Malaya Journal of Matematik, Vol. 5, No. 4, 660-666, 2017
https://doi.org/10.26637/MJM0504/0008

Harvesting model for fishery resource with reserve area of bird predator and modified effort function

Y. Louartassi[1]* and J. El Alami[2] and N. Elalami[3]

Abstract
The purpose of this paper is to study the dynamics of fisheries resources with reserve area, in the presence of bird predators. Aquatic area under investigation is divided into two areas : one for the free fishing and other limited for any type of fishing. In the project harvesting system according to the modified E effort is considered, which depends on the effect of resource density. The local and stability criteria, the overall stability and instability are established for the project model. Finally, the theoretical results are illustrated by numerical simulations in the last section.

Keywords
Fichery effort, Stability, Harvesting.

AMS Subject Classification
92D25.

[1] *Mohammed V University in Rabat, Superior School of Technology Salé, LASTIMI, Salé, Morocco.*
[2] *Mohammed V University in Rabat, Superior School of Technology Salé, LASTIMI, Salé, Morocco.*
[3] *Mohammed V University in Rabat, Mohammadia School of Engineers, Rabat, Morocco.*
***Corresponding author**: ylouartassi@gmail.com; [2]alamijamila1@gmail.com [3]prelalami@gmail.com
Article History: Received **07** June **2016**; Accepted **14** October **2017**
©201x MJM.

Contents

1. Introduction

Renewable resources are considered an important food source for the growth and survival of biological population. But the continued operation and unplanned these resources can lead to the extinction of resources and thus affecting the survival of species dependent on resources. For this, there has been a significant interest in modeling of renewable resources such as fisheries and forestry. Dynamic models for commercial fishing have been widely studied in the light of economic and ecological factors [6, 7]. In recent decades there has been considerable interest for modeling the dynamics of fisheries resources systems [2, 4, 5, 10, 15]. Chaudhuri [3] proposed a model for two species of fish competitors, each of which develops logistically. It examined the stability analysis and discussed the bionomic balance and optimal harvesting policy. It has also been shown [3] that there is no limit cycle in the positive quadrant. Fan and Wang [11] generalized the classical model of Clark [6, 7] by considering the time-dependent the Logistic equation with periodic coefficients and they showed that their model has a unique positive periodic solution, which is globally asymptotically stable for positive solutions. Dubey et al. [8] proposed a model where the fish population depends in part of a resource and is harvested. They examined the stability analysis. Dubey et al. [9] discussed a model of a fishery resource system in an aquatic environment has been divided into two areas : the free fishing area and the reserved area. They discussed the biological and bionomic balance and the optimal harvesting policy. Dubey et al. [10] also proposed and analyzed a shore-to-sea fishing model where the fish population is harvested in both zones. Then they studied the stability analysis and optimal harvest policy taking taxation as a control instrument. Louartassi et al. [16] has studied the coastal sea-fishing model where the fish population is harvested in two areas of Dubey et al. [10] where he propose a comprehensive state regulation by output feedback based on a Lyapunov function. Sharma et al. [19] proposed a study the dynamics of fishery resource with reserve area in the presence of bird predator. The aquatic region under investigation is divided into two zones : one free for fishing and another restricted for any kind of fishery. The criteria of biological and bionomic equilibrium of system are established. The points of local stability, global stability, and instability are obtained for the proposed model. An optimal harvesting policy is established using Pontryagin's maximum principle. The purpose of this paper is to study a mathematical model of fishery resource with reserve area in the presence of bird predator. In the proposed harvest model, a modified effort function E is considered that depends on the effect of the density of the biomass resource. Local stability criteria, global stability and instability are established for the proposed system. The obtained theoretical results are illustrated using numerical simulations in the last section.

2. The Model

By Sharma et al. [19], it is considered a fishery resource system consists of two zones : a free fishing and a reserve area where fishing is prohibited. Each zone is assumed to be homogeneous. There are also a bird predators feed on both, that is, fish reserved and restricted areas. It is assumed that the predator population is harvested in the area without reservation. We assume that the prey species migrate between the two areas in a random way. Growth prey in each zone in the absence of predator is assumed to be logistics. Taking these in order, the model becomes

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-m_1xz-q_1E_1x,\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y-m_2yz, \qquad (2.1)\\ \frac{dz}{dt} &= -dz+k_1m_1xz+k_2m_2yz-q_2E_2z. \end{aligned}$$

Here $x(t)$ and $y(t)$ are the respective biomass densities of the prey species inside the unreserved and reserved areas, respectively, at a time t, $z(t)$ is the biomass density of predator at time t; respectively; r and s are intrinsic growth rates of prey species inside the unreserved and reserved zones, respectively; K and L are the carrying capacities of prey species in the unreserved and reserved zones, respectively; σ_1 and σ_2 are migration rates from the unreserved area to reserved area and the reserved area to the unreserved area, respectively; d is death rate of predator; m_1 and m_2 are the capturing rates and k_1 and k_2 are the conversion rates of prey in unreserved and reserved zones, respectively; q_1 and q_2 are the catchability coefficient of prey and predator in unreserved zone, respectively; E_1 and E_2 are the efforts applied to harvest the fish population and predator in unreserved zone, respectively.
In this case the fishing effort E is simply taken as a function of t i.e. $E=E(t)$, which do not address the inverse effect of fish abundance on fishing effort. That is, it do not address the fact that higher the density of fishes, lesser the amount of effort needed to catch unit harvest. In order to overcome this deficiency, Idels et al. [14] proposed a modified effort function which is a function of t as well as x (respectively z) and is given by

$$E_1(t,x)=\alpha_1(t)-\beta_1(t)\frac{1}{x}\frac{dx}{dt}, \quad E_2(t,z)=\alpha_2(t)-\beta_2(t)\frac{1}{z}\frac{dz}{dt}, \qquad (2.2)$$

where $\alpha_i \geq 0, \beta_i \geq 0$ for $i=1,2$ are continuous functions of t. Incorporating (2.2), we get the following modified version of the model (2.1) as :

$$\begin{aligned} \frac{dx}{dt} &= \frac{r}{1-q_1\beta_1}x\left(1-\frac{x}{K}\right)-\frac{\sigma_1 x}{1-q_1\beta_1}+\frac{\sigma_2 y}{1-q_1\beta_1}\\ &- \frac{m_1xz}{1-q_1\beta_1}-\frac{q_1\alpha_1 x}{1-q_1\beta_1},\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y-m_2yz,\\ \frac{dz}{dt} &= \frac{-dz}{1-q_2\beta_2}+\frac{k_1m_1xz}{1-q_2\beta_2}+\frac{k_2m_2yz}{1-q_2\beta_2}-\frac{q_2\alpha_2 z}{1-q_2\beta_2}. \end{aligned} \qquad (2.3)$$

All the parameters are assumed to be positive. Here we observe that if there is no migration of fish population from the reserved area to the unreserved area (i.e., $\sigma_2=0$) and $r-\sigma_1-q_1\alpha_1<0$, then $\dot{x}<0$. Similarly, if there is no migration of fish population from the unreserved area to reserved area (i.e., $\sigma_1=0$) and $s-\sigma_2<0$, then $\dot{y}<0$. Hence, throughout our analysis, we assume that

$$r-\sigma_1-q_1\alpha_1>0,\ s-\sigma_2>0,\ 1-q_1\beta_1>0 \text{ and } 1-q_2\beta_2>0. \qquad (2.4)$$

To simplify the study of the proposed model in (2.3), we assume that catch rates are the same two reserves, which areas, $m_1=m_2=m$ and conversion rates of prey in unreserved and reserved zones are the same, that is, $k_1=k_2=k$. Thus, under these assumptions, the system (2.3) becomes

$$\begin{aligned} \frac{dx}{dt} &= \frac{r}{1-q_1\beta_1}x\left(1-\frac{x}{K}\right)-\frac{\sigma_1 x}{1-q_1\beta_1}+\frac{\sigma_2 y}{1-q_1\beta_1}\\ &- \frac{mxz}{1-q_1\beta_1}-\frac{q_1\alpha_1 x}{1-q_1\beta_1},\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y-myz,\\ \frac{dz}{dt} &= \frac{-dz}{1-q_2\beta_2}+\frac{\alpha\,(xz+yz)}{1-q_2\beta_2}-\frac{q_2\alpha_2 z}{1-q_2\beta_2}. \end{aligned} \qquad (2.5)$$

where $\alpha=km$.

First, the following Lemma says it is a region of attraction for the model system (2.5).

Lemma 2.1. *All the solutions of the system (2.5) which initiate in R^3_+ are uniformly bounded.*

Proof. We define a function $w(t) = x(t)+y(t)+z(t)$ and $\eta > 0$ be a constant.
Then, the time derivative of w along the solution of system (2.5) is

$$\begin{aligned}
& \frac{dw}{dt}+\eta w = \frac{dx}{dt}+\frac{dy}{dt}+\frac{dz}{dt}+\eta x+\eta y+\eta z,\\
= & -\tfrac{r}{K(1-q_1\beta_1)}\left(x-\tfrac{K}{2r}\left(r-q_1\alpha_1-q_1\beta_1\sigma_1+\eta\right)\right)^2\\
- & \tfrac{s}{L}\left(y-\tfrac{L}{2s}\left(s+\sigma_2+\eta\right)\right)^2\\
- & z\left(x\left(\tfrac{m}{1-q_1\beta_1}-\tfrac{\alpha}{1-q_2\beta_2}\right)+y\left(m-\tfrac{\alpha}{1-q_2\beta_2}\right)+\left(\tfrac{d}{1-q_2\beta_2}+\tfrac{q_2\alpha_2}{1-q_2\beta_2}-\eta\right)\right)\\
+ & \tfrac{K}{4r(1-q_1\beta_1)}\left(r-q_1\alpha_1-q_1\beta_1\sigma_1+\eta\right)^2+\tfrac{L}{4s}\left(s+\sigma_2+\eta\right)^2,\\
\leq & \tfrac{K}{4r(1-q_1\beta_1)}\left(r-q_1\alpha_1-q_1\beta_1\sigma_1+\eta\right)^2+\tfrac{L}{4s}\left(s+\sigma_2+\eta\right)^2\\
= & \mu.
\end{aligned} \tag{2.6}$$

By the thoery of differentiel inequality [1, 18], we have

$$0 < w(x(t),y(t),z(t)) \leq \frac{\eta}{\mu}\left(1-e^{-\eta t}\right)+w(x(0),y(0),z(0))e^{-\eta t}, \tag{2.7}$$

and for $t \to 0$, $0 < w \leq \mu/\eta$. This proves the lemma. □

3. Existence of Equilibria

The dynamical behavior of a system is studied at equilibrium points and equilibrium points of model (2.5) are obtained by solving $\dot{x} = \dot{y} = \dot{z} = 0$. This gives three possible steady states, namely, $P_0\,(0,0,0)$, $P_1\,(\bar{x},\bar{y},0)$ and $P^*\,(x^*,y^*,z^*)$.
At $P_0\,(0,0,0)$ the population is extinct and this equilibrium point always exists.
Now, consider the equilibrium point $P_1\,(\bar{x},\bar{y},0)$, where the predator is not present. Here $\bar{x}$ and $\bar{y}$ are the positive solutions of

$$\begin{aligned}
r\bar{x}\left(1-\frac{\bar{x}}{K}\right)-\sigma_1\bar{x}+\sigma_2\bar{y}-q_1\alpha_1\bar{x} &= 0,\\
s\bar{y}\left(1-\frac{\bar{y}}{L}\right)+\sigma_1\bar{x}-\sigma_2\bar{y} &= 0.
\end{aligned} \tag{3.1}$$

This system (3.1) is already solved by Dubey et. al. [10] and local and global stability results for the system at $P_1\,(\bar{x},\bar{y},0)$ are discussed there.

Now assume here that the interior equilibrium point $P^*\,(x^*,y^*,z^*)$ exists and is a solution of

$$\begin{aligned}
rx^*\left(1-\frac{x^*}{K}\right)-\sigma_1x^*+\sigma_2y^*-mx^*z^*-q_1\alpha_1x^* &= 0,\\
sy^*\left(1-\frac{y^*}{L}\right)+\sigma_1x^*-\sigma_2y^*-my^*z^* &= 0,\\
-dz^*+\alpha\left(x^*z^*+y^*z^*\right)-q_2\alpha_2z^* &= 0.
\end{aligned} \tag{3.2}$$

4. Stability Analysis

We now investigate the dynamical behaviour of system (2.5) at equilibrium points. The general variational matrix corresponding to the system (2.5) is given by

$$V(x,y,z) = \begin{pmatrix} V_1 & \frac{\sigma_2}{1-q_1\beta_1} & \frac{-mx}{1-q_1\beta_1}\\ \sigma_1 & V_2 & -my\\ \frac{\alpha z}{1-q_2\beta_2} & \frac{\alpha z}{1-q_2\beta_2} & V_3 \end{pmatrix} \tag{4.1}$$

where $V_1 = \frac{r-2rx-\sigma_1-q_1\alpha_1}{1-q_1\beta_1}-mz$, $V_2 = s-\frac{2s}{L}-\sigma_2-mz$ and $V_3 = \frac{-d+\alpha(x+y)-q_2\alpha_2}{1-q_2\beta_2}$.
Firstly, the equilibrium point $P_1\,(\bar{x},\bar{y},0)$ has the characteristic equation is $\lambda^2+a_1\lambda+a_2 = 0$,
where

$$\begin{aligned}
a_1 &= \tfrac{r}{K(1-q_1\beta_1)}\bar{x}+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{\bar{x}}{\bar{y}},\\
a_2 &= \left(\tfrac{r}{K(1-q_1\beta_1)}\bar{x}+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{\bar{x}}{\bar{y}}\right)\times\left(\tfrac{s}{L}\bar{y}+\sigma_1\tfrac{\bar{x}}{\bar{y}}\right)-\sigma_1\sigma_2.
\end{aligned} \tag{4.2}$$

Therefore

$$\begin{aligned}
\lambda_1+\lambda_2 &= -\left(\tfrac{r}{K(1-q_1\beta_1)}\bar{x}+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{\bar{x}}{\bar{y}}\right) < 0,\\
\lambda_1\times\lambda_2 &= \left(\tfrac{r}{K(1-q_1\beta_1)}\bar{x}+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{\bar{x}}{\bar{y}}\right)\times\left(\tfrac{s}{L}\bar{y}+\sigma_1\tfrac{\bar{x}}{\bar{y}}\right)-\sigma_1\sigma_2 > 0.
\end{aligned} \tag{4.3}$$

Therefore, all eigenvalues are negative and hence P_1 is locally asymptotically stable.

On the other hand, Let us now suppose that system (2.5) has a unique positive equilibrium $P^*\,(x^*,y^*,z^*)$. The variational matrix of (2.5) at P^* is

$$V(x,y,z) = \begin{pmatrix} \frac{1}{1-q_1\beta_1}\left(-\frac{r}{K}x^*-\sigma_2\frac{y^*}{x^*}\right) & \frac{\sigma_2}{1-q_1\beta_1} & \frac{-m}{1-q_1\beta_1}x^*\\ \sigma_1 & -\frac{s}{L}y^*-\sigma_1\frac{x^*}{y^*} & -my^*\\ \frac{\alpha z^*}{1-q_2\beta_2} & \frac{\alpha z^*}{1-q_2\beta_2} & 0 \end{pmatrix}$$

(4.4)

The characteristic equation of variational matrix of system (2.5) at P^* is given by $\lambda^3+b_1\lambda^2+b_2\lambda+b_3=0$, where

$$\begin{aligned} b_1 &= \tfrac{r}{K(1-q_1\beta_1)}x^*+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{y^*}{x^*}+\tfrac{s}{L}y^*+\sigma_1\tfrac{x^*}{y^*},\\ b_2 &= \tfrac{r\sigma_1}{K(1-q_1\beta_1)y^*}x^{*2}+rs\tfrac{x^*y^*}{KL(1-q_1\beta_1)}+\tfrac{\alpha m}{1-q_2\beta_2}\left(x^*z^*+y^*z^*\right)\\ &+ s\sigma_2\tfrac{y^{*2}}{Lx^*(1-q_1\beta_1)},\\ b_3 &= \tfrac{\alpha msx^*y^*z^*}{L(1-q_2\beta_2)}+\tfrac{\alpha mrx^*y^*z^*}{k(1-q_1\beta_1)(1-q_2\beta_2)}+\tfrac{\alpha m\sigma_1}{(1-q_2\beta_2)}\cdot\frac{x^{*2}z^*}{y^*}\\ &+ \tfrac{\alpha m\sigma_1}{1-q_2\beta_2}x^*z^*+\tfrac{\alpha m\sigma_2}{(1-q_1\beta_1)(1-q_2\beta_2)}y^*z^*\\ &+ \tfrac{\alpha m\sigma_2}{(1-q_1\beta_1)(1-q_2\beta_2)}\cdot\tfrac{y^{*2}z^*}{x^*}. \end{aligned} \tag{4.5}$$

According to Routh-Hurwitz criteria, the necessary and sufficient conditions for local stability of equilibrium point P^* are $b_1>0$, $b_3>0$, and $b_1b_2-b_3>0$.
It is evident that $b_1>0$ and $b_3>0$. Thus the stability of P^* is determined by the sign of $b_1b_2-b_3$. By direct calculation, we obtain

$$\begin{aligned} b_1b_2-b_3 &= \left(\tfrac{r}{K(1-q_1\beta_1)}x^*+\tfrac{\sigma_2}{(1-q_1\beta_1)}\cdot\tfrac{y^*}{x^*}+\tfrac{s}{L}y^*+\sigma_1\tfrac{x^*}{y^*}\right)\\ &\times \left(\tfrac{r\sigma_1}{K(1-q_1\beta_1)}x^{*2}+\tfrac{rsx^*y^*}{KL(1-q_1\beta_1)}+\tfrac{s\sigma_2}{L(1-q_1\beta_1)}\tfrac{y^{*2}}{x^*}\right)\\ &+ \tfrac{r}{K(1-q_1\beta_1)(1-q_2\beta_2)}x^{*2}z^*\\ &+ \alpha m\tfrac{s}{L(1-q_2\beta_2)}y^{*2}z^*+\alpha m>0, \end{aligned} \tag{4.6}$$

and hence $P^*\left(x^*,y^*,z^*\right)$ is locally asymptotically stable.

Now we will discuss the global stability of the endemic equilibrium point $P^*\left(x^*,y^*,z^*\right)$ of the system (2.5).

Theorem 4.1. *The equilibrium point $P^*\left(x^*,y^*,z^*\right)$ of system (2.5) is globally asymptotically stable if* $\left(1-\tfrac{y^*\sigma_2}{x^*(1-q_1\beta_1)\sigma_1}\right)(y-y^*)(z-z^*)<0.$

Proof. Let us consider the following Lyapunov function:

$$\begin{aligned} V(x,y,z) &= \left(x-x^*-x^*\ln\left(\tfrac{x}{x^*}\right)\right)+l_1\left(y-y^*-y^*\ln\left(\tfrac{y}{y^*}\right)\right)\\ &+ l_2\left(z-z^*-z^*\ln\left(\tfrac{z}{z^*}\right)\right), \end{aligned} \tag{4.7}$$

where l_1 and l_2 are positive constants to be chosen later on.
Differentiating $V(x,y,z)$ with respect to time t, we get

$$\frac{dV}{dt}=\left(\frac{x-x^*}{x}\right)\frac{dx}{dt}+l_1\left(\frac{y-y^*}{y}\right)\frac{dy}{dt}+l_2\left(\frac{z-z^*}{z}\right)\frac{dz}{dt}. \tag{4.8}$$

Choosing $l_1=(y^*/x^*)\cdot(\sigma_2/(1-q_1\beta_1)\sigma_1)$ and $l_2=(m(1-q_2\beta_2))/(\alpha(1-q_1\beta_1))$, a little algebraic manipulation yields

$$\begin{aligned} \frac{dV}{dt} &= -\tfrac{r}{K(1-q_1\beta_1)}(x-x^*)^2-\tfrac{s}{L}\tfrac{y^*\sigma_2}{x^*(1-q_1\beta_1)\sigma_1}(y-y^*)^2\\ &+ m\left(1-\tfrac{y^*\sigma_2}{x^*(1-q_1\beta_1)\sigma_1}\right)(y-y^*)(z-z^*)<0. \end{aligned} \tag{4.9}$$

Clearly $dV/dt<0$ if and only if
$\left(1-\dfrac{y^*\sigma_2}{x^*(1-q_1\beta_1)\sigma_1}\right)(y-y^*)(z-z^*)<0.$
Therefore, $P^*\left(x^*,y^*,z^*\right)$ is globally asymptotically stable. □

5. Numerical Simulation

To study dynamics of the system (2.5) with the numerical simulation, for this, we choose the values of the following parameters (see [19]) :

$$\begin{gathered} r=2.1,\quad s=1.7,\quad K=L=100,\\ \sigma_1=0.7,\quad \sigma_2=0.1,\quad q_1=0.02,\quad q_2=0.01,\\ \alpha=0.003,\quad m=0.02,\quad d=0.02,\\ \alpha_1=10,\quad \alpha_2=15,\quad \beta_1=1,\quad \beta_2=1. \end{gathered} \tag{5.1}$$

In appropriate units with initial conditions $(30,30,30)$. For the values of the parameters listed below in (5.1), the conditions (2.4) are satisfied. Also, according to the Theorem 4.1 the balance point $P^*\left(x^*,y^*,z^*\right)$ is both locally and globally asymptotically stable within the quadrant.
For the set of values of parameters given in (5.1), the behavior of x, y and z with respect to time t is plotted in Figure 1.
In Figure 1, firstly the biomass density of prey species in the region increases without reservation with respect to time and little decreases slightly and moved to its equilibrium level. Secondly, it is clear that the biomass density of the prey population in the reserved area increases abruptly near its carrying capacity, then moved to its equilibrium level near the carrying capacity of this zone. Finally, the same figure shows that the density of the biomass of predators increases with time in an almost linear way and tries to adjust to their equilibrium level.

We note that the α_i and β_i for $i=1,2$ are important parameters that govern the dynamics of the system. Therefore, we plotted the behavior of x, y and z with time t for different values of α_i and β_i in Figures 2–4.

6. Conclusion

In this paper, we have is a mathematical model of exploitation of fisheries resources with the reserve area, in the presence of bird predators and the function of the modified strain has been proposed. It was assumed that the aquatic ecosystem consists of two zones : a free fishing area and other restricted areas where fishing is prohibited. It was assumed that fish populations are logistic growth in both areas. Using the theory

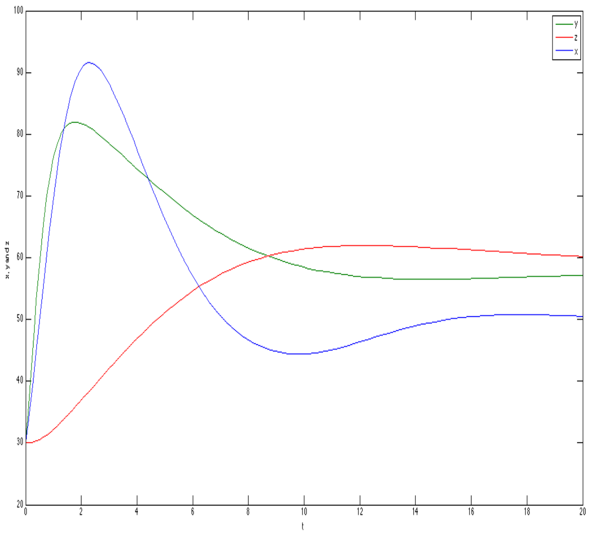

Figure 1. Plot x, y and z versus time t

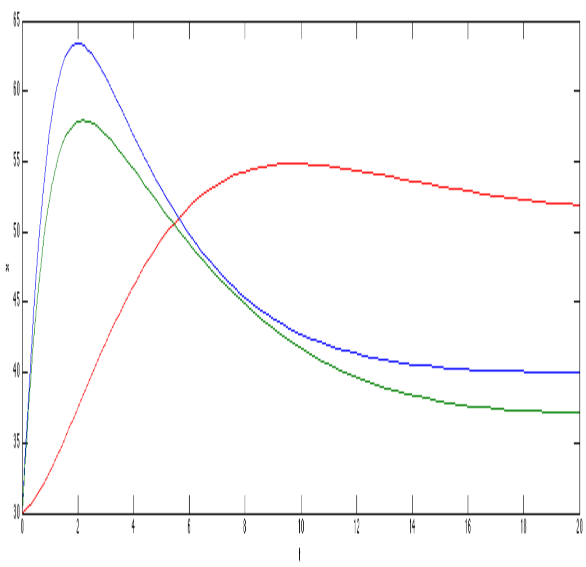

Figure 2. Plot of x verses time t to $\alpha_1 = 10$, $\alpha_2 = 20$, (red), $\alpha_2 = 40$, (blue), $\alpha_2 = 60$, (green), $\beta_1 = \beta_2 = 1$

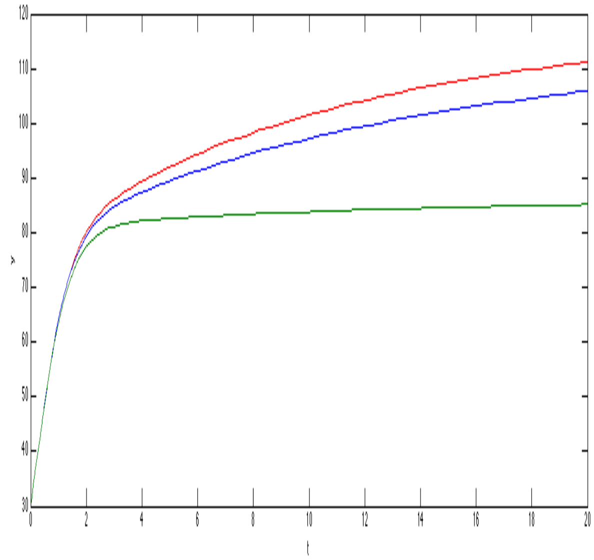

Figure 3. Plot of y verses time t to $\alpha_1 = 10$, $\alpha_2 = 20$, (red), $\alpha_2 = 40$, (blue), $\alpha_2 = 60$, (green), $\beta_1 = \beta_2 = 1$

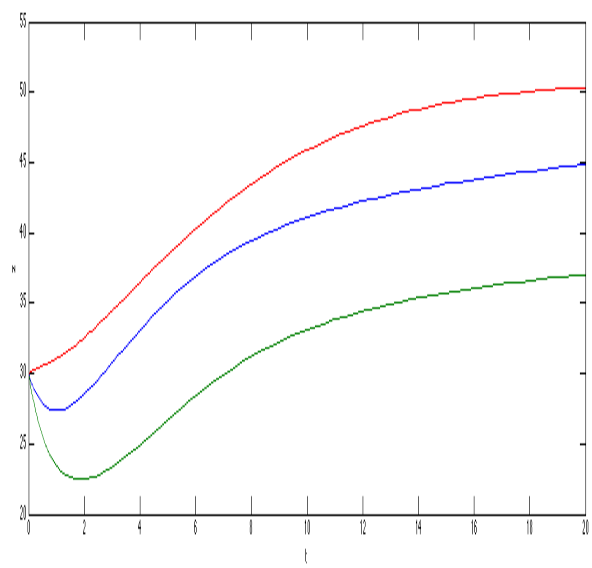

Figure 4. Plot of z verses time t to $\alpha_1 = 10$, $\alpha_2 = 20$, (red), $\alpha_2 = 40$, (blue), $\alpha_2 = 60$, (green), $\beta_1 = \beta_2 = 1$

of stability of ordinary differential equation, it has been proven that there is inner balance under certain conditions and it is globally asymptotically stable.

Acknowledgments

The authors would like to express sincere gratitude to the reviewers for his/her valuable suggestions.

References

[1] G. Birkoff and G. C. Rota, Ordinary Differential Equations, Ginn, 1982.

[2] D. K. Bhattacharya and S. Begum, Bionomic equilibrium of two-species system, Mathematical Biosciences, vol. 135, no. 2, pp. 111—127, 1996.

[3] K. S. Chaudhuri, A bioeconomic model of harvesting a multispecies fishery, Ecological Modelling, vol. 32, no. 4, pp. 267—279, 1986.

[4] K. S. Chaudhuri and S. Saha Roy, Bioeconomic exploitation of a lotka-volterra prey-predator system, Bulletin of Calcutta Mathematical Society, vol. 83, pp. 175—186, 1991.

[5] K. S. Chaudhuri and S. S. Ray, On the combined harvesting of a prey-predator system, Journal of Biological Systems, vol. 4, no. 3, pp. 376—389, 1996.

[6] C. W. Clark, Mathematical Bioeconomics : The Optimal Management of Renewable Resource, John Wiley & Sons, New York, NY, USA, 1976.

[7] C. W. Clark, Mathematical Bioeconomics : The Optimal Management of Renewable Resource, John Wiley & Sons, New York, NY, USA, 2nd edition, 1990.

[8] B. Dubey, P. Chandra and P. Sinha, A resource dependent fishery model with optimal harvesting policy, Journal of Biological Systems, vol. 10, no. 1, pp. 1—13, 2002.

[9] B. Dubey, P. Chandra, and P. Sinha, A model for an inshore-offshore fishery, Journal of Biological Systems, vol. 11, no. 1, pp. 27—41, 2003.

[10] B. Dubey, P. Chandra, and P. Sinha, A model for fishery resource with reserve area, Nonlinear Analysis: Real World Applications, vol. 4, no. 4, pp. 625-–637, 2003.

[11] M. Fan and K. Wang, Optimal harvesting policy for single population with periodic coefficients, Mathematical Biosciences, vol. 152, no. 2, pp. 165—177, 1998.

[12] H. I. Freedman and J. W. H. So, Global stability and persistence of simple food chain, Mathematical Biosciences, vol. 76, no. 1, pp. 69—86, 1983.

[13] B. Gupta and A. Sharma, Harvesting Model for Fishery Resource with Reserve Area and Modified Effort Function, Malaya J. Mat. 4(2), 255—262, 2016.

[14] L. V. Idels and M. Wang, Harvesting Fisheries Management Srategies with Modified Effort Function, International Journal of Modelling, Identification and Control, 3(1) : 83–87, 2008.

[15] T. K. Kar, S. Misra and B. Mukhopadhyay, A bioeconomic model of a ratio-dependent predator-prey system and optimal harvesting, Journal of Applied Mathematics and Computing, vol. 22, no. 1–2, pp. 387—401, 2006.

[16] Y. Louartassi, N. Elalami, and E. H. El Mazoudi, Stability analysis and static output feedback design for a model the fishery resource with reserve area, Applied Mathematical Sciences, vol. 6, no. 66, pp. 3289—3299, 2012.

[17] Y. Louartassi and N. Elalami, Observer Design for a Mathematical Model for the Management of a Renewable Resource Population, International Journal of Computing Science and Mathematics, Vol. 6, No. 1, pp. 1–8, 2015.

[18] Y. Louartassi Y,E El Mazoudi and N. Elalami, A new generalization of lemma Gronwall-Bellman, Applied Mathematical Sciences, 6(13) : 621–628, 2012.

[19] A. Sharma and B. Gupta, Harvesting Model for Fishery Resource with Reserve Area and Bird Predator, Journal of Marine Biology, Volume 2014, ID 218451, 7 pages, 2014.

[20] J. B. Shukla and B. Dubey, Modelling the depletion and conservation of forestry resources: effects of population and pollution, Journal of Mathematical Biology, vol. 36, no. 1, pp. 71—94, 1997.

[21] R. P. Agarwal, *Difference Equations and Inequalities: Theory, Methods and Applications*, Marcel Dekker, New York, 1999.

[22] I. Gyori and G. Ladas, *Oscillation Theory of Delay Differential Equations with Applications*, Clarendon Press, Oxford, 1991.

[23] D. A. Georgiu , E. A. Grove and G. Ladas, Oscillations of neutral difference equations, *Appl. Anal.*, 33(1989), 234–253.

[24] J. R. Graef and P. W. Spikes, Asymptotic decay of oscillatory solutions of forced nonlinear difference equations, *Dyn. Syst. Appl.*, 3(1994), 95–102.

[25] G. H. Hardy, J. E Little wood and G. Polya, *Inequalities*, The print of the 1952 edition, Cambridge University Press, Cambridge, UK, 1988.

[26] J. W. Hooker and W. T. Patula, A second order nonlinear difference equation: Oscillation and asymptotic behaviour, *J. Math. Anal. Appl.*, 91(1983), 9–29.

[27] V. Lakshmikantham and D. Trigiante, *Theory of Difference Equations: Numerical methods and Applications*, Academic press, New York, 1988.

[28] G. Ladas, Ch. G. Philos and Y.G. Cas, Sharp conditions for the oscillation of delay difference equations, *J. Appl. Math. Simulation*, 2(1989), 101–112.

[29] B. S. Lalli and B. G. Zhang, On existence of positive solutions and bounded oscillations for neutral difference equations, *J. Math. Anal. Appl.*, 166(1992), 272– 287.

[30] T. Li and Y. V. Rogovchenko, Oscillation theorems for second - order nonlinear neutral delay differential equations, *Abstr. Appl. Anal.*, Vol. 2014, Article ID 594190.

[31] H. J. Li and C. C. Yeh, Oscillation criteria for second-order neutral delay difference equations, *Comput. Math. Appl.*, 36(1998), 123–132.

[32] W. T. Li and S. S. Cheng, Oscillation criteria for a

nonlinear difference equation, *Comput. Math. Appl.*, 36(8)(1998), 87–94.

[33] A. Murugesan and K. Ammamuthu, Conditions for oscillation and convergence of solutions to second order neutral delay difference equations with variable coefficients, *Malaya J. Mat.*, 5(2)(2017), 367–377.

[34] S. H. Saker and S. S. Cheng, Oscillation criteria for difference equations with damping terms, *Appl. Math. Comput.*, 148(2004), 421–442.

[35] A. Sternal and B. Szmanda, Asymptotic and oscillatory behaviour of certain difference equations, *LEMATEMATICHE*, (1996), 77–86.

[36] Z. Szafranski and B. Szmanda, A note on the oscillation of some difference equations, *Fasc. Math.*, 21 (1990), 57–63.

[37] Z. Szafranski and B. Szmanda, Oscillations of some linear difference equations, *Fasc. Math.*, 25(1995), 165–174.

[38] B. Szmanda, Note on the behaviour of solutions of a second order nonlinear difference equation, *Atti. Acad. Naz. Lincei, Rend. Sci. Fiz. Mat.*, 69(1980), 120–125.

[39] B. Szmanda, Characterization of oscillation of second order nonlinear difference equations, *Bull. Polish. Acad. Sci. Math.*, 34(1986), 133–141.

[40] B. Szmanda, Oscillatory behaviour of certain difference equations, *Fasc. Math.*, 21(1990), 65–78.

[41] E. Thandapani and K. Mahalingam, Necessary and sufficient conditions for oscillation of second order neutral difference equations, *Tamkang J. Math.*, 34(2)(2003), 137–145.

[42] E. Thandapani, Asymptotic and oscillatory behaviour of solutions on nonlinear second order difference equations, *Indian J. Pure Appl. Math.*, 24(1993), 365– 372.

[43] E. Thandapani, Asymptotic and oscillatory behaviour of solutions of a second order nonlinear neutral delay difference equation, *Riv. Mat. Univ. Parma*, (5)(1)(1992), 105–113.

[44] B. G. Zhang and S. S. Cheng, Oscillation criteria and comparison theorems for delay difference equations, *Fasc. Math.*, 25(1995), 13–32.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆