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Intuitionistic Q-fuzzy ternary subhemiring of a hemiring

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Abstract

In this paper, a generalized intuitionistic Q-fuzzy ternary subhemiring of a hemiring is proposed. Further, some important notions and basic algebraic properties of intuitionistic fuzzy sets are discussed.

Keywords: Q-fuzzy subhemiring, Q-fuzzy ternary subhemiring, intuitionistic fuzzy ternary subhemiring, homomorphism, anti-homomorphism.

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1 Introduction

An algebra $(R;+;\cdot)$ is said to be a semiring if $(R;+)$ and $(R;\cdot)$ are semigroups satisfying $a.(b+c) = a.b + a.c$ and $(b+c).a = b.a + c.a$ for all a, b and c in R. A Semiring R is said to be additively commutative if $a+b = b+a$ for all a and b in R. Ternary rings are introduced by Lister [9]. And he investigated some of their properties and radical theory of such rings. A Semiring R may have an identity 1, defined by $1.a = a = a.1$ and a zero 0, defined by $0 + a = a = a + 0$ and $a.0 = 0 = 0.a$ for all a in R. Ternary semirings arise naturally as follows-consider the ring of integers Z which plays a vital role in the theory of ring. The concept of intuitionistic fuzzy subsets (IFS) was presented by K.T.Atanassov [5], as a generalization of the notion of fuzzy set. Solairaju.A and R.Nagarajan, have given a new structure in construction of Q-fuzzy groups [14]. Also Giri.R.D and Chide.B.R [8], given the structure of Prime Radical in Ternary Hemiring. In this paper, we introduce some properties and theorems in intuitionistic Q-fuzzy ternary subhemiring of a hemiring.

2 Preliminaries

Definition 2.1. *Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is function* $A : X \times Q \to [0,1]$.

Definition 2.2. *Let R be a hemiring. A fuzzy subset A of R is said to be a Q-fuzzy ternary subhemiring (FTSHR) of R if it satisfies the following conditions:*

(i) $A(x+y,q) \geq \min\{A(x,q), A(y,q)\}$,

(ii) $A(xyz,q) \geq \min\{A(x,q), A(y,q), A(z,q)\}$, *for all x, y and z in R and q in Q.*

Definition 2.3. *Let R be a hemiring. A Q-fuzzy subset A of R is said to be an anti Q-fuzzy subhemiring (AFTSHR) of R if it satisfies the following conditions:*

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(i) $A(x+y,q) \leq \max\{A(x,q), A(y,q)\}$,

(ii) $A(xyz,q) \leq \max\{A(x,q), A(y,q), A(z,q)\}$, *for all* x, y *and* z *in* R *and* q *in* Q.

Definition 2.4. *An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form* $A = \{< x, \mu_A(x), \mu_A(x) > / x \in X\}$, *where* $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *define the degree of membership and the degree of non-membership of the element* $x \in X$ *respectively and for every* $x \in X$ *satisfying* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.5. *Let R be a hemiring. An intuitionistic Q-fuzzy subset A of R is said to be an intuitionistic Q-fuzzy ternary subhemiring (IFTSHR) of R if it satisfies the following conditions:*

(i) $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$,

(ii) $\mu_A(xyz,q) \geq \min\{\mu_A(x,q), \mu_A(y,q), \mu_A(z,q)\}$,

(i) $\nu_A(x+y,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}$,

(ii) $\nu_A(xyz,q) \leq \max\{\nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\}$, *for all* x, y *and* z *in* R *and* q *in* Q.

Definition 2.6. *Let* $(R, +, \cdot)$ *and* $(R', +, \cdot)$ *be any two hemirings. Then the function* $f : R \to R'$ *is called a homomorphism if* $f(x+y,q) = f(x,q) + f(y,q)$ *and* $f(xyz,q) = f(x,q)f(y,q)f(z,q)$, *for all* x, y *and* z *in* R *and* q *in* Q.

Definition 2.7. *Let* $(R, +, \cdot)$ *and* $(R', +, \cdot)$ *be any two hemirings. Then the function* $f : R \to R'$ *is called an anti-homomorphism if* $f(x+y,q) = f(y,q) + f(x,q)$ *and* $f(xyz,q) = f(z,q)f(y,q)f(x,q)$, *for all* x, y *and* z *in* R *and* q *in* Q.

Definition 2.8. *Let* $(R, +, \cdot)$ *and* $(R', +, \cdot)$ *be any two hemirings. Then the function* $f : R \to R'$ *is called an isomorphism if* f *is bijection.*

Definition 2.9. *Let* $(R, +, \cdot)$ *and* $(R', +, \cdot)$ *be any two hemirings. Then the function* $f : R \to R'$ *is called an anti-isomorphism if* f *is bijection.*

3 Some properties of intuitionistic Q-fuzzy ternary subhemiring of a hemiring

Theorem 3.1. *If A is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring* $(R, +, \cdot)$, *then* $H = \{(x,q)/x \in R : \mu_A(x,q) = 1, \nu_A(x,q) = 0\}$ *is either empty or is a ternary subhemiring of R.*

Proof. If none of the elements satisfies this condition, then H is empty. If (x,q) and (y,q) in H, then $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\} = \min\{1,1\} = 1$. Therefore $\mu_A(x+y,q) = 1$, for all (x,q) and (y,q) in H. And $\mu_A(xyz,q) \geq \min\{\mu_A(x,q), \mu_A(y,q), \mu_A(z,q)\} = \min\{1,1,1\} = 1$. Therefore $\mu_A(xyz,q) = 1$, for all $(x,q), (y,q)$ and (z,q) in H. And $\nu_A(x+y,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\} = \max\{0,0\} = 0$. Therefore $\nu_A(x+y,q) = 0$, for all (x,q) and (y,q) in H. And $\nu_A(xyz,q) \leq \max\{\nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\} = \max\{0,0,0\} = 0$. Therefore $\nu_A(xyz,q) = 0$, for all $(x,q), (y,q)$ and (z,q) in H. Therefore H is a ternary subhemiring of R. Hence H is either empty or is a ternary subhemiring of R. □

Theorem 3.2. *If A is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring* $(R, +, \cdot)$, *then* $H = \{\langle (x,q), \mu_A(x,q)\rangle : 0 < \mu_A(x,q) \leq 1 \, and \, \nu_A(x,q) = 0\}$ *is either empty or is a Q-fuzzy ternary subhemiring of R.*

Proof. By using Theorem 3.1. □

Theorem 3.3. *If A is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring* $(R, +, \cdot)$, *then* $H = \{\langle (x,q), \mu_A(x,q)\rangle : 0 < \mu_A(x,q) \leq 1\}$ *is either empty or is a Q-fuzzy ternary subhemiring of R.*

Proof. By using Theorem.3.2. □

Theorem 3.4. *If A is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring $(R,+,\cdot)$, then A is an intuitionistic Q-fuzzy ternary subhemiring of R.*

Proof. Let A be an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. Consider $A = \{ \langle (x,q), \mu_A(x,q), \nu_A(x,q) \rangle \}$, for all x in R. $A = B = \{ \langle (x,q), \mu_B(x,q), \nu_B(x,q) \rangle \}$, where $\mu_B(x,q) = \mu_A(x,q), \nu_B(x,q) = 1 - \mu_A(x,q)$. clearly, $\mu_B(x+y,q) \geq \min\{\mu_B(x,q), \mu_B(y,q)\}$ and $\mu_B(xyz,q) \geq \min\{\mu_B(x,q), \mu_B(y,q), \mu_B(z,q)\}$. Since A is an intuitionistic Q-fuzzy ternary subhemiring of R, we have, $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$ for all x and y in R, which implies that $1 - \nu_B(x+y,q) \geq \min\{(1-\nu_B(x,q)),(1-\nu_B(y,q))\}$ which implies that $\nu_B(x+y,q) \leq 1 - \min\{(1-\nu_B(x,q)),(1-\nu_B(y,q))\} = \max\{\nu_B(x,q), \nu_B(y,q)\}$. Therefore, $\nu_B(x+y,q) \leq \max\{\nu_B(x,q), \nu_B(y,q)\}$ for all x and y in R and q in Q. And $\mu_A(xyz,q) \geq \min\{\mu_A(x,q), \mu_A(y,q), \mu_A(z,q)\}$ which implies that $1 - \nu_B(xyz,q) \geq \min\{(1-\nu_B(x,q)),(1-\nu_B(y,q)),(1-\nu_B(z,q))\}$ which implies that $\nu_B(xyz,q) \leq 1 - \min\{(1-\nu_B(x,q)),(1-\nu_B(y,q)),(1-\nu_B(z,q))\} = \max\{\nu_B(x,q), \nu_B(y,q), \nu_B(z,q)\}$. Therefore $\nu_B(xyz,q) \leq \max\{\nu_B(x,q), \nu_B(y,q), \nu_B(z,q)\}$, for all x, y and z in R. Hence $B = A$ is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. □

Remark 3.1. *The converse of the above theorem is not true. It is shown by the following example: Consider the hemiring $Z_5 = \{0,1,2,3,4\}$ with addition modulo and multiplicative modulo operation and $Q = \{p\}$. Then $A = \{ (\langle 0,0.7,0.2\rangle, p), (\langle 1,0.5,0.1\rangle, p), (\langle 2,0.5,0.4\rangle, p), (\langle 3,0.5,0.1\rangle, p), (\langle 4,0.5,0.4\rangle, p) \}$ is not an intuitionistic Q-fuzzy ternary subhemiring of Z_5, but $A = \{ (\langle 0,0.7,0.3\rangle, p), (\langle 1,0.5,0.5\rangle, p), (\langle 2,0.5,0.5\rangle, p), (\langle 3,0.5,0.5\rangle, p), (\langle 4,0.5,0.5\rangle, p) \}$ is an intuitionistic Q-fuzzy ternary subhemiring of Z_5.*

Theorem 3.5. *If A is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring $(R,+,\cdot)$, then A is an intuitionistic Q-fuzzy ternary subhemiring of R.*

Proof. Let A be an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. That is $A = \{ \langle (x,q), \mu_A(x,q)\nu_A(x,q) \rangle \}$, for all x in R and q in Q. Let $A = B = \{ \langle (x,q), \mu_B(x,q), \nu_B(x,q) \rangle \}$, where $\mu_B(x,q) = 1 - \nu_A(x,q), \nu_B(x,q) = \nu_A(x,q)$. clearly, $\nu_B(x+y,q) \leq \max\{\nu_B(x,q), \nu_B(y,q)\}$ and $\nu_B(xyz,q) \leq \max\{\nu_B(x,q), \nu_B(y,q), \nu_B(z,q)\}$ for all x, y and z in R. Since A is an intuitionistic Q-fuzzy ternary subhemiring of R, we have, $\nu_A(x+y,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}$ for all x and y in R, which implies that $1 - \mu_B(x+y,q) \leq \max\{(1-\mu_B(x,q)),(1-\mu_B(y,q))\}$ which implies that $\mu_B(x+y,q) \geq 1 - \max\{(1-\mu_B(x,q)),(1-\mu_B(y,q))\} = \min\{\mu_B(x,q), \mu_B(y,q)\}$. Therefore, $\mu_B(x+y,q) \geq \min\{(\mu_B(x,q)),(\mu_B(y,q))\}$ for all x and y in R and q in Q. And $\nu_A(xyz,q) \leq \max\{\nu_A(x,q), \nu_A(y,q), \nu_A(z,q)\}$ which implies that $1 - \mu_B(xyz,q) \leq \max\{(1-\mu_B(x,q)),(1-\mu_B(y,q)),(1-\nu_B(z,q))\}$ which implies that $\mu_B(xyz,q) \geq 1 - \max\{(1-\mu_B(x,q)),(1-\mu_B(y,q)),(1-\mu_B(z,q))\} = \min\{\mu_B(x,q), \mu_B(y,q), \mu_B(z,q)\}$. Therefore $\mu_B(xyz,q) \geq \min\{\mu_B(x,q), \mu_B(y,q), \mu_B(z,q)\}$, for all x, y and z in R. Hence $B = A$ is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. □

Remark 3.2. *The converse of the above theorem is not true. It is shown by the following example: Consider the hemiring $Z_5 = \{0,1,2,3,4\}$ with addition modulo and multiplicative modulo operation and $Q = \{p\}$. Then $A = \{ (\langle 0,0.5,0.1\rangle, p), (\langle 1,0.6,0.4\rangle, p), (\langle 2,0.5,0.4\rangle, p), (\langle 3,0.6,0.4\rangle, p), (\langle 4,0.5,0.4\rangle, p) \}$ is not an intuitionistic Q-fuzzy ternary subhemiring of Z_5, but $A = \{ (\langle 0,0.9,0.1\rangle, p), (\langle 1,0.6,0.4\rangle, p), (\langle 2,0.6,0.4\rangle, p), (\langle 3,0.6,0.4\rangle, p), (\langle 4,0.6,0.4\rangle, p) \}$ is an intuitionistic Q-fuzzy ternary subhemiring of Z_5.*

In The Following Theorem $\circ$ Is The Composition Operation of Functions:

Theorem 3.6. *Let A be an intuitionistic Q-fuzzy ternary subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then $A \circ f$ is an intuitionistic Q-fuzzy ternary subhemiring of R.*

Proof. Let x and y in R and A be an intuitionistic Q-fuzzy ternary subhemiring H. Then we have $(\mu_A \circ f)(x+y,q) = \mu_A(f(x+y,q)) = \mu_A(f(x,q)+f(y,q)) \geq \min\{\mu_A(f(x,q)), \mu_A(f(y,q))\}$ (as A is an IFTSHR of H) $\geq \min\{(\mu_A \circ f)(x,q), (\mu_A \circ f)(y,q)\}$ which implies that $(\mu_A \circ f)(x+y,q) \geq \{(\mu_A \circ f)(x,q), (\mu_A \circ f)(y,q)\}$, for all x and y in R and q in Q. And $(\mu_A \circ f)(xyz,q) = \mu_A(f(xyz,q)) = \mu_A(f(x,q)f(y,q)f(z,q))$, as f is an isomorphism $\geq \min\{\mu_A(f(x,q)), \mu_A(f(y,q)), \mu_A(f(z,q))\}$, as A is an IFTSHR of H $\geq \min\{(\mu_A \circ f)(x,q), (\mu_A \circ f)(y,q)\}$ which implies that $(\mu_A \circ f)(xyz,q) \geq \min\{(\mu_A \circ$

$f)(x,q),(\mu_A \circ f)(y,q),(\mu_A \circ f)(z,q)\}$, for all x, y and z in R. We have $(\nu_A \circ f)(x+y,q) = \nu_A(f(x+y,q)) = \nu_A(f(x,q)+f(y,q))$, as f is an isomorphism $\leq \max\{\nu_A(f(x,q)),\nu_A(f(y,q))\} \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q)\}$ which implies that $(\nu_A \circ f)(x+y,q) \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q)\}$, for all x and y in R. And $(\nu_A \circ f)(xyz,q) = \nu_A(f(xyz,q)) = \nu_A(f(x,q)f(y,q)f(z,q))$, as f is an isomorphism $\leq \max\{\nu_A(f(x,q)),\nu_A(f(y,q)),\nu_A(f(z,q))\} \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q),(\nu_A \circ f)(z,q)\}$ which implies that $(\nu_A \circ f)(xyz,q) \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q),(\nu_A \circ f)(z,q)\}$ for all x, y and z in R. Therefore $(A \circ f)$ is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. □

Theorem 3.7. *Let A be an intuitionistic Q-fuzzy ternary subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then $A \circ f$ is an intuitionistic Q-fuzzy ternary subhemiring of R.*

Proof. Let x and y in R and A be an intuitionistic Q-fuzzy ternary subhemiring H. Then we have $(\mu_A \circ f)(x+y,q) = \mu_A(f(x+y,q)) = \mu_A(f(y,q)+f(x,q))$, as f is an anti-homomorphism $\geq \min\{\mu_A(f(y,q)),\mu_A(f(x,q))\}$ as A is an IFTSHR of $H \geq \min\{(\mu_A \circ f)(x,q),(\mu_A \circ f)(y,q)\}$ which implies that $(\mu_A \circ f)(x+y,q) \geq \min\{(\mu_A \circ f)(x,q),(\mu_A \circ f)(y,q)\}$, for all x and y in R. And $(\mu_A \circ f)(xyz,q) = \mu_A(f(xyz,q)) = \mu_A(f(z,q)f(y,q)f(x,q))$, as f is an anti-isomorphism $\geq \min\{\mu_A(f(z,q)),\mu_A(f(y,q)),\mu_A(f(x,q))\}$, as A is an IFTSHR of $H \geq \min\{(\mu_A \circ f)(x,q),(\mu_A \circ f)(y,q)\}$ which implies that $(\mu_A \circ f)(xyz,q) \geq \min\{(\mu_A \circ f)(x,q),(\mu_A \circ f)(y,q),(\mu_A \circ f)(z,q)\}$ for all x, y and z in R. We have $(\nu_A \circ f)(x+y,q) = \nu_A(f(x+y,q)) = \nu_A(f(y,q)+f(x,q))$, as f is an anti-isomorphism $\leq \max\{\nu_A(f(y,q)),\nu_A(f(x,q))\} \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q)\}$ which implies that $(\nu_A \circ f)(x+y,q) \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q)\}$, for all x and y in R. And $(\nu_A \circ f)(xyz,q) = \nu_A(f(xyz,q)) = \nu_A(f(z,q)f(y,q)f(x,q))$, as f is an anti-isomorphism $\leq \max\{\nu_A(f(z,q)),\nu_A(f(y,q)),\nu_A(f(x,q))\} \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q),(\nu_A \circ f)(z,q)\}$ which implies that $(\nu_A \circ f)(xyz,q) \leq \max\{(\nu_A \circ f)(x,q),(\nu_A \circ f)(y,q),(\nu_A \circ f)(z,q)\}$ for all x, y and z in R and q in Q. Therefore $(A \circ f)$ is an intuitionistic Q-fuzzy ternary subhemiring of a hemiring R. □

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