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Some Existence results for implicit fractional differential equations with impulsive conditions

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Abstract

In this paper, we investigate the existence of solutions for implicit impulsive fractional order differential equations with non-local conditions. An example is included to prove the applicability of the results.

Keywords: Existence, Implicit Impulsive Fractional Differential Equations, Non-local Condition.

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1 Introduction

The theory of fractional differential equations is a new branch of mathematics by valuable tools in the modelling of many phenomena in various fields of science and engineering. Indeed, we can find numerous applications in viscoelasticity, electrochemistry, control, porous media, electromagnetic, etc.(For details [1–4, 6, 11–14, 18–21]).

Impulsive differential equations have become important in recent years as mathematical models of phenomena in both the physical and social sciences. There has a significant development in impulsive theory especially in the area of impulsive differential equations with fixed moments; see for instance the monographs by Benchohra *et al* [7], Lakshmikantham *et al* [17], and Samoilenko and Perestyuk [22], K.Balachandran and J.Y.Park [5] and the references therein [15, 16].

Benchohra et al. studied the following Fractional Differential Equations Caputo's derivative:

In [8], u is bounded on J, $t \in J = [0, \infty)$ and $1 < \alpha \leq 2$.

$$^{c}D^{\alpha}u(t) = f(t, u(t), {}^{c}D^{\alpha-1}u(t)),\ u(0) = u_0$$

In [9], The existence results for nonlinear implicit fractional-order differential equations given by

$$^{c}D^{\alpha}y(t) = f(t, y(t), {}^{c}D^{\alpha}y(t)), y(0) = y_0, t \in J = [0, T],\ 0 < \alpha \leq 1.$$

Inspiration by the above works, we study the existence of solutions for the implicit fractional order differential equations with impulsive and nonlocal conditions of the form

$$^{c}D^{\alpha}y(t) = f(t, y(t), {}^{c}D^{\alpha}y(t)), \quad t \in J' := J \backslash \{t_1, ..., t_m\}, J = [0, T], 0 < \alpha \leq 1. \tag{1}$$

$$y(t_k^{+}) = y(t_k^{-}) + y_k,\ k = 1, 2, ..., m \quad y_k \in X \tag{2}$$

$$y(0) = y_0 - \eta(t), \tag{3}$$

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where ${}^cD^\alpha$ is the Caputo fractional derivative, $f : J \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is a given function, $\eta : C \to \mathbb{X}$ is continuous, and $y_0 \in \mathbb{X}$ and t_k satisfy $0 = t_0 < t_1 < ... < t_m < t_{m+1} = T$.

In this paper is planned as shadows. Section 2 has definitions and elementary results of the fractional calculus. In section 3, implicit impulsive fractional differential equations is attained and proved the theorems on the existence and uniqueness of a solution to the problem (1.1 - 1.3). In section 4, an illustrative example is provided in support of the results of a problem (1.1 - 1.3).

2 Preliminaries

In this section, we introduce notations, definition and preliminary facts. We introduce the Banach space $PC(J,X) = \{x : J \to X : x \in C(t_k, t_{k+1}], X\}, k = 0,1,2,...,m$ and their exist $x(t_k^-)$ and $x(t_k^+), k = 0,1,2,..,m$ with $x(t_k^-) = x(t_k)$ with the norm $||x||_{PC} := sup\{||x(t)|| : t \in J\}$.

Definition 2.1. *The fractional order integral of the function $h \in L^1([0,T], \mathbb{X}_+)$ of order $\alpha \in \mathbb{X}_+$ is defined by*

$$I^\alpha h(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} h(s)ds,$$

where Γ is the gamma function.

Definition 2.2. *For a function h given on the interval $[0,T]$, the Caputo fractional order derivative of order α of h, is defined by*

$$({}^cD^\alpha h)(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} h^{(n)}(s)ds,$$

where $n = |\alpha| + 1$, $|\alpha|$ denoted the integral part of real number α, provided $h^{(n)}(t)$ exists.

Lemma 2.1. *Let a function $f(t,u,v) : J \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ be continuous. Then the problem (1.1)-(1.3) is equivalent to the problem:*

$$y(t) = \begin{cases} y_0 - \eta(t) + I^\alpha g(t), \ for \ t \in [0,t_1] \\ y_0 - \eta(t) + y_1 + I^\alpha g(t), \ for \ t \in (t_1,t_2] \\ y_0 - \eta(t) + y_1 + y_2 + I^\alpha g(t), \ for \ t \in (t_2,t_3] \\ \vdots \\ y_0 - \eta(t) + \sum_{i=1}^m y_i + I^\alpha g(t), \ for \ t \in (t_m,T] \end{cases} \tag{2.1}$$

where $g \in C(J,\mathbb{X})$ satisfies the functional equation

$$g(t) = \begin{cases} f(t, y_0 - \eta(t) + I^\alpha g(t), g(t)), \ for \ t \in [0,t_1] \\ f(t, y_0 - \eta(t) + y_1 + I^\alpha g(t), g(t)), \ for \ t \in (t_1,t_2] \\ f(t, y_0 - \eta(t) + y_1 + y_2 + I^\alpha g(t), g(t)), \ for \ t \in (t_2,t_3] \\ \vdots \\ f(t, y_0 - \eta(t) + \sum_{i=1}^m y_i + I^\alpha g(t), g(t)), \ for \ t \in (t_m,T] \end{cases}$$

Proof. If

$${}^cD^\alpha y(t) = g(t)$$

then

$$I^\alpha \, {}^cD^\alpha y(t) = I^\alpha g(t).$$

So we obtain for $t \in [t_0,t_1]$,

$$y(t) = y(0) - \eta(t) + I^\alpha g(t),$$
$$y(t) = y_0 - \eta(t) + I^\alpha g(t), \ for \ t \in [0,t_1]$$

For $t \in (t_1, t_2]$ we have

$$\begin{aligned} y(t) &= y(t_1^+) - \eta(t) - \frac{1}{\Gamma(\alpha)} \int_0^{t_1} (t_1 - s)^{\alpha-1} f(t, u(t), ^c D^\alpha u(t)) du + I^\alpha g(t), \\ &= y(t_1) - \eta(t) + y_1 + I^\alpha g(t), \\ y(t) &= y_0 - \eta(t) + y_1 + I^\alpha g(t), \ for \ t \in (t_1, t_2] \end{aligned}$$

Similarly $t \in (t_m, T]$ we get

$$y(t) = y_0 - \eta(t) + \sum_{i=1}^{m} y_i + I^\alpha g(t), \ for \ t \in (t_m, T]$$

Therefore, we have

$$y(t) = \begin{cases} y_0 - \eta(t) + I^\alpha g(t), \ for \ t \in [0, t_1] \\ y_0 - \eta(t) + y_1 + I^\alpha g(t), \ for \ t \in (t_1, t_2] \\ y_0 - \eta(t) + y_1 + y_2 + I^\alpha g(t), \ for \ t \in (t_2, t_3] \\ \vdots \\ y_0 - \eta(t) + \sum_{i=1}^{m} y_i + I^\alpha g(t), \ for \ t \in (t_m, T] \end{cases}$$

The proof is completed. □

Lemma 2.2. *[10] Let E be a Banach space, C a closed, convex subset of E, U an open subset of C and $0 \in U$. Suppose that $F : \overline{U} \to C$ is a continuous, compact map. Then either (i) F has a fixed point in $\overline{U}$, or (ii) there is a $u \in \partial U$ and $\lambda \in (0, 1)$ with $u = \lambda F(u)$.*

Theorem 2.1. (Krasnoselkii) *Let M be a closed convex and nonempty subset of a Banach space $\mathbb{X}$. Let A and B be two operators such that (i) $Ax + By \in M$ whenever $x,y \in M$; (ii) A is compact and continuous; (iii) B is a contraction mapping. Then there exists $z \in M$ such that $z = Az + Bz$.*

3 Main Results

To prove the main result we need the following assumptions :
(A_1) The function $f : J \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$, w are continuous.
(A_2) There exist constants $K_1 > 0$ and $0 < K_2 < 1$ such that

$$||f(t, u_1, v_1) - f(t, u_2, v_2)|| \leq K_1 ||u_1 - u_2|| + K_2 ||v_1 - v_2||$$

for any $u_1, u_2, v_1 \& v_2 \in \mathbb{X}$ and $t \in J$.
(A_3) η is continuous, and there exists a constant $b < 1$ such that

$$||\eta(y_1) - \eta(y_2)|| \leq b ||y_1 - y_2||$$

$$\forall \ y_1, y_2 \in \mathbb{X},$$

(A_4) The function $f : J \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$, w are continuous and $\eta : C \to \mathbb{X}$ is continuous.

Theorem 3.2. *Assume that the assumptions $(A_1) - (A_3)$ holds. If*

$$\sum_{i=1}^{m} x_i + \frac{\eta K_1 T^\alpha}{(1 - K_2)\Gamma(\alpha + 1)} < 1 \tag{3.1}$$

then there exists a unique solution for $(1.1) - (1.3)$ on J

Proof. Define the operator $M : C(J, \mathbb{X}) \to C(J, \mathbb{X})$ by

The formula of solutions for equation $(1.1) - (1.3)$ should be

$$M(y)(t) = \begin{cases} y_0 - \eta(t) + I^\alpha g(t), \ for \ t \in [0, t_1] \\ y_0 - \eta(t) + y_1 + I^\alpha g(t), \ for \ t \in (t_1, t_2] \\ y_0 - \eta(t) + y_1 + y_2 + I^\alpha g(t), \ for \ t \in (t_2, t_3] \\ \vdots \\ y_0 - \eta(t) + \sum_{i=0}^{m} y_i + I^\alpha g(t), \ for \ t \in (t_m, T] \end{cases} \tag{3.2}$$

$$\text{where} \quad g(t) = f(t, y(t), g(t)), \quad g \in C(J, \mathbb{X}).$$

In general case $t \in (t_m, T]$):

$$M(y)(t) = y_0 - \eta(t) + \sum_{i=1}^{m} y_i + I^\alpha g(t), \quad \text{for} \quad t \in (t_m, T]) \tag{3.3}$$

where

$$g(t) = f(t, y(t), g(t)), \quad g \in C(J, \mathbb{X}).$$

Clearly, the fixed points of operation M are solutions of problem $(1.1) - (1.3)$.

Let $y_1, y_2 \in C(J, \mathbb{X})$. Then for $t \in J$, we have

$$(My_1)(t) - (My_2)(t) = \eta(y_1) - \eta(y_2) + \sum_{i=1}^{m} y_i + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} (g(s) - h(s)) ds,$$

where $g, h \in C(J, \mathbb{X})$ be such that

$$g(t) = f(t, y_1(t), g(t)),$$
$$h(t) = f(t, y_2(t), h(t)),$$

Then, for $t \in J$

$$||(My_1)(t) - (My_2)(t)|| = ||\eta(y_1) - \eta(y_2)|| + \sum_{i=1}^{m} ||y_i|| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} ||(g(s) - h(s))|| ds \tag{3.4}$$

By (A_2) we have

$$\begin{aligned} ||g(t) - h(t)|| &= ||f(t, y_1(t), g(t)) - f(t, y_2(t), h(t))|| \\ &\le K_1 ||y_1(t) - y_2(t)|| + K_2 ||y_1(t) - y_2(t)|| \\ &\le \frac{K_1}{1 - K_2} ||y_1(t) - y_2(t)|| \end{aligned}$$

Therefore (3.4)

$$\begin{aligned} ||(My_1)(t) - (My_2)(t)|| &\le \sum_{i=1}^{m} ||y_i|| + \frac{b||y_1 - y_2||K_1}{(1 - K_2)\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} ||y_1(t) - y_2(t)|| ds, \\ &\le \sum_{i=1}^{m} ||y_i|| + \frac{bK_1 T^\alpha}{(1 - K_2)\Gamma(\alpha + 1)} ||y_1 - y_2||_\infty. \end{aligned}$$

Thus

$$||My_1 - My_2||_\infty \le \sum_{i=1}^{m} ||y_i|| + \frac{bK_1 T^\alpha}{(1 - K_2)\Gamma(\alpha + 1)} ||y_1 - y_2||_\infty.$$

By (3.2), the operator M is a continuous. Hence, by Banach's contraction principle, M has a unique fixed point which is a unique solution of the problem $(1.1) - (1.3)$. The proof is completed. □

Theorem 3.3. *Assume the $(A_1) - (A_3)$. Then the problem (1.1)-(1.3) has at least one solution on $[0, T]$.*

Proof. Choose

$$\frac{||y||}{y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{\varphi(||x||)||p||_{L_1}}{\Gamma(\alpha+1)} T^{\alpha}} \leq 1.$$

Case: (i) M maps bounded sets (balls) into bounded sets in $C([0, T], \mathbb{X})$.

For a positive number r, let $B_r = \{x \in C([0, T], \mathbb{X}) : ||x|| \leq r\}$ be a bounded ball in $C([0, T], \mathbb{X})$. Then for $t \in (t_m, T]$ we have

$$\begin{aligned} ||M(y)(t)|| &\leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} ||g(s)|| ds, \\ &\leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} ||f(t, y(t), g(t))|| ds, \\ &\leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{\varphi(||x||)||p||_{L_1}}{\Gamma(\alpha+1)} T^{\alpha} \end{aligned}$$

Consequently

$$||M(y)|| \leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{\varphi(r)||p||_{L_1}}{\Gamma(\alpha+1)} T^{\alpha}.$$

Case: (ii) M maps bounded sets (balls) into equicontinuous sets in $C([0, T], \mathbb{X})$.

Let $\sup_{(t,x)\in[0,T]\times B_r} ||f(t, u, v)|| = f^* < \infty, \quad \mu_1, \mu_2 \in [0, T]$ with $\mu_1, \mu_2 \in (t_m, T]$ and $x \in B_r$. Then we have

$$\begin{aligned} ||M(y)(\mu_1) - M(y)(\mu_2)|| &= y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{1}{\Gamma(\alpha)} \int_0^{\mu_1} (\mu_1 - s)^{\alpha-1} ||f(t, y(t), g(t))|| ds \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_{\mu_1}^{\mu_2} (\mu_2 - s)^{\alpha-1} ||f(t, y(t), g(t))|| ds, \\ &\leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{f^*}{\Gamma(\alpha+1)} ||\mu_2^{\alpha} - \mu_1^{\alpha}|| \end{aligned}$$

Obvisously the right-hand side of the above inequality tends to zero independently of $x \in B_r$ as $\mu_2 - \mu_1 \to 0$. As M satisfies the above assumptions, therefore it follows by the Arzela-Ascoli theorem that $M : C([0, T], \mathbb{X}) \to C([0, T], \mathbb{X})$ is completely continuous. Let y be a solution. Then, for $t \in [0, T]$ and following the similar computations as in the first step, we have

$$||y|| \leq y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{\varphi(||x||)||p||_{L_1}}{\Gamma(\alpha+1)} T^{\alpha}.$$

Consequently, we have

$$\frac{||y||}{y_0 - \eta(t) + \sum_{i=1}^{m} ||y_i|| + \frac{\varphi(||x||)||p||_{L_1}}{\Gamma(\alpha+1)} T^{\alpha}} \leq 1.$$

There exist N^* such that $||x|| \neq N^*$. Let us set c

$$U = \{x \in C([0, T], \mathbb{X}) : ||x|| < N^*\}.$$

Note that the operator $M : \overline{U} \to C([0, T], \mathbb{X})$ is continuous and completely continuous. Consequently, by the nonlinear alternative of Lerary-Schauder type, we deduce that M has fixed point $y \in \overline{U}$ which is a solution of the problem $(1.1) - (1.3)$. The proof is completed. □

Theorem 3.4. *(Existence results via Krasnoselskii's fixed point theorem) Assume that* $|f(t,u,v)| \leq \mu||u-v||, \quad \mu \in C([0,T],\mathbb{X}^+)$. *Then the problem (1.1-(1.3) has at least one solution on* $[0,T]$ *if*

$$L\sum_{i=1}^{m}||y_i|| < 1 \tag{3.5}$$

Proof. Choose a suitable constant r as

$$r \geq \frac{(\mu-b)T^\alpha}{\Gamma(\alpha+1)}||u-v|| + y_0 + \sum_{i=1}^{m}||y_i||$$

Define the operators $\mathcal{P}$ and $\mathcal{Q}$ on $B_r = \{y \in C([0,T],\mathbb{X}) : ||y|| \leq r\}$ as

$$(\mathcal{P}y)(t) = I^\alpha g(t)$$

$$(\mathcal{Q}y)(t) = y_0 - \eta(t) + \sum_{i=1}^{m} y_i$$

For $u, v \in B_r$, we obtain

$$\begin{aligned}
||\mathcal{P}y + \mathcal{Q}y|| &\leq \frac{\mu T^\alpha}{\Gamma(\alpha+1)}||u-v|| + y_0 - b||u-v|| + \sum_{i=1}^{m}||y_i|| \\
&\leq \frac{(\mu-b)T^\alpha}{\Gamma(\alpha+1)}||u-v|| + y_0 + \sum_{i=1}^{m}||y_i|| \\
&\leq r
\end{aligned}$$

Thus, $\mathcal{P}x + \mathcal{Q}y \in B_r$. It follows from the assumption to gether with (3.5) that $\mathcal{Q}$ is a contraction mapping. Continuity of f implies that the operator $\mathcal{P}$ is continuous. Also, $\mathcal{P}$ is uniformly bounded on B_r as

$$||\mathcal{P}x|| \leq \frac{(\mu-b)T^\alpha}{\Gamma(\alpha+1)}||u-v||$$

Now we prove the compactness of the operator $\mathcal{P}$.

Let $\sup\limits_{(t,x)\in[0,T]\times B_r} ||f(t,u,v)|| = f^* < \infty, \quad \mu_1, \mu_2 \in [0,T]$ with $\mu_1, \mu_2 \in (t_m, T]$ and $x \in B_r$. Then we have

$$\begin{aligned}
||\mathcal{P}(y)(\mu_1) - \mathcal{P}(y)(\mu_2)|| &= \frac{1}{\Gamma(\alpha)}\int_0^{\mu_1}(\mu_1-s)^{\alpha-1}||f(t,y(t),g(t))||ds \\
&\quad + \frac{1}{\Gamma(\alpha)}\int_{\mu_1}^{\mu_2}(\mu_2-s)^{\alpha-1}||f(t,y(t),g(t))||ds, \\
&\leq \frac{f^*}{\Gamma(\alpha+1)}||\mu_2^\alpha - \mu_1^\alpha||
\end{aligned}$$

which is independent of y and tends to zero as $\mu_2 - \mu_1 \to 0$. Thus, $\mathcal{P}$ is equicontinuous. So $\mathcal{P}$ is relatively compact on B_r. Hence, by the Arzela-Ascoli theorem, $\mathcal{P}$ is compact on B_r. Thus all the assumations of Theorem 1 are satisfied. So the conclusion of Theorem 1 implies that the impulsive implicit fractional non-local problem (1.1)-(1.3) has at least one solution on $[0,T]$. The proof is completed. □

4 Example

Consider the following Implicit fractional differential equation with nonlocal impulsive condition of the form

$${}^cD^\alpha y(t) = \frac{1}{(t+2)^2}\left[\frac{|y(t)|}{1+|y(t)|} - \frac{|D^\alpha y(t)|}{1+|D^\alpha y(t)|}\right] \tag{4.1}$$

$$y(t_k^+) = y(t_k^-) + \frac{1}{4}, \tag{4.2}$$

$$y(0) = y_0 - \sum_{i=1}^{m} c_i y(t_i) \tag{4.3}$$

Take $J=[0,1]$. Set

$$f(t,y(t),^{c}D^{\alpha}y(t))=\frac{1}{(t+2)^2}\left[\frac{|y(t)|}{1+|y(t)|}-\frac{|D^{\alpha}y(t)|}{1+|D^{\alpha}y(t)|}\right],t\in J^{'}.x\in X$$

Let $y_1,y_2\in X$ and $t\in J^{'}$. Then we have

$$||f(t,y_1(t),^{c}D^{\alpha}y_1(t))-f(t,y_2(t),^{c}D^{\alpha}y_2(t))||\leq\frac{K_1}{4(1-K_2)}||y_1-y_2||$$

Hence the condition $(A_1)-(A_3)$ hold. Note that $K_1=\frac{1}{4}$ and $K_2=\frac{1}{8}$. Then by Theorem 2, the problem equations $(1.1)-(1.3)$ has an unique solution on $[0,1]$ for the values of α satisfying equation (4.1).

5 Conclusion

We have proven an existence result for implicit fractional differential equations with impulsive condition. In the future, we will extend the results to other fractional derivatives and boundary value problems.

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