

## Soft Almost Semi-Continuous Mappings

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### Abstract

In the present paper the concept of soft almost semi-continuous mappings and soft almost semi-open mappings in soft topological spaces have been introduced and studied.

*Keywords:* Soft regular open set, Soft semi open set, Soft almost continuous mappings, Soft semi-continuous mappings, Soft almost semi-continuous mappings and Soft almost semi-open mappings.

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## 1 Introduction

The theory of soft set was proposed by Molodtsov in 1999 [10]. It is a method for handling uncertain data. In 2011 Shabir and Naz [11] initiated the study of soft topological spaces. Many researchers worked on the findings of structures of soft set theory, soft topology and applied to many problems having uncertainties. Theoretical study of soft sets and soft topological spaces have been by some authors in [1, 3, 5–7, 10, 11, 13–15]. In 2013, Chen [2] introduced the concept of soft semi-open sets and soft-semi-closed sets in soft topological spaces. The section 2, of this paper gives the basic concept of soft set theory and soft topology. In section 3, we define the concepts of soft almost continuous mappings. It is shown that every soft almost continuous mapping is soft almost semi continuous and the example shows that the converse may not be true. Several characterization and properties of soft almost continuous mappings in soft topological spaces have been studied in this section. Section 4, introduces and studied soft almost open mappings. Last section give the conclusion of this paper.

## 2 Preliminaries

Let  $U$  is an initial universe set,  $E$  be a set of parameters,  $P(U)$  be the power set of  $U$  and  $A \subseteq E$ .

**Definition 2.1.** [10] A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For all  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Definition 2.2.** [6] For two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , we say that  $(F, A)$  is a soft subset of  $(G, B)$ , denoted by  $(F, A) \subseteq (G, B)$ , if:

- (a)  $A \subseteq B$  and
- (b)  $F(e) \subseteq G(e)$  for all  $e \in E$ .

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**Definition 2.3.** [6] Two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  are said to be soft equal denoted by  $(F, A) = (G, B)$  If  $(F, A) \subseteq (G, B)$  and  $(G, B) \subseteq (F, A)$ .

**Definition 2.4.** [7] The complement of a soft set  $(F, A)$ , denoted by  $(F, A)^c$ , is defined by  $(F, A)^c = (F^c, A)$ , where  $F^c : A \rightarrow P(U)$  is a mapping given by  $F^c(e) = U - F(e)$ , for all  $e \in E$ .

**Definition 2.5.** [6] Let a soft set  $(F, A)$  over  $U$ .

- (a) Null soft set denoted by  $\phi$  if for all  $e \in A$ ,  $F(e) = \phi$ .  
 (b) Absolute soft set denoted by  $\tilde{U}$ , if for each  $e \in A$ ,  $F(e) = U$ .

Clearly,  $\tilde{U}^c = \phi$  and  $\phi^c = \tilde{U}$ .

**Definition 2.6.** [1] Union of two sets  $(F, A)$  and  $(G, B)$  over the common universe  $U$  is the soft  $(H, C)$ , where  $C = A \cup B$ , and for all  $e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

**Definition 2.7.** [1] Intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , is the soft set  $(H, C)$  where  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for each  $e \in E$ .

Let  $X$  and  $Y$  be an initial universe sets and  $E$  and  $K$  be the non empty sets of parameters,  $S(X, E)$  denotes the family of all soft sets over  $X$  and  $S(Y, K)$  denotes the family of all soft sets over  $Y$ .

**Definition 2.8.** [11] A subfamily  $\tau$  of  $S(X, E)$  is called a soft topology on  $X$  if:

1.  $\tilde{\phi}, \tilde{X}$  belong to  $\tau$ .
2. The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
3. The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . The members of  $\tau$  are called soft open sets in  $X$  and their complements called soft closed sets in  $X$ .

**Definition 2.9.** If  $(X, \tau, E)$  is soft topological space and a soft set  $(F, E)$  over  $X$ .

- (a) The soft closure of  $(F, E)$  is denoted by  $Cl(F, E)$  is defined as the intersection of all soft closed super sets of  $(F, E)$  [11].  
 (b) The soft interior of  $(F, E)$  is denoted by  $Int(F, E)$  is defined as the soft union of all soft open subsets of  $(F, E)$  [14].

**Definition 2.10.** [14] The soft set  $(F, E) \in S(X, E)$  is called a soft point if there exist  $x \in X$  and  $e \in E$  such that  $F(e) = \{x\}$  and  $F(e') = \phi$  for each  $e' \in E - \{e\}$ , and the soft point  $(F, E)$  is denoted by  $(x_e)_E$ .

**Definition 2.11.** [14] The soft point  $(x_e)_E$  is said to be in the soft set  $(G, E)$ , denoted by  $(x_e)_E \in (G, E)$  if  $(x_e)_E \subseteq (G, E)$ .

**Definition 2.12.** [2, 13] A soft set  $(F, E)$  in a soft topological space  $(X, \tau, E)$  is said to be :

- (a) Soft regular open if  $(F, E) = Int(Cl(F, E))$ .
- (b) Soft regular closed if its complement is soft regular open.
- (c) Soft semi-open if  $(F, E) \subseteq Cl(Int(F, E))$ .
- (d) Soft semi-closed if its complement is soft semi-open.

**Remark 2.13.** [4, 13] Every soft regular open (resp. soft regular closed) set is soft open (resp. closed) and every soft open (resp. closed) set is soft semi-open (resp. semi-closed) but the converses may not be true.

**Definition 2.14.** [2] Let  $(F, E)$  be a soft set in a soft topological space  $(X, \tau, E)$ .

- (a) The soft semi-closure of  $(F, E)$  is denoted by  $sCl(F, E)$  is defined as the smallest soft semi-closed set over which contains  $(F, E)$ .
- (b) The soft semi-interior of  $(F, E)$  is denoted by  $sInt(F, E)$  is defined as the largest soft semi-open set over which is contained in  $(F, E)$ .

**Definition 2.15.** [5] Let  $S(X,E)$  and  $S(Y,K)$  be families of soft sets. Let  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  be mappings. Then a mapping  $f_{pu}: S(X, E) \rightarrow S(Y, K)$  is defined as :

(i) Let  $(F, A)$  be a soft set in  $S(X, E)$ . The image of  $(F, A)$  under  $f_{pu}$ , written as  $f_{pu}(F, A) = (f_{pu}(F), p(A))$ , is a soft set in  $S(Y,K)$  such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(F(e)), & p^{-1}(k) \cap A \neq \phi \\ \phi, & p^{-1}(k) \cap A = \phi \end{cases}$$

For all  $k \in K$ .

(ii) Let  $(G, B)$  be a soft set in  $S(Y, K)$ . The inverse image of  $(G, B)$  under  $f_{pu}$ , written as

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e) \in B \\ \phi, & \text{otherwise} \end{cases}$$

For all  $e \in E$ .

**Definition 2.16.** [8, 12] Let  $(X,\tau,E)$  and  $(Y,\nu,K)$  be a soft topological spaces. A soft mapping  $f_{pu}: (X,\tau,E) \rightarrow (Y,\nu,K)$  is said to be :

- Soft almost continuous if  $f_{pu}^{-1}(G, K)$  is soft open in  $X$ , for all soft regular open set  $(G,K)$  in  $Y$ .
- Soft almost open if  $f_{pu}(F, E)$  is soft open in  $Y$ , for all soft regular open set  $(F, E)$  in  $X$ .
- Soft semi-continuous mapping if  $f_{pu}^{-1}(G, K)$  is soft semi-open in  $X$ , for all soft open set  $(G,K)$  in  $Y$ .
- Soft semi-open if  $f_{pu}(F, E)$  is soft semi-open in  $Y$ , for all soft open set  $(F, E)$  in  $X$ .
- Soft semi-irresolute if  $f_{pu}^{-1}(G, K)$  is soft semi-open in  $X$ , for all soft semi-open set  $(G, K)$  in  $Y$ .

### 3 Soft Almost Semi-Continuous Mappings

**Definition 3.1.** A soft mapping  $f_{pu}: (X,\tau,E) \rightarrow (Y,\nu,K)$  is said to be soft almost semi-continuous if the inverse image of every soft regular open set over  $Y$  is soft semi-open over  $X$ .

**Remark 3.2.** Every soft almost continuous mapping is soft almost semi-continuous but converse may not be true.

**Example 3.3.** Let  $X = \{x_1, x_2\}$ ,  $E = \{e_1, e_2\}$  and  $Y = \{y_1, y_2\}$ ,  $K = \{k_1, k_2\}$ . The soft sets  $(F_1,E), (F_2,E), (F_3,E), (G_1,K), (G_2,K)$  are defined as follows :

$$\begin{aligned} F_1(e_1) &= \phi, & F_1(e_2) &= \{x_1\}, \\ F_2(e_1) &= \{x_1\}, & F_2(e_2) &= \phi, \\ F_3(e_1) &= \{x_1\}, & F_3(e_2) &= \{x_1\}, \\ G_1(k_1) &= \{y_1\}, & G_1(k_2) &= \{y_2\}, \\ G_2(k_1) &= \{y_2\}, & G_2(k_2) &= \{y_1\} \end{aligned}$$

Let  $\tau = \{\phi, (F_1,E), (F_2,E), (F_3,E), \tilde{X}\}$  and  $\nu = \{\phi, (G_1,K), (G_2,K), \tilde{Y}\}$  are topologies on  $X$  and  $Y$  respectively. Then soft mapping  $f_{pu}: (X,\tau,E) \rightarrow (Y,\nu,K)$  defined by  $u(x_1) = y_1$ ,  $u(x_2) = y_2$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft almost semi-continuous mapping not soft almost continuous.

**Remark 3.4.** Every soft semi-continuous mapping is soft almost semi-continuous but converse may not be true.

**Example 3.5.** Let  $X = \{x_1, x_2, x_3\}$ ,  $E = \{e_1, e_2\}$  and  $Y = \{y_1, y_2, y_3\}$ ,  $K = \{k_1, k_2\}$ . The soft set  $(G,K)$  is defined as follows :

$$G(k_1) = \{y_1\}, \quad G(k_2) = \phi$$

Let  $\tau = \{\phi, \tilde{X}\}$ , and  $\nu = \{\phi, (G,K), \tilde{Y}\}$  are topologies on  $X$  and  $Y$  respectively. Then soft mapping  $f_{pu}: (X,\tau,E) \rightarrow (Y,\nu,K)$  defined by  $u(x_1) = y_1$ ,  $u(x_2) = y_2$ ,  $u(x_3) = y_3$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft almost semi-continuous but not soft semi-continuous.

**Theorem 3.6.** Let  $f_{pu}: (X,\tau,E) \rightarrow (Y,\nu,K)$  be a soft mapping. Then the following conditions are equivalent:

- $f_{pu}$  is soft almost semi-continuous.
- $f_{pu}^{-1}(G,K)$  is soft semi-closed set in  $X$  for every soft regular closed set  $(G,K)$  in  $Y$ .
- $f_{pu}^{-1}(A,K) \subset sInt(f_{pu}^{-1}(Int(Cl(A,K))))$  for every soft open set  $(A,K)$  in  $Y$ .
- $sCl(f_{pu}^{-1}(Cl(Int(G,K)))) \subset f_{pu}^{-1}(G,K)$  for every soft closed set  $(G,K)$  in  $Y$ .
- For each soft point  $(x_e)_E$  over  $X$  and each soft regular open set  $(G,K)$  over  $Y$  containing  $f_{pu}((x_e)_E)$ , there exists a soft semi-open set  $(F,E)$  over  $X$  such that  $(x_e)_E \in (F,E)$  and  $(F,E) \subset f_{pu}^{-1}(G,K)$ .

(f) For each soft point  $(x_e)_E$  over  $X$  and each soft regular open set  $(G,K)$  over  $Y$  containing  $f_{pu}((x_e)_E)$ , there exists a soft semi-open set  $(F,E)$  over  $X$  such that  $(x_e)_E \in (F,E)$  and  $f_{pu}(F,E) \subset (G,K)$ .

*Proof:* (a) $\Leftrightarrow$ (b) Since  $f_{pu}^{-1}((G,K)^C) = (f_{pu}^{-1}(G,K))^C$  for every soft set  $(G,K)$  over  $Y$ .

(a) $\Rightarrow$ (c) Since  $(A,K)$  is soft open set over  $Y$ ,  $(A,K) \subset \text{Int}(\text{Cl}(A,K))$  and hence,  $f_{pu}^{-1}(A,K) \subset f_{pu}^{-1}(\text{Int}(\text{Cl}(A,K)))$ . Now  $\text{Int}(\text{Cl}(A,K))$  is a soft regular open set over  $Y$ . By (a),  $f_{pu}^{-1}(\text{Int}(\text{Cl}(A,K)))$  is soft semi-open set over  $X$ . Thus,  $f_{pu}^{-1}(A,K) \subset f_{pu}^{-1}(\text{Int}(\text{Cl}(A,K))) = \text{sInt}(f_{pu}^{-1}(\text{Int}(\text{Cl}(A,K))))$ .

(c) $\Rightarrow$ (a) Let  $(A,K)$  be a soft regular open set over  $Y$ , then we have  $f_{pu}^{-1}(A,K) \subset \text{sInt}(f_{pu}^{-1}(\text{Int}(\text{Cl}(A,K)))) = \text{sInt}(f_{pu}^{-1}(A,K))$ . Thus,  $f_{pu}^{-1}(A,K) = \text{sInt}(f_{pu}^{-1}(A,K))$  shows that  $f_{pu}^{-1}(A,K)$  is a soft semi-open set over  $X$ .

(b) $\Rightarrow$ (d) Since  $(G,K)$  is soft closed set over  $Y$ ,  $\text{Cl}(\text{Int}(G,K)) \subset (G,K)$  and  $f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \subset f_{pu}^{-1}(G,K)$ .  $\text{Cl}(\text{Int}(G,K))$  is soft regular closed set over  $Y$ . Hence,  $f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K)))$  is soft semi-closed set over  $X$ . Thus,  $\text{sCl}(f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K)))) = f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \subset f_{pu}^{-1}(G,K)$ .

(d) $\Rightarrow$ (b) Let  $(G,K)$  be a soft regular closed set over  $Y$ , then we have  $\text{sCl}(f_{pu}^{-1}(G,K)) = \text{sCl}(f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K)))) \subset f_{pu}^{-1}(G,K)$ . Thus,  $\text{sCl}(f_{pu}^{-1}(G,K)) \subset f_{pu}^{-1}(G,K)$ , shows that  $f_{pu}^{-1}(G,K)$  is soft semi-closed set over  $X$ .

(a) $\Rightarrow$ (e) Let  $(x_e)_E$  be a soft point over  $X$  and  $(G,K)$  be a soft regular open set over  $Y$  such that  $f_{pu}((x_e)_E) \in (G,K)$ . Put  $(F,E) = f_{pu}^{-1}(G,K)$ . Then by (a),  $(F,E)$  is soft semi-open set,  $(x_e)_E \in (F,E)$  and  $(F,E) \subset f_{pu}^{-1}(G,K)$ .

(e) $\Rightarrow$ (f) Let  $(x_e)_E$  be a soft point over  $X$  and  $(G,K)$  be a soft regular open set over  $Y$  such that  $f_{pu}((x_e)_E) \in (G,K)$ . By (e) there exists a soft semi-open set  $(F,E)$  such that  $(x_e)_E \in (F,E)$ ,  $(F,E) \subset f_{pu}^{-1}(G,K)$ . And so, we have  $(x_e)_E \in (F,E)$ ,  $f_{pu}(F,E) \subset f_{pu}(f_{pu}^{-1}(G,K)) \subset (G,K)$ .

(f) $\Rightarrow$ (a) Let  $(G,K)$  be a soft regular open set over  $Y$  and  $(x_e)_E$  be a soft point over  $X$  such that  $(x_e)_E \in f_{pu}^{-1}(G,K)$ . Then  $f_{pu}((x_e)_E) \in f_{pu}(f_{pu}^{-1}(G,K)) \subset (G,K)$ . By (f), there exists a soft semi-open set  $(F,E)$  such that  $(x_e)_E \in (F,E)$  and  $f_{pu}(F,E) \subset (G,K)$ . This shows that  $(x_e)_E \in (F,E) \subset f_{pu}^{-1}(G,K)$ . It follows that  $f_{pu}^{-1}(G,K)$  is soft semi-open set and hence  $f_{pu}^{-1}$  is soft almost semi-continuous.

**Definition 3.7.** A soft topological space  $(X, \tau, E)$  is said to be soft semiregular if for each soft open set  $(F,E)$  and each soft point  $(x_e)_E \in (F,E)$ , there exists a soft open set  $(G,E)$  such that  $(x_e)_E \in (G,E)$  and  $(G,E) \subset \text{Int}(\text{Cl}(G,E)) \subset (F,E)$ .

**Theorem 3.8.** Let  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  be a soft mapping from a soft topological space  $(X, \tau, E)$  to a soft semiregular space  $(Y, \vartheta, K)$ . Then  $f_{pu}$  is soft almost semi-continuous if and only if  $f_{pu}$  is soft semi-continuous.

*Proof:* Necessity: Let  $(x_e)_E$  be a soft point in  $X$  and  $(F,K)$  be a soft open set in  $Y$  such that  $f_{pu}((x_e)_E) \in (F,K)$ . Since  $(Y, \vartheta, K)$  is soft semiregular there exists a soft open set  $(G,K)$  in  $Y$  such that  $f_{pu}((x_e)_E) \in (G,K)$  and  $(G,K) \subset \text{Int}(\text{Cl}(G,K)) \subset (F,K)$ . Since  $\text{Int}(\text{Cl}(G,K))$  is soft regular open in  $Y$  and  $f_{pu}$  is soft almost semi-continuous, by theorem 3.6 (f) there exists a soft semi-open set  $(A,E)$  in  $X$  such that  $(x_e)_E \in (A,E)$  and  $f_{pu}(A,E) \subset \text{Int}(\text{Cl}(G,K))$ . Thus,  $(A,E)$  is soft semi-open set such that  $(x_e)_E \in (A,E)$  and  $f_{pu}(A,E) \subset (F,K)$ . Hence by theorem [26] [2],  $f_{pu}$  is soft semi-continuous.

Sufficiency: Obvious.

**Lemma 3.9.** If  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  be a soft mapping and  $f_{pu}$  is a soft open and soft continuous mapping then  $f_{pu}^{-1}(G,K)$  is soft semi-open in  $X$  for every  $(G,K)$  is soft semi-open in  $Y$ .

*Proof:* Let  $(G,K)$  is soft semi-open in  $Y$ . Then,  $(G,K) \subseteq \text{Int}(\text{Cl}(\text{Int}(G,K)))$ . Since  $f_{pu}$  is soft continuous we have,  $f_{pu}^{-1}(G,K) \subseteq f_{pu}^{-1}(\text{Int}(\text{Cl}(\text{Int}(G,K)))) \subseteq \text{Int}(f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))))$ . By the openness of  $f_{pu}$ , we have  $f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \subseteq \text{Cl}(f_{pu}^{-1}(\text{Int}(G,K)))$ . Again  $f_{pu}$  is soft continuous  $f_{pu}^{-1}(\text{Int}(G,K)) \subseteq \text{Int}(f_{pu}^{-1}(G,K))$ . Thus,  $f_{pu}^{-1}(G,K) \subseteq \text{Int}(\text{Cl}(\text{Int}(f_{pu}^{-1}(G,K))))$ .

Consequently,  $f_{pu}^{-1}(G,K)$  is soft semi-open in  $X$ .

**Theorem 3.10.** If soft mapping  $f_{p_1 u_1} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  is soft open soft continuous and soft mapping  $g_{p_2 u_2} : (Y, \vartheta, K) \rightarrow (Z, \eta, T)$  is soft almost semi-continuous, then  $g_{p_2 u_2} \circ f_{p_1 u_1} : (X, \tau, E) \rightarrow (Z, \eta, T)$  is soft almost semi-continuous.

*Proof:* Suppose  $(U,T)$  is a soft regular open set in  $Z$ . Then  $g_{p_2 u_2}^{-1}(U,T)$  is a soft semi-open set in  $Y$  because  $g_{p_2 u_2}$  is soft almost semi-continuous. Since  $f_{p_1 u_1}$  being soft open and continuous. By lemma 3.9  $f_{p_1 u_1}^{-1}(g_{p_2 u_2}^{-1}(U,T))$  is soft semi-open in  $X$ . Consequently,  $g_{p_2 u_2} \circ f_{p_1 u_1} : (X, \tau, E) \rightarrow (Z, \eta, T)$  is soft almost semi-continuous.

**Lemma 3.11.** If  $(A,E)$  be a soft semi-open set over  $X$  and  $(Y,E)$  is soft open in a soft topological space  $(X, \tau, E)$ . Then  $(A,E) \cap (Y,E)$  is soft semi-open in  $(Y,E)$ .

*Proof:* Obvious.

**Theorem 3.12.** Let  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  be a soft almost semi-continuous mapping and  $(A,E)$  is soft open set in  $X$ . Then  $f_{pu}/(A,E)$  is soft almost semi-continuous.

*Proof:* Let  $(G,K)$  be a soft regular open set in  $Y$  then  $f_{pu}^{-1}(G,K)$  is soft semi-open in  $X$ . Since  $(A,E)$  is soft open in  $X$ , By lemma 3.11  $(A,E) \cap f_{pu}^{-1}(G,K) = [f_{pu}/(A,E)]^{-1}(G,K)$  is soft semi-open in  $(A,E)$ . Therefore,  $f_{pu}/(A,E)$  is soft almost semi-continuous.

## 4 Soft Almost Semi-Open Mappings

**Definition 4.1.** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  is said to be soft almost semi-open if for each soft regular open set  $(F, E)$  in  $X$ ,  $f_{pu}(F, E)$  is soft semi-open in  $Y$ .

**Remark 4.2.** Every soft almost open is soft almost semi-open but converse may not be true.

**Example 4.3.** Let  $X = \{x_1, x_2\}$ ,  $E = \{e_1, e_2\}$  and  $Y = \{y_1, y_2\}$ ,  $K = \{k_1, k_2\}$ . The soft sets  $(F_1, E)$ ,  $(F_2, E)$ ,  $(G, K)$  are defined as follows :

$$\begin{aligned} F_1(e_1) &= \{x_1\}, & F_1(e_2) &= \{x_2\}, \\ F_2(e_1) &= \{x_2\}, & F_2(e_2) &= \{x_1\}, \\ G_1(k_1) &= \phi & G_1(k_2) &= \{y_1\}, \\ G_2(k_1) &= \{y_1\} & G_2(k_2) &= \phi, \\ G_3(k_1) &= \{y_1\} & G_3(k_2) &= \{y_1\} \end{aligned}$$

Let  $\tau = \{\phi, (F_1, E), (F_2, E), \tilde{X}\}$ , and  $\nu = \{\phi, (G_1, K), (G_2, K), (G_3, K), \tilde{Y}\}$  are topologies on  $X$  and  $Y$  respectively. Then soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (Y, \nu, K)$  defined by  $u(x_1) = y_1$ ,  $u(x_2) = y_2$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft almost semi-open mapping but not soft almost open.

**Remark 4.4.** Every soft semi-open mappings is soft almost semi-open but converse may not be true.

**Example 4.5.** Let  $X = \{x_1, x_2, x_3\}$ ,  $E = \{e_1, e_2\}$  and  $Y = \{y_1, y_2, y_3\}$ ,  $K = \{k_1, k_2\}$ . The soft sets  $(F, E)$  is defined as follows :

$$F(e_1) = \{x_1\}, \quad F(e_2) = \phi,$$

Let  $\tau = \{\phi, (F, E), \tilde{X}\}$  and  $\nu = \{\phi, \tilde{Y}\}$  are topologies on  $X$  and  $Y$  respectively. Then soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (Y, \nu, K)$  defined by  $u(x_1) = y_1$ ,  $u(x_2) = y_2$  and  $p(e_1) = k_1$ ,  $p(e_2) = k_2$  is soft almost semi-open mapping but not soft semi-open.

**Theorem 4.6.** Let  $f_{p_1u_1} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  and  $g_{p_2u_2} : (Y, \vartheta, K) \rightarrow (Z, \eta, T)$  be two soft mappings, If  $f_{p_1u_1}$  is soft almost open and  $g_{p_2u_2}$  is soft semi-open. Then the soft mapping  $g_{p_2u_2} \circ f_{p_1u_1} : (X, \tau, E) \rightarrow (Z, \eta, T)$  is soft almost semi-open.

*Proof :* Let  $(F, E)$  be soft regular open in  $X$ . Then  $f_{p_1u_1}(F, E)$  is soft open in  $Y$  because  $f_{p_1u_1}$  is soft almost open. Therefore,  $g_{p_2u_2}(f_{p_1u_1}(F, E))$  is soft semi-open in  $Z$ . Because  $g_{p_2u_2}$  is soft semi-open. Since  $(g_{p_2u_2} \circ f_{p_1u_1})(F, E) = (g_{p_2u_2}(f_{p_1u_1}(F, E)))$ , it follows that the soft mapping  $(g_{p_2u_2} \circ f_{p_1u_1})$  is soft almost semi-open.

**Definition 4.7.** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  is said to be soft semi-irresolute if the inverse image of soft semi-open set of  $Y$  is soft semi-open set in  $X$ .

**Theorem 4.8.** Let  $f_{p_1u_1} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  and  $g_{p_2u_2} : (Y, \vartheta, K) \rightarrow (Z, \eta, T)$  be two soft mappings, such that  $g_{p_2u_2} \circ f_{p_1u_1} : (X, \tau, E) \rightarrow (Z, \eta, T)$  is soft almost semi-open and  $g_{p_2u_2}$  is soft semi-irresolute and injective then  $f_{p_1u_1}$  is soft almost semi-open.

*Proof :* Suppose  $(F, E)$  is soft regular open set in  $X$ . Then  $g_{p_2u_2} \circ f_{p_1u_1}(F, E)$  is soft semi-open in  $Z$  because  $g_{p_2u_2} \circ f_{p_1u_1}$  is soft almost semi-open. Since  $g_{p_2u_2}$  is injective, we have  $(g_{p_2u_2}^{-1}(g_{p_2u_2} \circ f_{p_1u_1}(F, E))) = f_{p_1u_1}(F, E)$ . Therefore  $f_{p_1u_1}(F, E)$  is soft semi-open in  $Y$ , because  $g_{p_2u_2}$  is soft semi-irresolute. This implies  $f_{p_1u_1}$  is soft almost semi-open.

**Theorem 4.9.** Let soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  be soft almost semi-open mapping. If  $(G, K)$  is soft set of  $Y$  and  $(F, E)$  is soft regular closed set of  $X$  containing  $f_{pu}^{-1}(G, K)$  then there is a soft semi-closed set  $(A, K)$  of  $Y$  containing  $(G, K)$  such that  $f_{pu}^{-1}(A, K) \subset (F, E)$ .

*Proof :* Let  $(A, K) = (f_{pu}(F, E))^C$ . Since  $f_{pu}^{-1}(G, K) \subset (F, E)$  we have  $f_{pu}(F, E)^C \subset (G, K)$ . Since  $f_{pu}$  is soft almost semi-open then  $(A, K)$  is soft semi-closed set of  $Y$  and  $f_{pu}^{-1}(A, K) = (f_{pu}^{-1}(f_{pu}(F, E)^C))^C \subset ((F, E)^C)^C = (F, E)$ . Thus,  $f_{pu}^{-1}(A, K) \subset (F, E)$ .

## 5 Conclusion

Continuity of soft mappings played very important role in the development of soft topology. In this paper we have introduced soft almost semi-continuous (resp. soft almost semi-open) mappings and it is shown by the examples that the class of soft almost semi-continuous (resp. soft almost semi-open) mappings properly contains the class of all soft almost continuous (resp. soft almost open) mappings. Various properties and characterization of these soft mappings have been studied. The class of all soft almost mappings introduced in this paper will be useful to study various strong and weak forms of soft separation axioms, soft connectedness and soft compactness in soft topology.

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