text{ as } k \to \infty. \end{cases}$$

Thus, (4.1) is holds. But we have $\beta(x) = \left(\frac{x}{3}, \frac{x}{3}\right)$ has the property

$$P'_{\xi,\zeta}\left(\delta_i^2 \beta(\delta_i x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\beta(x), t\right), \ \forall\, x \in X, t > 0.$$

Hence,

$$P'_{\xi,\zeta}\left(\beta(x), t\right) = P'_{\xi,\zeta}\left(\frac{x}{3}, \frac{x}{3}, t\right) = \begin{cases} P'_{\xi,\zeta}\left(\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(\left\|\frac{x}{3}\right\|^s + \left\|\frac{x}{3}\right\|^s\right), t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left\|\frac{x}{3}\right\|^s\left\|\frac{x}{3}\right\|^s, t\right); \\ P'_{\xi,\zeta}\left(\epsilon\left(\left\|\frac{x}{3}\right\|^s\left\|\frac{x}{3}\right\|^s + \left\|\frac{x}{3}\right\|^{2s} + \left\|\frac{x}{3}\right\|^{2s}\right), t\right); \end{cases}$$

$$= \begin{cases} P'_{\xi,\zeta}\left(\epsilon, t\right); \\ P'_{\xi,\zeta}\left(\frac{2\epsilon}{3^s}||x||^s, t\right); \\ P'_{\xi,\zeta}\left(\frac{\epsilon}{3^{2s}}||x||^{2s}, t\right); \\ P'_{\xi,\zeta}\left(\frac{3\epsilon}{3^{2s}}||x||^{2s}, t\right). \end{cases}$$

for all $x \in X$ and $t > 0$. Now,

$$P'_{\xi,\zeta}\left(\delta_i^2 \beta(\delta_i x), t\right) = \begin{cases} P'_{\xi,\zeta}\left(\delta_i^2 \epsilon, t\right); \\ P'_{\xi,\zeta}\left(\frac{2\epsilon}{3^s}\delta_i^2||\delta_i x||^s, t\right); \\ P'_{\xi,\zeta}\left(\frac{\epsilon}{3^{2s}}\delta_i^2||\delta_i x||^{2s}, t\right); \\ P'_{\xi,\zeta}\left(\frac{3\epsilon}{3^{2s}}\delta_i^2||\delta_i x||^{2s}, t\right); \end{cases} = \begin{cases} P'_{\xi,\zeta}\left(\delta_i^2 \epsilon, t\right); \\ P'_{\xi,\zeta}\left(\frac{2\epsilon}{3^s}\delta_i^{2+s}||x||^s, t\right); \\ P'_{\xi,\zeta}\left(\frac{\epsilon}{3^{2s}}\delta_i^{2+2s}||x||^{2s}, t\right); \\ P'_{\xi,\zeta}\left(\frac{3\epsilon}{3^{2s}}\delta_i^{2+2s}||x||^{2s}, t\right); \end{cases}$$

$$= \begin{cases} P'_{\xi,\zeta}\left(\delta_i^2 \beta(x), t\right); \\ P'_{\xi,\zeta}\left(\delta_i^{2+s} \beta(x), t\right); \\ P'_{\xi,\zeta}\left(\delta_i^{2+2s} \beta(x), t\right); \\ P'_{\xi,\zeta}\left(\delta_i^{2+2s} \beta(x), t\right), \end{cases}$$

for all $x \in X$ and $t > 0$. Now, from (4.6), we prove the following cases:
Case:1 $L = 3^2$ if $i = 0$;

$$P_{\mu,\nu}\left(f(x) - R(x), r\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{L}{1-L}\beta(x), t\right) = P'_{\xi,\zeta}\left(\frac{-9}{8}\epsilon, t\right).$$

Case:2 $L = 3^{-2}$ if $i = 1$;

$$P_{\mu,\nu}\left(f(x) - R(x), r\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{1}{1-L}\beta(x), 2t\right) = P'_{\xi,\zeta}\left(\frac{9}{8}\epsilon, t\right).$$

Case:3 $L = 3^{s+2}$ for $s < -2$ if $i = 0$;

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\frac{3^{s+2}\epsilon}{1-3^{s+2}}\beta(x)||x||^s, t\right) = P'_{\xi,\zeta}\left(\frac{18\epsilon}{1-3^{s+2}}||x||^s, t\right).$$

Case:4 $L = 3^{-s-2}$ for $s > -2$ if $i = 1$;

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\left(\frac{\epsilon}{1-3^{-s-2}}\right)\beta(x)||x||^s, t\right) = P'_{\xi,\zeta}\left(\frac{18\epsilon}{3^{s+2}-1}||x||^s, t\right).$$

Case:5 $L = 3^{2s+2}$ for $s < -1$ if $i = 0$;

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\left(\frac{3^{2s+2}\epsilon}{1-3^{2s+2}}\right)\beta(x)||x||^{2s}, t\right) = P'_{\xi,\zeta}\left(\frac{9\epsilon}{1-3^{2s+2}}||x||^{2s}, t\right).$$

Case:6 $L = 3^{-2s-2}$ for $s > -1$ if $i = 1$;

$$P_{\mu,\nu}\left(f(x) - R(x), t\right) \geq_{L^*} P'_{\xi,\zeta}\left(\left(\frac{\epsilon}{1-3^{-2s-2}}\right)\beta(x)||x||^{2s}, t\right) = P'_{\xi,\zeta}\left(\frac{9\epsilon}{3^{2s+2}-1}||x||^{2s}, t\right).$$

Hence complete the proof. □

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