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α-Stable local necks of fuzzy automata

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Abstract

In this paper we introduce α-stable local necks, α-stable monogenically directable, α-stable monogenically strongly directable, α-stable monogenically trap directable, α-stable uniformly monogenically directable, α-stable uniformly monogenically strongly directable, α-stable uniformly monogenically trap-directable fuzzy automata. We have shown that α-stable local necks of fuzzy automaton exists then it is α-stable subautomaton. Further we prove a some equivalent conditions on fuzzy automaton.

Keywords

α-Stable local necks, α-Stable monogenically directable, α-Stable monogenically trap-directable, α-Stable uniformly monogenically trap-directable.

AMS Subject Classification

03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

Fuzzy set was introduced by L. A. Zadeh in 1965 [8]. The fuzzy set is a simple mathematical tool for representing the inevitability of vagueness, uncertainty, and imprecision in everyday life. W.G. Wee extended the fuzzy idea to automata in 1967 [7]. Later, numerous academics adapted the fuzzy notion to a wide range of domains, and it has a wide range of applications. J.N. Mordeson and D. S. Malikgave a detailed account of fuzzy automata and languages in their book [6].

T. Petkovic et al. [1] discussed directable automata, monogenically directable, generalized directable using necks. T. Petkovic et al.[3] introduce and studied trap-directable, trapped automata and other related automata. Also, we refer the survey paper Directable automata and their generalizations were investigated by S. Bogdanovic et al [2]. Further the necks and local necks of fuzzy automata were studied and discussed in [4, 5]. In this paper we introduce α-stable local necks, α-stable monogenically directable, α-stable monogenically strongly directable, α-stable monogenically trap directable, α-stable uniformly monogenically directable, α-stable uniformly monogenically strongly directable, α-stable uniformly monogenically trap-directable fuzzy automata. We have shown that α-stable local necks of fuzzy automata exists then it is α-stable subautomata. Further we prove a some equivalent conditions on fuzzy automaton.

2. Preliminaries

Definition 2.1. *[6] A fuzzy automaton $S = (D,\ I,\ \psi)$, where,*

D - set of states $\{d_0,\ d_1,\ d_2, \ldots,\ d_n\}$,

I - alphabets (or) input symbols,

ψ - function from $D \times I \times D \to [0,1]$,

The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $t \in I^*$ is denoted by $|t|$.

Definition 2.2. *[6] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. The extended transition function is defined by $\psi^* : D \times I^* \times D \to [0,1]$ and is given by*

$$\psi^*(d_i,\ \lambda,\ d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases}$$

$\psi^*(d_i,tt',d_j) = \vee_{q_r\in D}\{\psi^*(d_i,t,d_r)\wedge\psi(d_r,t',d_j)\}, t\in I^*, t'\in I$.

Definition 2.3. *[4] Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. Let $D'\subseteq D$. Let ψ' is the restriction of ψ and let $S'=(D',\ I,\ \psi')$. The fuzzy automaton S' is called a subautomaton of S if*
(i) $\psi' : D'\times I\times D'\to[0,1]$ and
(ii) For any $d_i\ \in\ D'$ and $\psi'(d_i,t,d_j)>0$ for some $t\ \in\ I^*$, then $d_j\ \in\ D'$.

Definition 2.4. *[6] Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is said to be strongly connected if for every $d_i,\ d_j\in D$, there exists $t\in I^*$ such that $\psi^*(d_i,\ t,\ d_j)>0$. Equivalently, S is strongly connected if it has no proper subautomaton.*

Definition 2.5. *[4] Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j\in D$ is called a neck of S if there exists $t\in I^*$ such that $\psi^*(d_i,\ t,\ d_j)>0$ for every $d_i\in D$.*
In that case d_j is also called t-neck of S and the word t is called a directing word of S.
If S has a directing word, then we say that S is a directable fuzzy automaton.

Definition 2.6. *[5] Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. If $d_i\in Q$ is called local neck of S, if it is neck of some directable subautomaton of S. The set of all local necks of S is denoted by LN(S).*

3. α-Stable Local Necks of Fuzzy Automata

Definition 3.1. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. If S is said to be α-stable fuzzy automaton then $\psi(d_i,t',d_j)\geq\alpha>0, \forall t'\in I, \alpha=$ Fixed value in [0,1].*

Definition 3.2. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton and let $d_i\in D$. The α-stable subautomaton of S generated by d_i is denoted by $\langle d_i\rangle$. It is given by*
$\langle d_i\rangle=\{\ d_j\mid\psi^*(d_i,\ t,\ d_j)\geq\alpha>0,\ t\in I^*\}$. If it exists, then it is called the α-stable least subautomaton of S containing d_i.

Definition 3.3. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. For any non-empty $D'\subseteq D$, the α-stable subautomaton of S generated by D' is denoted by $\langle D'\rangle$ and is given by*
$\langle D'\rangle=\{\{\ d_j\mid\psi^*(d_i,\ t,\ d_j)\geq\alpha\}>0,\ d_i\in D',\ t\in I^*\}$. It is called the α-stable least subautomaton of S containing D'. The α-stable least subautomaton of a fuzzy automaton S if it exists is called the α-stable kernel of S.

Definition 3.4. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_i\ \in\ D$ is called α-stable local neck of S if it is α-stable neck of some α-stable directable subautomaton of S. The set of all α-stable local necks of S is denoted by $\alpha SLN(S)$.*

Definition 3.5. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable monogenically directable if every monogenic subautomaton of S is α-stable directable.*

Definition 3.6. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable monogenically strongly directable if every monogenic subautomaton of M is α-stable strongly directable.*

Definition 3.7. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable monogenically trap-directable if every monogenic subautomaton of S has a single α-stable neck.*

Definition 3.8. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. If $t\ \in\ I^*$ is α-stable common directing word of S if t is a α-stable directing word of every monogenic subautomaton of S. The set all α-stable common directing words of S will be denoted by $\alpha SCDW(S)$. In other words, $\alpha SCDW(S)=\cap_{d_i\ \in\ D}\alpha SDW(\langle d_i\rangle)$.*

Definition 3.9. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable uniformly monogenically directable fuzzy automaton if every monogenic subautomaton of S is α-stable directable and have atleast one β-weak common directing word.*

Definition 3.10. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable uniformly monogenically strongly directable fuzzy automaton if every monogenic subautomaton of S is strongly α-stable directable and have atleast one α-stable common directing word.*

Definition 3.11. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton. S is called α-stable uniformly monogenically trap directable fuzzy automaton if every monogenic subautomaton of S has a single α-stable neck and have atleast one α-stable common directing word.*

4. Properties of α-Stable Local Necks of Fuzzy Automata

Theorem 4.1. *Let $S=(D,\ I,\ \psi)$ be a fuzzy automaton and $d_i\in D$. Then the following conditions are equivalent:*
(i) d_i is a α-stable local neck;
(ii)$\langle d_i\rangle$ is a strongly α-stable directable fuzzy automaton;
(iii) for every $t'\ \in\ I^*$, there exists $t\ \in\ I^*$ such that $\psi^*(d_i,\ t't,\ d_i)\geq\alpha>0$.

Proof. $(i)\Rightarrow(ii)$
Let d_i be a α-stable local neck of S. Then there exists a α-stable directable subautomaton S' of S such that $d_i\in\alpha SN(S')$. Thus $\alpha SN(S')$ is a strongly α-stable directable fuzzy automaton. Also, $\langle d_i\rangle\subseteq\alpha SN(S')$, and $\alpha SN(S')$ is strongly connected, then $\langle d_i\rangle=\alpha SN(S')$. Therefore, $\langle d_i\rangle$ is a

strongly α-stable directable fuzzy automaton.
$(ii) \Rightarrow (iii)$

Let $\langle d_i \rangle$ be a strongly α-stable directable fuzzy automaton. Then d_i is a t-α-stable neck of $\langle d_i \rangle$ for some $t \in I^*$. Since $\langle d_i \rangle$ is strongly α-stable directable, for every $t' \in I^*$, there exists some $d_l \in \langle d_i \rangle$ such that $\{\psi^*(d_i, t', d_l) \geq \alpha\} > 0$. Now,
$\psi^*(d_i, t't, d_i) = \{\wedge_{d_l \in D}\{\psi^*(d_i, t', d_l), \psi^*(d_l, t, d_i)\}\} \geq \alpha > 0$.
$(iii) \Rightarrow (i)$

(iii) clearly shows that d_i is a $t-$ α-stable neck of $\langle d_i \rangle$, and hence, it is a α-stable local neck of S. □

Theorem 4.2. *Let $S = (D, I, \psi)$ be a fuzzy automaton. If $\alpha SLN(S) \neq \phi$, then $\alpha SLN(S)$ is a α-stable subautomaton of S.*

Proof. Let $d_i \in \alpha SLN(S)$ and $t \in I$. Then, the monogenic α-stable subautomaton $\langle d_i \rangle$ of S is strongly α-stable directable. Now, $\langle d_i \rangle \subseteq \langle d_l \rangle$, for some $d_l \in \langle d_i \rangle$. Since $\langle d_i \rangle$ is strongly connected, $\langle d_i \rangle = \langle d_l \rangle$. Therefore, d_l is also a α-stable local neck of S, i.e., $d_l \in \alpha SLN(S)$. Hence, $\alpha SLN(S)$ is a α-stable subautomaton of S. □

Theorem 4.3. *Let $S = (D, I, \psi)$ be a fuzzy automaton. Then the following conditions are equivalent:*
(i) Every state of D in S is a α-stable local neck;
(ii) S is α-stable monogenically strongly directable;
(iii) S is α-stable monogenically directable and α-stable reversible;
(iv) S is a direct sum of α-stable strongly directable fuzzy automata;
(v) $(\forall d_i \in D)(\exists t \in I^*)(\forall t' \in I^*)$ such that $\psi^*(d_i, t't, d_i) \geq \alpha > 0$.

Proof. $(i) \Rightarrow (ii)$

If every state $d_i \in D$ is a α-stable local neck of S. Then we have that for every $d_i \in D$ the α-stable monogenic subautomaton $\langle d_i \rangle$ of D in S is α-stable strongly directable. Hence, S is α-stable monogenically strongly directable.
$(ii) \Rightarrow (iii)$

If S is α-stable monogenically strongly directable, then it is α-stable monogenically directable. Now, every α-stable monogenic subautomaton of S is strongly connected, hence S is α-stable reversible.
$(iii) \Rightarrow (iv)$

If S is α-stable reversible, then it is a direct sum of α-stable strongly connected fuzzy automata S_β, $\beta \in Y$. Let $\beta \in Y$ and $d_i \in D_\beta$. Then $\langle d_i \rangle = S_\beta$. Since S_β is strongly connected, and by the α-stable monogenic directability of S we have that $S_\beta = \langle d_i \rangle$ is α-stable directable. Therefore, S_β is α-stable strongly directable, for any $\beta \in Y$.
$(iv) \Rightarrow (i)$

Let S be a direct sum of α-stable strongly directable fuzzy automata S_β, $\beta \in Y$. Then for each state $d_i \in D$, there exists $\beta \in Y$ such that $d_i \in D_\beta$, that is, $d_i \in S_\beta = \alpha SN(S_\beta)$, so d_i is a α-stable local neck of S.
$(i) \Rightarrow (v)$

Since, every state of S is a α-stable local neck, for any $d_i \in D$, $\langle d_i \rangle$ is α-stable monogenically strongly directable. Hence, $\langle d_i \rangle$ is α-stable reversible.
$(v) \Rightarrow (i)$

This is an immediate consequence of proof of the Theorem 4.1. □

5. Conclusion

We introduce α-stable local necks, α-stable monogenically directable, α-stable monogenically strongly directable, α-stable monogenically trap-directable, α-stable uniformly monogenically directable, α-stable uniformly monogenically strongly directable, α-stable uniformly monogenically trap-directable fuzzy automata. We have shown that α-stable local necks of fuzzy automata exists then it is α-stable subautomata. Further we prove a some equivalent conditions on fuzzy automaton.

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