

## Anti $S$ -fuzzy soft subhemirings of a hemiring

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### Abstract

In this paper, we have studied the algebraic operations of anti  $S$ -fuzzy soft sets to establish their basic properties. We have discussed different algebraic structures of anti  $S$ -fuzzy soft sets under the restricted and extended operations of union and intersection in a comprehensive manner. Logical equivalences have also been made in order to give a complete overview of these structures.

*Keywords:* Fuzzy soft set, anti  $S$  fuzzy soft subhemiring, pseudo anti  $S$  fuzzy soft coset.

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## 1 Introduction

Many fields deal daily with the uncertain data that may not be successfully modeled by the classical mathematics. There are some mathematical tools for dealing with uncertainties; two of them are fuzzy set theory, developed by Zadeh (1965), and soft set theory, introduced by Molodtsov (1999), that are related to this work. At present, work on the soft set theory is progressing rapidly. Maji et al (2003) defined operations of soft sets to make a detailed theoretical study on the soft sets. By using these definitions, the applications of soft set theory have been studied increasingly. Soft decision making (Cagman & Enginoglu 2010a, 2010b, Cagman et al 2010a, 2010b, Chen et al 2005, Feng et al 2010, Herawan & Deris 2009b, Herawan et al 2009, Kong et al 2008, 2009, Maji et al 2002, Xiao et al 2003, Majumdar & Samanta 2008, 2010. Cagman & Enginoglu (2010a) redefine the operations of soft sets to make them more functional for improving several new results. By using these new definitions they also construct a uni-int decision making method which selects a set of optimum elements from the alternatives. Cagman & Enginoglu (2010b) introduced a matrix representation of this work that gives several advantages to compute applications of the soft set theory. Later than the preamble of fuzzy sets as a result of L.A. Zadeh [16], more than a few scholars developed lying on the overview of the notion of fuzzy sets. Ali, M.I., M. Shabir and M. Naz, [5] developed the algebraic structures of soft sets associated with new operations, Fuzzy Sets and Systems: Theory and Applications was urbanized by Dubois, D. and Prade, H.[8], also Maji, P.K., R. Biswas and A.R. Roy, [13] have been produced Fuzzy Soft Sets. In this article, we introduce some properties and theorems in anti  $S$ -fuzzy soft subhemirings of a hemiring.

## 2 Preliminaries

**Definition 2.1.** A  $S$ -norm is a binary operation  $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$  satisfying the following requirements:

- (i)  $0Sx = x, 1Sx = 1$  (boundary conditions)

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(ii)  $xSy = ySx$  (commutativity)

(iii)  $xS(ySz) = (xSy)Sz$  (associativity)

(iv) If  $x \in y$  and  $w \in z$ , then  $xSw \in ySz$  (monotonicity).

**Definition 2.2.** Let  $(R, +, \cdot)$  be a hemiring.  $(F, A)$ -fuzzy subset of  $R$  is said to be an anti S-fuzzy soft subhemiring (anti fuzzy soft subhemiring with respect to S-norm) of  $R$  if it satisfies the following conditions:

(i)  $\mu_{(F,A)}(x + y) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$

(ii)  $\mu_{(F,A)}(xy) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , for all  $x$  and  $y$  in  $R$ .

**Definition 2.3.** Let  $(F, A)$  and  $(F, B)$  be fuzzy subsets of sets  $G$  and  $H$ , respectively. The anti-product of  $(F, A)$  and  $(F, B)$ , denoted by  $(F, A \times B)$  is defined as  $(F, A \times B) = \{ \langle (x, y), \mu_{(F,A \times B)}(x, y) \rangle / \text{for all } x \text{ in } G \text{ and } y \text{ in } H \}$ , where  $\mu_{(F,A \times B)}(x, y) = \max \{ \mu_{(F,A)}(x), \mu_{(F,B)}(y) \}$ .

**Definition 2.4.** Let  $(F, A)$  be a fuzzy subset in a set  $S$ , the anti-strongest relation fuzzy relation on  $S$ , that is fuzzy relation on  $(F, A)$  is  $(G, V)$  given by  $\mu_{(G,V)}(x, y) = \max \{ \mu_{(F,A)}(x), \mu_{(F,B)}(y) \}$  for all  $x$  and  $y$  in  $S$ .

**Definition 2.5.** Let  $(R, +, \cdot)$  and  $(R', +, \cdot)$  be any two hemirings. Let  $f : R \rightarrow R'$  be any function and  $(F, A)$  be an anti S fuzzy soft subhemiring in  $R$ ,  $(G, V)$  be an anti S- fuzzy soft subhemiring in  $f(R) = R'$ , defined by  $\mu_{(G,V)}(y) = \inf_{x \in f^{-1}(y)} \mu_{(F,A)}(x)$  for all  $x$  in  $R$  and  $y$  in  $R'$ . Then  $(F, A)$  is called a preimage of  $(G, V)$  under  $f$  and is denoted by  $f^{-1}((G, V))$ .

**Definition 2.6.** Let  $(F, A)$  be an anti S fuzzy soft subhemiring of a hemiring  $(R, +, \cdot)$  and  $a$  in  $R$ . Then the pseudo anti S fuzzy soft coset  $(a(F, A))^p$  is defined by  $((a\mu_{(F,A)})^p)(x) = p^{(a)}\mu_{(F,A)}(x)$ , for every  $x$  in  $R$ , and for some  $p$  in  $P$ .

### 3 Properties of anti S-fuzzy soft subhemiring of hemiring

**Theorem 3.1.** Union of any two anti S fuzzy soft subhemiring of a hemiring  $R$  is an anti S fuzzy soft subhemiring of  $R$ .

*Proof.* Let  $(F, A)$  and  $(G, B)$  be any two S-fuzzy soft subhemirings of a hemiring  $R$  and  $x$  and  $y$  in  $R$ . Let  $(F, A) = \{ \langle (x), \mu_{(F,A)}(x) \rangle / x \in R \}$  and  $(G, B) = \{ \langle (x), \mu_{(G,B)}(x) \rangle / x \in R \}$  and also let  $(H, C) = (F, A) \cup (G, B) = \{ \langle (x), \mu_{(H,C)}(x) \rangle / x \in R \}$ , where  $\max \{ \mu_{(F,A)}(x), \mu_{(G,B)}(x) \} = \mu_{(H,C)}(x)$ ,

now,  $\mu_{(H,C)}(x + y, q) \leq \max \{ S(\mu_{(F,A)}(x), \mu_{(F,A)}(y)), S(\mu_{(G,B)}(x), \mu_{(G,B)}(y)) \} \leq S(\mu_{(H,C)}(x), \mu_{(H,C)}(y))$ .

Therefore,  $\mu_{(H,C)}(x + y) \leq S(\mu_{(H,C)}(x), \mu_{(H,C)}(y))$ , for all  $x$  and  $y$  in  $R$ . And,  $\mu_{(H,C)}(xy) \leq \max \{ S(\mu_{(F,A)}(x), \mu_{(F,A)}(y)), S(\mu_{(G,B)}(x), \mu_{(G,B)}(y)) \} \leq S(\mu_{(H,C)}(x), \mu_{(H,C)}(y))$ . Therefore,  $\mu_{(H,C)}(xy) \leq S(\mu_{(H,C)}(x), \mu_{(H,C)}(y))$ , for all  $x$  and  $y$  in  $R$ . Therefore  $(H, C)$  is an an anti S-fuzzy soft subhemiring of a hemiring  $R$ . □

**Theorem 3.2.** The Union of a family of anti S-fuzzy soft subhemirings of hemiring  $R$  is an anti S-fuzzy soft subhemiring of  $R$ .

*Proof.* It is trivial. □

**Theorem 3.3.** If  $(F, A)$  and  $(F, B)$  are two anti S-fuzzy soft subhemirings of the hemirings  $R_1$  and  $R_2$  respectively, then anti product  $(F, A \times B)$  is an anti S-fuzzy soft subhemiring of  $R_1 \times R_2$ .

*Proof.* Let  $(F, A)$  and  $(G, B)$  be two anti S-fuzzy subhemirings of the hemirings  $R_1$  and  $R_2$  respectively. Let  $x_1$  and  $x_2$  be in  $R_1$ ,  $y_1$  and  $y_2$  in  $R_2$ . Then  $(x_1, y_1)$  and  $(x_2, y_2)$  are in  $R_1 \times R_2$ . Now,

$$\begin{aligned} &\mu_{(F,A \times B)}[(x_1, y_1) + (x_2, y_2)] \\ &\leq \max \{ S(\mu_{(F,A)}(x_1), \mu_{(F,A)}(x_2)), S(\mu_{(F,B)}(y_1), \mu_{(F,B)}(y_2)) \} \\ &\leq S(\mu_{(F,A \times B)}(x_1, y_1), \mu_{(F,A \times B)}(x_2, y_2)). \end{aligned}$$

Therefore,

$$\mu_{(F,A \times B)}[(x_1, y_1) + (x_2, y_2)] \leq S(\mu_{(F,A \times B)}(x_1, y_1), \mu_{(F,A \times B)}(x_2, y_2)).$$

Also,

$$\begin{aligned} & \mu_{(F,A \times B)}[(x_1, y_1)(x_2, y_2)] \\ & \leq \max \{S(\mu_{(F,A)}(x_1), \mu_{(F,A)}(x_2)), S(\mu_{(F,B)}(y_1), \mu_{(F,B)}(y_2))\} \\ & \leq S(\mu_{(F,A \times B)}(x_1, y_1), \mu_{(F,A \times B)}(x_2, y_2)). \end{aligned}$$

Therefore,

$$\mu_{(F,A \times B)}[(x_1, y_1)(x_2, y_2)] \leq S(\mu_{(F,A \times B)}(x_1, y_1), \mu_{(F,A \times B)}(x_2, y_2)).$$

Hence  $(F, A \times B)$  is an anti S-fuzzy soft subhemiring of a hemiring  $R_1 \times R_2$ .  $\square$

**Theorem 3.4.** Let  $(F, A)$  be a fuzzy soft subset of a hemiring  $R$  and  $(G, V)$  be the anti strongest fuzzy relation of  $R$ . Then  $(F, A)$  is an anti S fuzzy soft subhemiring of  $R$  if and only if  $(G, V)$  is an anti S-fuzzy soft sub hemiring of  $R \times R$ .

*Proof.* Suppose that  $(F, A)$  is an anti S-fuzzy soft subhemiring of a hemiring  $R$ . Then for any  $X = (x_1, x_2)$  and  $Y = (y_1, y_2)$  are in  $R \times R$ , We have

$$\begin{aligned} \mu_{(G,V)} & \leq \max \{S(\mu_{(F,A)}(x_1, y_1), \mu_{(F,A)}(x_2, y_2))\} \\ & \leq \max \{S(\mu_{(F,A)}(x_1), \mu_{(F,A)}(y_1)), S(\mu_{(F,A)}(x_2), \mu_{(F,A)}(y_2))\} \\ & \leq S(\mu_{(G,V)}((x_1, x_2)), \mu_{(G,V)}((y_1, y_2))) \\ & \leq S(\mu_{(G,V)}(X), \mu_{(G,V)}(Y)), \text{ for all } X \text{ and } Y \text{ in } R \times R. \end{aligned}$$

Therefore,  $\mu_{(G,V)}(XY) \leq S(\mu_{(G,V)}(X), \mu_{(G,V)}(Y))$ , for all  $X$  and  $Y$  in  $R \times R$ . This proves that  $(G, V)$  is an anti S fuzzy soft subhemiring of a hemiring of  $R \times R$ . Conversely assume that  $(G, V)$  is an anti S-fuzzy soft subhemiring of a hemiring of  $R \times R$ , then for any  $X = (x_1, x_2)$  and  $Y = (y_1, y_2)$  are in  $R \times R$ . We have

$$\begin{aligned} & \max \{S(\mu_{(F,A)}(x_1 + y_1), \mu_{(F,A)}(x_2 + y_2))\} \\ & = \mu_{(G,V)}(X + Y) \\ & \leq S(\mu_{(G,V)}(X), \mu_{(G,V)}(Y)) \\ & = S(\mu_{(G,V)}((x_1, y_1)), \mu_{(G,V)}((y_1, y_2))) \\ & = S(\max(\mu_{(F,A)}(x_1), \mu_{(F,A)}(y_1)), \max(\mu_{(F,A)}(x_2), \mu_{(F,A)}(y_2))). \end{aligned}$$

If  $X_2 = 0, y_2 = 0$ , we get  $\mu_{(F,A)}(x_1 + y_1) \leq S(\mu_{(F,A)}(x_1), \mu_{(F,A)}(y_1))$  for all  $x_1$  and  $y_1$  in  $R$ . And

$$\begin{aligned} & \max \{S(\mu_{(F,A)}(x_1 y_1), \mu_{(F,A)}(x_2 y_2))\} \\ & = \mu_{(G,V)}(x, y) \\ & \leq S(\mu_{(G,V)}(X), \mu_{(G,V)}(Y)) \\ & = S(\mu_{(G,V)}((x_1, y_1)), \mu_{(G,V)}((y_1, y_2))) \\ & = S(\max\{\mu_{(F,A)}(x_1), \mu_{(F,A)}(x_2)\}, \max\{\mu_{(F,A)}(y_1), \mu_{(F,A)}(y_2)\}). \end{aligned}$$

If  $x_2 = 0, y_2 = 0$ . We get  $\mu_{(F,A)}(x_1 y_1) \leq S(\mu_{(F,A)}(x_1), \mu_{(F,A)}(y_1))$  for all  $x_1$  and  $y_1$  in  $R$ . Therefore  $(F, A)$  is an anti S-fuzzy soft subhemiring of  $R$ .  $\square$

**Theorem 3.5.** If  $(F, A)$  is an anti S-fuzzy soft subhemiring of a hemiring  $(R, +, \cdot)$  if and only if  $\mu_{(F,A)}(x + y) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y), \mu_{(F,A)}(xy)) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$  for all  $x$  and  $y$  in  $R$ .

*Proof.* It is trivial.  $\square$

**Theorem 3.6.** *If  $(F, A)$  is an anti S-fuzzy soft subhemiring of a hemiring  $(R, +, \cdot)$ , then  $H = \{x/x \in R : \mu_{(F,A)}(x) = 0\}$  is either empty or is a subhemiring of  $R$ .*

*Proof.* It is trivial. □

**Theorem 3.7.** *If  $(F, A)$  is an anti S-fuzzy soft subhemiring of a hemiring  $(R, +, \cdot)$ . If  $\mu_{(F,A)}(x + y) = 1$ , then either  $\mu_{(F,A)}(x) = 1$  or  $\mu_{(F,A)}(y) = 1$ , for all  $x$  and  $y$  in  $R$ .*

*Proof.* It is trivial. □

**Theorem 3.8.** *If  $(F, A)$  is an anti S-fuzzy soft subhemiring of a hemiring  $(R, +, \cdot)$ , then the pseudo anti S-fuzzy coset  $(a(F, A))^p$  is an anti S-soft subhemiring of a hemiring  $R$ , for every  $a$  in  $R$ .*

*Proof.* Let  $(F, A)$  is an anti S-fuzzy subhemiring of a hemiring  $R$ . For every  $x$  and  $y$  in  $R$ , we have  $((a\mu_{(F,A)})^p)(x + y) \leq p(a)S(\mu_{(F,A)}(x), \mu_{(F,A)}(y)) \in S(p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y)) = S((a\mu_{(F,A)})^p(x), (a\mu_{(F,A)})^p(y))$ . Therefore,  $((a\mu_{(F,A)})^p)(x + y) \leq S((a\mu_{(F,A)})^p(x), (a\mu_{(F,A)})^p(y))$ . Now,  $((a\mu_{(F,A)})^p)(xy) \leq p(a)S(\mu_{(F,A)}(x), \mu_{(F,A)}(y)) \in S(p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y)) = S((a\mu_{(F,A)})^p(x), (a\mu_{(F,A)})^p(y))$ . Therefore,  $((a\mu_{(F,A)})^p)(xy) \leq S((a\mu_{(F,A)})^p(x), (a\mu_{(F,A)})^p(y))$ . Hence  $(a(F, A))^p$  is an anti S-fuzzy soft subhemiring of a hemiring  $R$ . □

**Theorem 3.9.** *Let  $(R, +, \cdot)$  and  $(R', +, \cdot)$  be any two hemi rings. The homomorphic image of an anti S-fuzzy soft subhemiring of  $R$  is an anti S-fuzzy soft subhemiring of  $R'$ .*

*Proof.* Let  $f : R \rightarrow R'$  be a homomorphism. Then  $f(x + y) = f(x) + f(y)$  and  $f(xy) = f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $(G, V) = f((F, A))$ , where  $(F, A)$  is an anti S-fuzzy soft subhemiring of  $R$ . Now, for  $f(x), f(y)$  in  $R'$ ,  $\mu_{(G,V)}((f(x)) + (f(y))) \leq \mu_{(F,A)}(x + y) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(G,V)}((f(x)) + (f(y))) \leq S(\mu_{(G,V)}((f(x))), \mu_{(G,V)}((f(y))))$ . Again,  $\mu_{(G,V)}((f(x))(f(y))) \leq \mu_{(F,A)}(xy) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ . Hence  $(G, V)$  is an anti S-fuzzy soft subhemiring of hemiring  $R'$ . □

**Theorem 3.10.** *Let  $(R, +, \cdot)$  and  $(R', +, \cdot)$  be any two hemirings. The homomorphic preimage of an anti S-fuzzy soft subhemiring of  $R'$  is an anti S-fuzzy soft subhemiring of  $R$ .*

*Proof.* Let  $f : R \rightarrow R'$  be a homomorphism. Then,  $f(x + y) = f(x) + f(y)$  and  $f(xy) = f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $(G, V) = f((F, A))$ , where  $(G, V)$  is an anti S- fuzzy soft subhemiring of  $R'$ . Now, for all  $x, y$  in  $R$ ,  $\mu_{(F,A)}((x) + (y)) = \mu_{(G,V)}((f(x)) + (f(y))) \leq S(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y))) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(F,A)}((x) + (y)) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ . Again,  $\mu_{(F,A)}((x)(y)) = \mu_{(G,V)}(f(x)f(y)) \leq S(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y))) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(F,A)}((x)(y)) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ . Hence  $(F, A)$  is an anti S fuzzy soft subhemiring of hemiring  $R$ . □

**Theorem 3.11.** *Let  $(R, +, \cdot)$  and  $(R', +, \cdot)$  be any two hemirings. The anti-homomorphic image of an anti S-fuzzy soft subhemiring of  $R$  is an anti S-fuzzy soft subhemiring of  $R'$ .*

*Proof.* Let  $f : R \rightarrow R'$  be a homomorphism. Then,  $f(x + y) = f(x) + f(y)$  and  $f(xy) = f(x)f(y)$ , for all  $x$  and  $y$  in  $R$ . Let  $(G, V) = f((F, A))$ , where  $(F, A)$  is an anti S- fuzzy soft subhemiring of  $R$ .  $\mu_{(G,V)}((f(x)) + (f(y))) \leq \mu_{(F,A)}(y + x) \leq S(\mu_{(F,A)}(y), \mu_{(F,A)}(x)) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(G,V)}((f(x)) + (f(y))) \leq S(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y)))$ . Again,  $\mu_{(G,V)}((f(x))(f(y))) \leq \mu_{(F,A)}(yx) \leq S(\mu_{(F,A)}(y), \mu_{(F,A)}(x)) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(G,V)}((f(x))(f(y))) \leq S(\mu_{(G,V)}(f(x)), \mu_{(G,V)}(f(y)))$ . Hence  $(G, V)$  is an anti S fuzzy soft subhemiring of hemiring  $R'$ . □

**Theorem 3.12.** *Let  $(R, +, \cdot)$  and  $(R', +, \cdot)$  be any two hemirings. The anti-homomorphic preimage of an anti S-fuzzy soft subhemiring of  $R'$  is an anti S-fuzzy soft subhemiring of  $R$ .*

*Proof.* Let  $(G, V) = f((F, A))$  where  $(G, V)$  is an anti S fuzzy subhemiring of  $R'$ . Let  $x$  and  $y$  in  $R$ . Then  $\mu_{(F,A)}((x) + (y)) = \mu_{(G,V)}((f(x)) + (f(y))) \leq S(\mu_{(G,V)}(f(y)), \mu_{(G,V)}(f(x))) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that,  $\mu_{(F,A)}((x) + (y)) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ . Again  $\mu_{(F,A)}((x)(y)) = \mu_{(G,V)}((f(x))(f(y))) \leq S(\mu_{(G,V)}(f(y)), \mu_{(G,V)}(f(x))) = S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ , which implies that  $\mu_{(F,A)}((x)(y)) \leq S(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$ . Hence  $(F, A)$  is an anti S fuzzy soft subhemiring of hemiring  $R$ . □

In the following Theorem  $\circ$  is the composition operation of functions:

**Theorem 3.13.** *Let  $(F, A)$  be an anti S-fuzzy soft subhemiring of hemiring  $H$  and  $f$  is an isomorphism from a hemiring  $R$  onto  $H$ . Then  $(F, A) \circ f$  is an anti S-fuzzy soft subhemiring of  $R$ .*

*Proof.* Let  $x$  and  $y$  in  $R$ . Then we have,  $(\mu_{(F,A)} \circ f)((x) + (y)) = \mu_{(F,A)}((f(x)) + (f(y))) \leq S(\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ , which implies that  $(\mu_{(F,A)} \circ f)((x) + (y)) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ . And  $(\mu_{(F,A)} \circ f)((x)(y)) = \mu_{(F,A)}((f(x))(f(y))) \leq S(\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ , which implies that  $(\mu_{(F,A)} \circ f)((x)(y)) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ . Therefore  $(F, A) \circ f$  is an anti S fuzzy soft subhemiring of hemiring  $R$ . □

**Theorem 3.14.** *Let  $(F, A)$  be an anti S-fuzzy soft subhemiring of hemiring  $H$  and  $f$  is an anti-isomorphism from a hemiring  $R$  onto  $H$ . Then  $(F, A) \circ f$  is an anti S-fuzzy soft subhemiring of  $R$ .*

*Proof.* Let  $x$  and  $y$  in  $R$ . Then we have,  $(\mu_{(F,A)} \circ f)((x) + (y)) = \mu_{(F,A)}((f(y)) + (f(x))) \leq S(\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ , which implies that  $(\mu_{(F,A)} \circ f)((x) + (y)) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ . And  $(\mu_{(F,A)} \circ f)((x)(y)) = \mu_{(F,A)}((f(y))(f(x))) \leq S(\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ , which implies that  $(\mu_{(F,A)} \circ f)((x)(y)) \leq S((\mu_{(F,A)} \circ f)(x), (\mu_{(F,A)} \circ f)(y))$ . Therefore  $(F, A) \circ f$  is an anti S fuzzy soft subhemiring of hemiring  $R$ . □

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