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α-Stable necks of fuzzy automata

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Abstract
In this paper we introduce α-stable necks, α-stable directable, α-stable trap-directable fuzzy automata. Further we show that α-stable necks of fuzzy automaton exists then it is α-stable subautomaton, α-stable kernel and discuss some of their properties. Finally we prove a fuzzy automaton is α-stable directable if and only if it is an extension of a α-stable strongly directable fuzzy automaton by a α-stable trap-directable fuzzy automaton.

Keywords
α-stable necks, α-stable directable, α-stable trap-directable.

AMS Subject Classification
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1. Introduction

Lofti A. Zadeh invented fuzzy set theory in 1965 [8], which is a generalization of classical set theory. The fuzzy set is a simple mathematical tool for representing the inevitability of vagueness, uncertainty, and imprecision in everyday life. W.G. Wee extended the fuzzy idea to automata in 1967 [7]. Later, numerous academics adapted the fuzzy notion to a wide range of domains, and it has a wide range of applications. J.N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and languages in their book [6].

T. Petkovic et al. [1] discussed directable automata, monogenically directable, generalized directable using necks. T. Petkovic et al.[3] introduce and studied trap-directable, trapped automata and other related automata. Also, we refer the survey paper Directable automata and their generalizations were investigated by S. Bogdanovic et al [2]. Further the necks of fuzzy automata were studied and discussed in [4]. In this paper we introduce α-stable necks, α-stable directable, α-stable trap-directable fuzzy automata. Further we show that α-stable necks of fuzzy automata exists then it is α-stable subautomaton, α-stable kernel and discuss some of their properties. Consequently, we prove α-stable directable fuzzy automaton is an extension of a α-stable strongly directable fuzzy automaton by a α-stable trap-directable fuzzy automaton.

2. Preliminaries

Definition 2.1. *[6] A fuzzy automaton $S=(D,\ I,\ \psi)$, where,*

D - set of states $\{d_0,\ d_1,\ d_2, ..., \ d_n\}$,
I - alphabets (or) input symbols,
ψ - function from $D\times I\times D \to [0,1]$,

The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $t\in I^*$ is denoted by $|t|$.

Definition 2.2. *[6] Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. The extended transition function is defined by $\psi^*: D\times I^*\times D \to [0,1]$ and is given by*

$$\psi^*(d_i,\ \lambda,\ d_j)=\begin{cases}1 & if\ d_i=d_j\\ 0 & if\ d_i\neq d_j\end{cases}$$

$\psi^*(d_i,tt',d_j)=\vee_{q_r\in D}\{\psi^*(d_i,t,d_r)\wedge\psi(d_r,t',d_j)\}, t\in I^*, t'\in I$.

Definition 2.3. [4] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $D' \subseteq D$. Let ψ' is the restriction of ψ and let $S' = (D',\ I,\ \psi')$. The fuzzy automaton S' is called a subautomaton of S if*

(i) $\psi' : D' \times I \times D' \to [0,1]$ and

(ii) For any $d_i \in D'$ and $\psi'(d_i,t,d_j) > 0$ for some $t \in I^*$, then $d_j \in D'$.

Definition 2.4. [6] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is said to be strongly connected if for every $d_i,\ d_j \in D$, there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$. Equivalently, S is strongly connected if it has no proper subautomaton.*

Definition 2.5. [5] *A relation R on a set D is said to be equivalence relation if it is reflexive, symmetric and transitive.*

Definition 2.6. [5] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. An equivalence relation R on D in S is called congruence relation if $\forall d_i, d_j \in D$ and $t \in I, d_iRd_j$ implies that, then there exists $d_l, d_k \in D$ such that $\psi(d_i,t,d_l) > 0, \psi(d_j,t,d_k) > 0$ and d_lRd_k.*

Definition 2.7. [4] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $S' = (D',\ I,\ \psi')$ be a subautomaton of S. A relation $R_{S'}$ on S is defined as follows. For any $d_i, d_j \in D$, we say that $(d_i,d_j) \in R_{S'}$ if and only if either $d_i = d_j$ or $d_i, d_j \in D'$.*

This relation is clearly an equivalence relation and it is also congruence. This relation is called Rees congruence relation on D in S determined by S'. A fuzzy automaton S/S' is called Rees factor fuzzy automaton determined by the relation $R_{S'}$ and it is defined as $S/S' = (\ \overline{D},\ I,\ \psi_{s/S'})$, where $\overline{D} = \{\ [d_i]\ /\ d_i \in D\}$ and $\psi_{S/S'} : \overline{D} \times I \times \overline{D} \to [0,1]$.

Definition 2.8. [4] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a neck of S if there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$ for every $d_i \in D$.*
In that case d_j is also called t-neck of S and the word t is called a directing word of S.
If S has a directing word, then we say that S is a directable fuzzy automaton.

Remark 2.9. *In this paper we consider only deterministic fuzzy automaton.*

3. α-Stable Necks of Fuzzy Automata

Definition 3.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If S is said to be α-stable fuzzy automaton then $\psi(d_i,t',d_j) \geq \alpha > 0, \forall t' \in I, \alpha =$ Fixed value in [0,1].*

Definition 3.2. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton and let $d_i \in D$. The α-stable subautomaton of S generated by d_i is denoted by $\langle d_i \rangle$. It is given by*
$\langle d_i \rangle = \{\ d_j \mid \psi^*(d_i,\ t,\ d_j) \geq \alpha,\ t \in I^*,\ \alpha =$ Fixed value in$[0,1]\}$.
If it exists, then it is called the α-stable least subautomaton of S containing d_i.

Definition 3.3. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. For any non-empty $D' \subseteq D$, the α-stable subautomaton of S generated by D' is denoted by $\langle D' \rangle$ and is given by*
$\langle D' \rangle = \{\ d_j \mid \psi^*(d_i,\ t,\ d_j) \geq \alpha,\ d_i \in D',\ t \in I^*\}$. It is called the α-stable least subautomaton of S containing D'. The α-stable least subautomaton of a fuzzy automaton S if it exists is called the α-stable kernel of S.

Definition 3.4. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a α-stable neck of S if there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) \geq \alpha, \alpha \in [0,1]$ for every $d_i \in D$.*
In that case d_j is also called t-α-stable neck of S and the word t is called a α-stable directing word of S.
If S has a α-stable directing word, then we say that S is a α-stable directable fuzzy automaton.

Remark 3.5. *1) The set of all α-stable necks of a fuzzy automaton S is denoted by $\alpha SN(S)$.*

2) The set of all α-stable directing words of a fuzzy automaton S is denoted by $\alpha SDW(S)$.

3) A fuzzy automaton S is called α-stable strongly directable if $D = \alpha SN(S)$.

Definition 3.6. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a α-stable trap of S if $\psi^*(d_j,\ t,\ d_j) \geq \alpha, \forall t \in I^*$.*

If S has exactly one α-stable trap, then S is called one α-stable trap fuzzy automaton. The set of all α-stable traps of a fuzzy automaton S is denoted by $\alpha STR(S)$.

A fuzzy automaton S is called a α-stable trapped fuzzy automaton, for each $d_i \in D$,if there exists a word $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) \geq \alpha,\ \ d_j \in \alpha STR(S)$.

Definition 3.7. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If S has a single α-stable neck, then S is called a α-stable trap-directable fuzzy automaton.*

Definition 3.8. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_i \in D$ is called α-stable reversible if for every word $t' \in I^*$, there exists a word $t \in I^*$ such that*
$\psi^*(d_i,\ t't,\ d_i) \geq \alpha$ and the set of all $alpha$-stable reversible states of S is called the $alpha-$ stable reversible part of S. It is denoted by $\alpha SR(S)$. If it is non-empty, $\alpha SR(S)$ is a α-stable subautomaton of S.

Definition 3.9. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A fuzzy automaton is called a direct sum of its α-stable subautomata $S_\beta,\ \beta \in Y$, if $S = \cup_{\beta \in Y} S_\beta$ and $S_\beta \cap S_\gamma = \phi$, for every $\beta,\ \gamma \in Y$ such that $\beta \neq \gamma$.*

Definition 3.10. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $d_i, d_j \in D$ are said to be α-stable mergeable if there exists a word $t \in I^*$ and $d_k \in D$ such that $\psi^*(d_i,t,d_k) \geq \alpha \Leftrightarrow \psi^*(d_j,t,d_k) \geq \alpha$.*

4. Properties of α-Stable Necks of Fuzzy Automata

Theorem 4.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $\alpha SN(S) \neq \phi$, then $\alpha SN(S)$ is a α-stable subautomaton of S.*

Proof. Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. Let $d_j\ \in \alpha SN(S)$ and $t'\in I^*$. Assume that d_j is a t-α-stable neck of S, for some $t\in I^*$. Then for each $d_i\in D$ we have
$\psi^*(d_i,\ tt',\ d_k)=\wedge_{d_j\ \in\ D}\{\psi^*(d_i,\ t,\ d_j),\psi^*(d_j,\ t',\ d_k)\}\geq\alpha$, it means that d_k is a tt'-α-stable neck of S and hence, $d_k\in\alpha SN(S)$. Therefore, $\alpha SN(S)$ is a α-subautomaton of S. □

Theorem 4.2. *Let $S=(D,\ I,\ \psi)$ be α-stable directable fuzzy automaton. Then $\alpha SN(M)$ is the α-stable kernel of S and $\alpha SN(S)=\alpha SR(S)$.*

Proof. Let $S=(D,\ I,\ \psi)$ be a α-stable directable fuzzy automaton. Let $d_j\in\alpha SN(S)$ and $d_i\in D$. Then $\psi^*(d_i,\ t,\ d_j)\geq\alpha$, for every $t\in\alpha SDW(S)$ and hence $d_j\in\langle d_i\rangle$. Therefore, $\alpha SN(S)\subseteq\langle d_i\rangle$ for every $d_i\in D$. This means that $\alpha SN(S)$ is a α-stable subautomaton contained in every other α-stable subautomaton of S. Thus, $\alpha SN(S)$ is the α-stable kernel of S.
On the other hand, $d_j\in\alpha SR(S)$. Then for every $t'\in I^*$, there exists $t\in I^*$ such that $\psi^*(d_j,\ t't,\ d_j)\geq\alpha$. Consider t as a α-stable directing word.
$\psi^*(d_j,t't,\ d_j)\geq\alpha=\wedge_{d_i\in D}\left\{\psi^*(d_j,\ t',\ d_i),\psi^*(d_i,\ t,\ q_j)\right\}\geq\alpha$
$\Longrightarrow\psi^*(d_i,\ t,\ d_j)\geq\alpha$, for every $d_i\in D$.
$\Longrightarrow d_j\in\alpha SN(S)$
$\Longrightarrow\alpha SR(S)\subseteq\alpha SN(S)$.
Let $d_j\ \in\ \alpha SN(S)$ and let $t'\in I^*$. Then $\psi^*(d_i,\ t,\ d_j)\geq\alpha$, for every $d_i\in D$.
Now, $\psi^*(d_j,\ t',\ d_k)\geq\alpha$ for some $d_k\in D$ and $\psi(d_k,\ t,\ d_j)\geq\alpha$
$\Longrightarrow\psi(d_j,\ t't,\ d_j)\geq\alpha$
$\Longrightarrow d_j\in\alpha SR(S)$
$\Longrightarrow\alpha SN(S)\subseteq\alpha SR(S)$. Therefore, $\alpha SN(S)=\alpha SR(S)$. □

Theorem 4.3. *A fuzzy automaton $S=(D,\ I,\ \psi)$ is α-stable strongly directable fuzzy automaton if and only if it is strongly connected and α-stable directable.*

Proof. Let $S=(D,\ I,\ \psi)$ be a α-stable strongly directable fuzzy automaton. It is clearly α-stable directable. Now we will prove it is strongly connected. It is enough to show that for any $d_i,d_j\in D$, there exists $t\in I^*$ such that $\psi^*(d_i,t,d_j)\geq\alpha>0$. Since $d_j\in\alpha SN(S)$ $[\alpha SN(S)=D]$, $\psi^*(d_k,t,d_j)\geq\alpha>0$, for every $d_k\ \in S$.
Therefore, $\psi^*(d_i,t,d_j)\geq\alpha>0$. Thus, S is strongly connected.
Conversely, let S be strongly connected and α-stable directable. Then $\alpha SN(S)\neq\phi$ and by Theorem 4.1, $\alpha SN(S)$ is α-stable subautomaton of S. Since S is strongly connected, there is no proper subautomaton. Hence, $D=\alpha SN(S)$. Thus, S is strongly α-stable directable fuzzy automaton. □

Theorem 4.4. *A fuzzy automaton $S=(D,\ I,\ \psi)$ is α-stable directable if and only if it is an extension of a strongly α-stable directable fuzzy automaton S' by a α-stable trap-directable fuzzy automaton S''.*
(i) $\alpha SDW(S'').\alpha SDW(S')\subseteq\alpha SDW(S)\subseteq\alpha SDW(S'')\cap\alpha SDW(S')$;
(ii) $\alpha SN(S)=S'$.

Proof. Let S be α-stable directable fuzzy automaton. Then $\alpha SN(S)$ is non-empty and by Theorem 4.1, $\alpha SN(S)$ is a α-stable subautomaton of S.
The Rees factor fuzzy automaton $S/\alpha SN(S)$ is also α-stable directable.
Further, by Rees factor, $S/\alpha SN(S)$ is a α-stable trap-directable fuzzy automaton and hence, S is an extension of a strongly α-stable directable fuzzy automaton $\alpha SN(S)$ by a α-stable trap-directable fuzzy automaton $S/\alpha SN(S)$.
Conversely, let S be an extension of strongly α-stable directable fuzzy automaton S' by a α-stable trap-directable fuzzy automaton S''. Let $t\in\alpha SDW(S'')$ and $t'\in\alpha SDW(S')$. Then for all $d_i,\ d_j\in D$ we have that $\psi^*(d_i,\ t,\ d_k)\geq\alpha$, $\psi^*(d_j,\ t,\ d_k)\geq\alpha$, where $d_k\in S'$.
Hence, $\psi^*(d_i,\ tt',\ d_m)=\wedge\{\psi^*(d_i,\ t,\ d_k),\psi^*(d_k,\ t',\ d_m)\}\geq\alpha$
Thus, $tt'\in\alpha SDW(S)$ and hence, S is a α-stable directable fuzzy automaton.
If $t\in\alpha SDW(S'')$ and $t'\in\alpha SDW(S')$, then $tt'\in\alpha SDW(S)$.
Therefore, $\alpha SDW(S'').\alpha SDW(S')\subseteq\alpha SDW(S)$.
Let $t\in\alpha SDW(S)$. Since S is an extension of a strongly α-stable directable fuzzy automaton S' by a α-stable trap-directable fuzzy automaton S''.
Therefore, t is a α-stable directing word of S' and S''.
Hence, $\alpha SDW(S)\subseteq\alpha SDW(S')\cap\alpha SDW(S'')$.
Thus, (i) holds.
By Theorem 4.2, $\alpha SN(S)$ is the α-stable kernel of S, so $\alpha SN(S)\subseteq S'$.
Conversely, assume that $d_j\in S'$. Since S' is strongly α-stable directable, we conclude that there exists $t'\in\alpha SDW(S')$ such that $\psi^*(d_i,\ t',\ d_j)\geq\alpha$, for every $d_i\in S'$. Hence, for every $d_i\in D$ and $t\in\alpha SDW(S'')$, $\psi^*(d_i,\ t,\ d_l)\geq\alpha$, where $d_l\in S'$.
Now, $\psi^*(d_i,\ tt',\ d_j)=\wedge_{d_l\ \in\ S'}\{\psi^*(d_i,\ t,\ q_l),\psi^*(d_l,\ t',\ d_j)\}\geq\alpha$. Therefore, $d_j\in\alpha SN(S)$ and hence, $\alpha SN(S)=S'$. □

5. Conclusion

We introduce α-stable necks, α-stable directable, α-stable trap-directable fuzzy automata. We show that α-stable necks of fuzzy automaton exists then it is α-stable subautomaton,α-stable kernel. Finally we prove a fuzzy automaton is α-stable directable if and only if it is an extension of a strongly α-stable directable fuzzy automaton by a α-stable trap-directable fuzzy automaton.

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