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Some Remarks on Semi A-Segal Algebras

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Abstract

In the present paper, we investigate some results on A-Segal algebras. Furthermore, the notion of a semi A-Segal algebra is introduced and some results are given. As an important result, we prove that if B is a finite-dimensional Banach algebra so that $\dim B > 1$, then there is an ideal B' of B such that it is semi B-Segal.

Keywords and Phrases: Semi A-Segal algebra, A-Segal algebra, division algebra.

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1 Introduction

The concept of Segal and abstract Segal algebras were first introduced and studied in [5]. Many authors have considered this notion ever since and investigated several properties of these algebras such as different kinds of amenability, BSE property, Arens regularity and so on (see for instance [1], [3] and [4]). An abstract Segal algebra is a Banach algebra which is perceived as a certain ideal in another Banach algebra. In this paper we focus our attention on a new concept called semi A-Segal algebra and obtain some results on the kind of algebras. Our work is motivated by relative completion of an A-Segal algebra defined by Burnham [2].

2 Main Results

It is known and easy to show that the complex Banach space $\mathbb{C}^n$ is a $\mathbb{R}^{2n}$-Segal algebra. Now, according to Burnham's notation [2]. we have

$$\widetilde{\mathbb{C}^n}^{\mathbb{R}^{2n}} = \bigcup_{\eta>0} \overline{S_{\mathbb{C}^n}(\eta)}^{\mathbb{R}^{2n}}$$

Where $S_{\mathbb{C}^n}(\eta) = \{x = (x_1, \dots, x_n) \in \mathbb{C}^n : \|x\|_{\mathbb{C}^n} \leq \eta\}$.

Notice that since $\mathbb{C}^n$ is isometrically isomorphic to $\mathbb{R}^{2n}$, thus we have

$$\|\cdot\|_{\mathbb{C}^n} = \|\cdot\|_{\mathbb{R}^{2n}}.$$

Moreover, the following is a norm on $\widetilde{\mathbb{C}^n}^{\mathbb{R}^{2n}}$. We leave verification of the properties to the reader.

$$|||x||| = \inf\left\{t : x \in \overline{S_{\mathbb{C}^n}(t)}^{\mathbb{R}^{2n}}\right\}$$

Now, as an important result, we have:

Theorem 2.1. $\widetilde{\mathbb{C}^n}^{R^{2n}} = \mathbb{R}^{2n}$.

Proof. Since $\bigcup_{\eta>0} \overline{S_{\mathbb{C}^n}(\eta)}^{\mathbb{R}^{2n}} = \mathbb{R}^{2n}$, hence the proof is finished. □

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As a corollary, we obtain the following.

Corollary 2.2. *$\widetilde{\mathbb{C}^n}^{\mathbb{R}^{2n}}$ is a $\mathbb{R}^{2n}$-Segal algebra.*

Proof. Since $\mathbb{C}^n$ is a $\mathbb{R}^{2n}$-Segal algebra, so by [2], $\widetilde{\mathbb{C}^n}^{\mathbb{R}^{2n}}$ is also. □

Next, we obtain an interesting result on division algebras.

Theorem 2.3. *Every division algebra is a $\mathbb{R}^{2n}$-Segal algebra.*

Proof. Let B be a division algebra. Therefore, by [5] B is a copy of the complex plane $\mathbb{C}$ and in view of proof of the Corollary 2.2 (with $n = 1$), we conclude that it is a $\mathbb{R}^2$-Segal algebra. □

Theorem 2.4. *For any $x \in \mathbb{C}^n$, we have*

$$\|x\|_{\mathbb{C}^n} = |||x|||$$

Proof. Since $\mathbb{C}^n$ is a $\mathbb{R}^{2n}$-Segal algebra, thus proof follows from the Theorem6 of [2]. Furthermore, $\|x\|_{\mathbb{R}^{2n}} = |||x|||$. □

Theorem 2.5. $\bigcup\limits_{\eta>0} S_{\mathbb{C}^n}(\eta) = \mathbb{R}^{2n}$.

Proof. According to Theorem 7 of [2]. because of $\mathbb{C}^n$ is a $\mathbb{R}^{2n}$-Segal algebra, we have

$$S_{\mathbb{C}^n}(\eta) = \overline{S_{\mathbb{C}^n}(\eta)}^{\mathbb{R}^{2n}} \cap \mathbb{C}^n$$

Therefore,

$$\bigcup_{\eta>0} S_{\mathbb{C}^n}(\eta) = \left(\bigcup_{\eta>0} \overline{S_{\mathbb{C}^n}(\eta)}^{\mathbb{R}^{2n}} \right) \cap \mathbb{C}^n = \mathbb{R}^{2n} \cap \mathbb{C}^n = \mathbb{R}^{2n}$$

This completes the proof. □

The next result is about the singularity of $\mathbb{C}^n$ as a $\mathbb{R}^{2n}$–Segal algebra.

Theorem 2.6. *$\mathbb{C}^n$ is singular.*

Proof. The proof follows the fact that $\mathbb{C}^n \subseteq \widetilde{\mathbb{C}^n}^{\mathbb{R}^{2n}} = \mathbb{R}^{2n}$ as well as Theorem11 of [2]. □

Here, we wish to introduce a new notion so-called semi A-Segal algebra.

Definition 2.7. *Let $(A, \|\cdot\|_A)$ be a Banach algebra. A subalgebra B of A is called a semi A-Segal algebra with respect to $\|\cdot\|_B$ when the following conditions are satisfied:*

(i) B is an ideal(not necessarily dense) in A such that it is a Banach algebra with respect to $\|\cdot\|_B$;

(ii) Natural injection from B into A is continuous and the product is a jointly continuous function from $A \times B$ into B.

Clearly, every A-Segal algebra is also semi A-Segal. But the converse need not be true; for instance, $\mathbb{R}$ is semi $\mathbb{R}^2$-Segal but not $\mathbb{R}^2$-Segal.

Now, we give some results on semi A-Segal algebras.

Theorem 2.8. *Suppose that m and n are two positive integers with $n > m$. Then $\mathbb{R}^m$ is a semi $\mathbb{R}^n$-Segal algebra.*

Proof. The natural embedding from $\mathbb{R}^m$ into $\mathbb{R}^n$ shows that $\mathbb{R}^m$ is an ideal in $\mathbb{R}^n$. The other conditions of Definition 2.7 are easy to verification. □

Theorem 2.9. *Let B be a finite-dimensional Banach algebra with an even dimension. Then there is an ideal B' of B such that it is B-Segal.*

Proof. Suppose that $\dim B = 2k$, for some positive integer k. It is easy to show that B is linearly isomorphic to $\mathbb{R}^k$. Now, by taking $B' = \mathbb{C}^k$, one can arrive at the desired result. □

The following holds for semi A-Segal algebras.

Theorem 2.10. *Let B be a finite-dimensional Banach algebra so that* $\dim B > 1$*. Then there is an ideal B' in B such that it is semi B-Segal.*

Proof. Assume that

$$\dim B = k \quad (k \in N, k > 1).$$

As before, B is linearly isomorphic to $\mathbb{R}^k$. We set $B' = \mathbb{R}^{k-1}$. Now, Theorem 2.8 completes the proof. □

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