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On $\mathbb{S}$ fuzzy soft sub hemi rings of a hemi ring

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Abstract

In this expose, on endeavors equipped on the way to achieve comprehension of the arithmetical character of $\mathbb{S}$-fuzzy soft sub hemi rings of a hemi ring.

Keywords: Fuzzy soft set, $\mathbb{S}$ fuzzy soft sub hemi ring, anti-$\mathbb{S}$-fuzzy soft sub hemi ring, and pseudo Fuzzy soft co-set.

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1 Introduction

A small amount of researchers done their works in near rings and a few kinds of semi rings contain conventional part. Semi rings emerge in a natural approach in a few applications in the theory of automata and formal languages. Soft set premise as a novel mathematical device means and deals with uncertainty which seems to be gratis from the intrinsic difficulties disturbing the obtainable works. The introduction of fuzzy sets as a result of Zadeh. L.A [16], a few scholars developed fuzzy concepts lying on the impression of the concept of fuzzy sets. Dubois.D and Prade. H [8], were urbanized the concept of fuzzy Sets and Systems: Theory and Applications. Aktas. H, CaSman.N [3] were developed by Soft sets and soft groups. In this article, $\mathbb{S}$-Fuzzy soft sub hemi ring of a hemi ring is initiated in addition to the theorems in the company of various example.

2 Preliminaries

Definition 2.1. *Let $\mathbb{R}$ be a hemi ring. A Fuzzy soft sub set (H,C) of $\mathbb{R}$ is supposed to be a $\mathbb{S}$-Fuzzy soft sub hemi ring (SFSHR) of $\mathbb{R}$ if it satisfies the subsequent circumstances:*

(i) $\mu_{(H,C)}(a,b) \geq \mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)}(b)\}$,

(ii) $\mu_{(H,C)}(ab) \geq \mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)}(b)\}$, *in favor of each and every one a and b in $\mathbb{R}$.*

Definition 2.2. *Let $(\mathbb{R},+,\cdot)$ be a hemi ring. A $\mathbb{S}$-Fuzzy soft sub hemi ring (H,C) of $\mathbb{R}$ is said to be an Fuzzy soft normal sub hemi ring (SFSNSHR) of $\mathbb{R}$ if it satisfies the subsequent conditions:*

(i)$\mu_{(H,C)}(ab) = \mu_{(H,C)}(ba)$ *on behalf of all a and b in $\mathbb{R}$.*

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Definition 2.3. *Let $\mathbb{R}$ and $\mathbb{R}^1$ be some two hemi rings. Assent to f is a mapping from $\mathbb{R}$ to $\mathbb{R}^1$ be any function and let B be a $\mathbb{S}$- Fuzzy soft sub hemi ring in $\mathbb{R}$, V be an $\mathbb{S}$-Fuzzy soft sub hemi ring in $f(\mathbb{R}) = \mathbb{R}^1$, defined by $\mu_{V(b)} = \sup_{(a)\in f^{-1}(b)}(\mu_{(H,C)})(a)$ intended for every a within $\mathbb{R}$ as well as b in $\mathbb{R}^1$. After that B is called a pre image of V under f and it is denoted by $f^{-1}(V)$.*

Definition 2.4. *Let (H, C) be an $\mathbb{S}$-Fuzzy soft sub hemi ring of a hemi ring $(\mathbb{R}, +, \cdot)$ and a in $\mathbb{R}$. Then the pseudo $\mathbb{S}$-Fuzzy soft coset $(x(H, B))^p$ obviously $((x\mu_{(H,C)})^p)(a) = p(x)\mu_{(H,C)}(a)$, for every x in $\mathbb{R}$ and for some p in P.*

3 A few proofs associated by way of $\mathbb{S}$-fuzzy soft sub hemi rings of a hemi ring

Theorem 3.1. *If (H, C) is an $\mathbb{S}$-Fuzzy soft sub hemi ring of a hemi ring $(\mathbb{R}, +, \cdot)$, then (H, C) is an $\mathbb{S}$-Fuzzy soft sub hemi ring of $\mathbb{R}$.*

Proof. Allow (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. Think about $(H, C) = \left\{ \langle a, \mu_{(H,C)}, (a)\rangle \right\}$, despite a in $\mathbb{R}$, we obtain $(H, C) = (H, D) = \left\{ \langle a, \mu_{(H,D)}, (a)\rangle \right\}$, somewhere $\mu_{(H,D)}(a) = \mu_{(H,C)}(a)$, visibly, $\mu_{(H,D)}(a + b) \geq \mathbb{S}\{\mu_{(H,D)}(a), \mu_{(H,D)}(b)\}$, in spite of a as well as b in $\mathbb{R}$ in addition to $\mu_{(H,D)}(ab) \geq \mathbb{S}\{\mu_{(H,D)}(a), \mu_{(H,D)}(b)\}$, for all that a moreover b in $\mathbb{R}$. While B is an $\mathbb{S}$-fuzzy soft sub hemi ring of $\mathbb{R}$, we encompass $\mu_{(H,C)}(a + b) \geq \mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)}(b)\}$, for all that a in addition to b in $\mathbb{R}$. Also $\mu_{(H,C)}(ab) \geq \mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)}(b)\}$, every a along with b in $\mathbb{R}$. For this reason $(H, D) = (H, C)$ is an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. □

Theorem 3.2. *If (H, C) is an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $(\mathbb{R}, +, \cdot)$, then (H, C) is an $\mathbb{S}$-fuzzy soft sub hemi ring of $\mathbb{R}$.*

Proof. Conset to (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. With the purpose of $(H, C) = \left\{ \langle a, \mu_{(H,C)}(a)\rangle \right\}$, in favor of every one a in $\mathbb{R}$. Let $(H, C) = (H, D) = \left\{ \langle a, \mu_{(H,D)}(a)\rangle \right\}$, designed for the entire a along with b in $\mathbb{R}$. In view of the fact that (H, B) is an $\mathbb{S}$- fuzzy soft sub hemi ring of $\mathbb{R}$, which implies to facilitate $1-\mu_{(H,D)}(ab) \leq \mathbb{S}\{(1-\mu_{(H,D)}(a)), (1-\mu_{(H,D)}(b))\}$, which implies so as to $\mu_{(H,D)}(ab) \geq 1 - \mathbb{S}\{(1 - \mu_{(H,D)}(a)), (1 - \mu_{(H,D)}(b))\} = \mathbb{S}\{\mu_{(H,D)}(a), \mu_{(H,D)}(b)\}$. As a result, $\mu_{(H,D)}(ab) \geq \mathbb{S}\{\mu_{(H,D)}(a), \mu_{(H,D)}(b)\}$, intended for every one of a furthermore b in $\mathbb{R}$. Consequently $(H, D) = (H, C)$ is an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. □

Theorem 3.3. *Accede to $(\mathbb{R}, +, \cdot)$ survice a hemi ring and (H, C) be present a non unfilled subset of $\mathbb{R}$. Then (H, C) is a sub hemi ring of $\mathbb{R}$merely if $(H, D) = \left\langle \chi_{(H,C)}, \overline{\chi}_{(H,C)} \right\rangle$ is a $\mathbb{S}$-fuzzy soft sub hemi ring of $\mathbb{R}$, where $\chi_{(H,C)}$ is the characteristic function.*

Proof. Allow $(\mathbb{R}, +, \cdot)$ be a hemi ring in addition to (H, C) be a unbalance subset of $\mathbb{R}$. Primary agree to (H, C) be a sub hemi ring of $\mathbb{R}$. Obtain a with b in $\mathbb{R}$.

Case (i): Condition a furthermore b in (H, C) afterward $a + b, ab$ inside (H, C), given that (H, C) is a sub hemi ring of $\mathbb{R}, \chi_{(H,C)}(a) = \chi_{(H,C)}(b) = \chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$ with $\chi_{(H,C)}(a) = \chi_{(H,C)}(b) = \chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 0$. As a result, $\chi_{(H,C)}(a + b) \geq \mathbb{S}\{(a), \mu_{(H,C)}\chi(b)\}$, meant for every one of a also b within $\mathbb{R}$, $\chi_{(H,C)}(ab) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, behalf of all a along with b inside $\mathbb{R}$. Subsequently, $\chi_{(H,C)}(a + b) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, in favor of every part of a in addition to b into $\mathbb{R}, \chi_{(H,C)}(ab) \leq \{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, intended for each and every one a as well as b within $\mathbb{R}$.

Case (ii) Either a or b in (H, C), then $a + b, ab$ may or may not be in (H, C), $\chi_{(H,C)}(a) = 1, \chi_{(H,C)}(b) = 0$ (or) $\chi_{(H,C)}(a) = 0, \chi_{(H,C)}(b) = 1), \chi_{(H,C)}(a + b) = 1, \chi_{(H,C)}(ab) = 1$ (or 0) and $\chi_{(H,C)}(a) = 0, \chi_{(H,C)}(a)(b) = 1$ (or) $\chi_{(H,C)}(a) = 1, \chi_{(H,C)}(b) = 0)\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 0$ (or 1). Obviously $\chi_{(H,C)}(a + b) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all that a along with b in $\mathbb{R}, \chi_{(H,C)}(ab) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, intended for every a and b in $\mathbb{R}$, and $\chi_{(H,C)}(a + b) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}\chi_{(H,C)}(ab) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}$.

Case (iii) If a and b somewhere else (H, C), at that time $a+b, ab$ may well otherwise may not in $(H, C), \chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 0, \chi_{(H,C)}(a+b) = \chi_{(H,C)}(ab) = 1$ or 0 and $\chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 1, \chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 0$ or 1. Evidently $\chi_{(H,C)}(a + b) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}\chi_{(H,C)}(ab) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}$ and $\chi_{(H,C)}(a + b) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}\chi_{(H,C)}(ab) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\}$, for all a and b in $\mathbb{R}$. Subsequently in above conditions we comprise B is a fuzzy soft sub hemi ring of (H, C) hemi ring $\mathbb{R}$. On the contrary, Accede to a and b in (H, C), In view of the fact that (H, C) is non blank subset of $\mathbb{R}$, thus $\chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 1, \chi_{(H,C)}(a) = \chi_{(H,C)}(b) = 0$. While $B = \left\langle \chi_{(H,C)}, \overline{\chi}_{(H,C)} \right\rangle$ is a fuzzy soft sub hemi ring of $\mathbb{R}$, we have $\chi_{(H,C)}(a + b) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = \mathbb{S}\{1, 1\} = 1, \chi_{(H,C)}(ab) \geq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(ab)\} = \mathbb{S}\{1, 1\} = 1$. For that reason $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 1$. and $\chi_{(H,C)}(a + b) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = \max\{0, 0\} = 0, \chi_{(H,C)}(ab) \leq \mathbb{S}\{\chi_{(H,C)}(a), \chi_{(H,C)}(b)\} = \max\{0, 0\} = 0$. Therefore $\chi_{(H,C)}(a + b) = \chi_{(H,C)}(ab) = 0$. Thus $a + b$ as well as ab in (H, C), as a result (H, C) is a sub hemi ring of $\mathbb{R}$. □

Theorem 3.4. *Let (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring H and f is an isomorphism from a hemi ring $\mathbb{R}$ onto H. Then $(H, C) \circ f$ is an $\mathbb{S}$- fuzzy soft sub hemi ring of $\mathbb{R}$.*

Proof. Consent to a and b in $\mathbb{R}$ as well as (H, C) be an fuzzy soft sub hemi ring of a hemi ring H. Subsequently, it is encompassed, $(\mu_{(H,C)} \circ f)(a + b) = \mu_{(H,C)} f(a + b) = \mu_{(H,C)}\{f(a) + f(b)\}$, as f is an isomorphism $\geq \mathbb{S}\{\mu_{(H,C)} f(a), \mu_{(H,C)} f(b)\}, = \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$, which implies so to $(\mu_{(H,C)} \circ f)(a + b) \geq \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$. And $(\mu_{(H,C)} \circ f)(ab) = \mu_{(H,C)}(f(ab)) = \mu_{(H,C)}(f(a)f(b))$, as f is an isomorphism $\geq \mathbb{S}\{\mu_{(H,C)} f(a), \mu_{(H,C)} f(b)\} = \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$, which implies that $(\mu_{(H,C)} \circ f)(ab) \geq \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$. For that reason $(H, C) \circ f$ is a $\mathbb{S}$- Fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. □

Theorem 3.5. *Let (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a heim ring h and f is an anti-isomorphism from a hemi ring r onto h. Then $(H, C) \circ f$ is a $\mathbb{S}$-fuzzy soft sub hemi ring of $\mathbb{R}$.*

Proof. Accede to a and b in $\mathbb{R}$ in addition to (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring H. Afterward we have $(\mu_{(H,C)} \circ f)(a + b) = \mu_{(H,C)}(f(a + b)) = \mu_{(H,C)}(f(b) + f(a))$, as f is an anti-isomorphism $\geq \min\{\mu_{(H,C)} f(a), \mu_{(H,C)} f(b)\} = \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$, which implies to facilitate $(\mu_{(H,C)} \circ f)(a + b) \geq \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$. In addition to, $(\mu_{(H,C)} \circ f)(ab) = \mu_{(H,C)} \circ f(ab) = \mu_{(H,C)} \circ (f(b)f(a))$, as f is an anti-isomorphism $\geq \mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)} f(b)\} = \mathbb{S}\{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$, which implies to $(\mu_{(H,C)} \circ f)(ab) \geq \{(\mu_{(H,C)} \circ f)(a), (\mu_{(H,C)} \circ f)(b)\}$. Thus $(G, B) \circ f$ is an $\mathbb{S}$-fuzzy soft sub hemi ring of the hemi ring $\mathbb{R}$. □

Theorem 3.6. *Let (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $(R, +, \cdot)$, then the pseudo fuzzy soft co-set $(x(H, C))^p$ is an $\mathbb{S}$-fuzzy soft hemi ring of a hemi ring $\mathbb{R}$, for every x in $\mathbb{R}$.*

Proof. Consent to (H, C) be an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. In favor of each a and b in $\mathbb{R}$, we have

$$\begin{aligned}((x\mu_{(H,C)})^p)(a+b) &= p(x)\mu_{(H,C)}(a+b) \geq p(x)\mathbb{S}\{(\mu_{(H,C)}(a), (\mu_{(H,C)}(b)\} \\ &= \mathbb{S}\{p(x)(\mu_{(H,C)}(a), (\mu_{(H,C)}(b)\} \\ &= \mathbb{S}\{((x\mu_{(H,C)})^p)(a), ((x\mu_{(H,C)})^p)(b)\}.\end{aligned}$$

As a result, $((x\mu_{(H,C)})^p)(a+b) \geq \mathbb{S}\{((x\mu_{(H,C)})^p)(a), ((x\mu_{(H,C)})^p)(b)\}$. At this istant

$$\begin{aligned}((x\mu_{(H,C)})^p)(ab) &= p(x)\mu_{(H,C)}(ab) \geq p(x)\mathbb{S}\{\mu_{(H,C)}(a), \mu_{(H,C)}(b)\} \\ &= \mathbb{S}\{p(x)\mu_{(H,C)}(a), p(x)\mu_{(H,C)}(b)\} \\ &= \mathbb{S}\{((x\mu_{(H,C)})^p)(a), ((x\mu_{(H,C)})^p)(b)\}.\end{aligned}$$

Consequently, $((x\mu_{(H,C)})^p)(ab) \geq \mathbb{S}\{((x\mu_{(H,C)})^p)(a), ((x\mu_{(H,C)})^p)(b)\}$. From now $(x(H, C))^p$ is an $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring $\mathbb{R}$. □

4 Conclusion

In the current work, a novel concept of $\mathbb{S}$-fuzzy soft sub hemi ring of a hemi ring which are defined with some properties and related theorems are studied.

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