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Binary soft locally closed sets

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Abstract. This work focuses on the concept of binary soft locally closed sets. Binary soft submaximal spaces are defined via binary soft locally closed sets. In addition, a new class of binary soft functions namely, BSLC-continuous, BSLC-irresolute, BScoLC-continuous and BScoLC-irresolute functions are defined and their characterizations are investigated.

AMS Subject Classifications: 54A05, 54A99, 54C08.

Keywords: Binary soft locally closed sets, binary soft contra locally closed sets, binary soft submaximal spaces.

Contents

1. Introduction and Background

In 1989, Ganster and Reilly [4] studied the notion of locally closed sets in topological spaces, which is defined by Bourbaki [3] as a subset of a topological space (X, τ) is locally closed if it is the intersection of an open and a closed set of X.

In this paper we have extend the notion of locally closed sets in the area of binary soft topological spaces. The notion of binary soft topological spaces is one of the latest topics, which is a combination of two popular ideas, binary topological spaces and soft topological spaces. Jothi and Thangavelu [5] introduced the concept of topology between two sets, known as binary topology. Binary topology is a structure which carries the subsets of two universal sets. The pioneer work of Molodtsov [7] on soft sets act as a successful mathematical tool over fuzzy mathematics, interval mathematics and theory of probability. In 2016, Acikgoz and Tas [1] defined binary soft sets as, (A, ρ) is a binary soft set over the two universal sets U_1, U_2 if $A : \rho \to P(U_1) \times P(U_2)$, $A(\varrho) = (X, Y)$ for every $\varrho \in \rho$ and $X \subseteq U_1$, $Y \subseteq U_2$, where $P(U_1)$ and $P(U_2)$ represents the power sets of U_1 and U_2 respectively and ρ is a set of constraints. Further, some set operations on binary soft sets namely, complement of a binary soft set, union, intersection and difference of binary soft sets are defined, and also, the notions of binary soft subset, binary absolute and null soft sets are initiated by [1].

1. (G, ρ) is called a binary soft subset of (H, ρ) over U_1, U_2 if $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ where $G(\varrho) = (X_1, Y_1)$ and $H(\varrho) = (X_2, Y_2)$ for all $\varrho \in \rho$ and is denoted by $(G, \rho) \subseteq (H, \rho)$.

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2. (G, ρ) over U_1, U_2 is called a binary absolute soft set if $G(\varrho) = (U_1, U_2)$ for all $\varrho \in \rho$ and is denoted by $\widetilde{\widetilde{\rho}}$.

3. (H, ρ) over U_1, U_2 is called a binary null soft set if $H(\varrho) = (\emptyset, \emptyset)$ for all $\varrho \in \rho$ and is denoted by $\widetilde{\widetilde{\emptyset}}$.

In 2017, Benchalli et al. [2] coined the concept of binary soft topological spaces and stated the definition of binary soft topology as, a collection τ of binary soft subsets over U_1, U_2 is a binary soft topology over U_1, U_2 if $\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}} \in \tau$ and τ is closed under arbitrary union and finite intersection of binary soft sets. The members of τ are binary soft open sets and their family is denoted by $BSO(U_1, U_2)$. The complements of binary soft open sets are binary soft closed sets, and the structure (U_1, U_2, τ, ρ) is a binary soft topological space. Also the notions of binary soft interior and binary soft closure of binary soft sets are introduced by [2]. Let (A, ρ) be a binary soft subset in (U_1, U_2, τ, ρ), then:

1. binary soft interior of (A, ρ) is denoted by $(A, \rho)^{\circledcirc}$ and is given by the union of all binary soft open sets contained in (A, ρ).

2. binary soft closure of (A, ρ) is denoted by $\overline{\overline{(A, \rho)}}$ and is given by the intersection of all binary soft closed sets containing (A, ρ).

Patil et al. [9], [10] studied new separation axioms in binary soft topological spaces as well as introduced and investigated binary soft functions in binary soft topological spaces. A binary soft function $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ is called binary soft continuous [10], if $f^{-1}(V, \omega) \in BSO(U_1, U_2)$ for all $(V, \omega) \in BSO(V_1, V_2)$. In continuation, Patil et al. [11] studied the compactness and connectedness properties of binary soft topological spaces by introducing the notion of infiniteness in binary soft sets.

The main aim of this work is to study the concept of binary soft locally closed sets and hence to define binary soft submaximal spaces. A new type of binary soft functions namely, BSLC-continuous and BSLC-irresolute functions are defined, which are the generalizations of binary soft continuity. Further, the notions of binary soft contra locally closed sets are introduced and via these ideas BScoLC-continuous and BScoLC-irresolute functions are introduced.

2. Binary Soft Locally Closed Sets

Definition 2.1. *A binary soft set (L, ρ) over U_1, U_2 is said to be a binary soft locally closed set in (U_1, U_2, η, ρ) if $(L, \rho) = (O, \rho) \cap (C, \rho)$, where (O, ρ), $(C, \rho)' \in \eta$.*

The family of all binary soft locally closed sets in (U_1, U_2, η, ρ) is denoted by $BSLC(U_1, U_2)$.

Example 2.2. *Let $U_1 = \{a_1, a_2\}$, $U_2 = \{b_1, b_2\}$ and $\rho = \{\varrho_1, \varrho_2\}$ with*

$$\eta = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}}, \{(\varrho_1, (\{a_1\}, \{b_1\})), (\varrho_2, (\{a_1\}, \{b_1\}))\}, \{(\varrho_1, (\{a_2\}, \{b_1\})), (\varrho_2, (\{a_2\}, \{b_1\}))\}, \{(\varrho_1, (\{a_1, a_2\}, \{b_1\})), (\varrho_2, (\{a_1, a_2\}, \{b_1\}))\}, \{(\varrho_1, (\emptyset, \{b_1\})), (\varrho_2, (\emptyset, \{b_1\}))\}\}.$$

Then, (U_1, U_2, η, ρ) is a binary soft topological space. Here,

$$\begin{aligned} BSLC(U_1, U_2) = \{&\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}}, \{(\varrho_1, (\{a_1\}, \{b_1\})), (\varrho_2, (\{a_1\}, \{b_1\}))\}, \{(\varrho_1, (\{a_2\}, \{b_1\})), (\varrho_2, (\{a_2\}, \{b_1\}))\}, \\ &\{(\varrho_1, (\{a_1, a_2\}, \{b_1\})), (\varrho_2, (\{a_1, a_2\}, \{b_1\}))\}, \{(\varrho_1, (\emptyset, \{b_1\})), (\varrho_2, (\emptyset, \{b_1\}))\}, \\ &\{(\varrho_1, (\{a_2\}, \{b_2\})), (\varrho_2, (\{a_2\}, \{b_2\}))\}, \{(\varrho_1, (\{a_1\}, \{b_2\})), (\varrho_2, (\{a_1\}, \{b_2\}))\}, \\ &\{(\varrho_1, (\emptyset, \{b_2\})), (\varrho_2, (\emptyset, \{b_2\}))\}\{(\varrho_1, (\{a_1, a_2\}, \{b_2\})), (\varrho_2, (\{a_1, a_2\}, \{b_2\}))\}, \\ &\{(\varrho_1, (\{a_1\}, \emptyset)), (\varrho_2, (\{a_1\}, \emptyset))\}, \{(\varrho_1, (\{a_2\}, \emptyset)), (\varrho_2, (\{a_2\}, \emptyset))\}, \\ &\{(\varrho_1, (\{a_1, a_2\}, \emptyset)), (\varrho_2, (\{a_1, a_2\}, \emptyset))\}\}. \end{aligned}$$

Remark 2.3. *Every binary soft open and binary soft closed sets are binary soft locally closed.*

Remark 2.4. *A binary soft subset (L, ρ) over U_1, U_2 is binary soft locally closed in (U_1, U_2, η, ρ) if and only if $(L, \rho)'$ is the union of binary soft open and binary soft closed set.*

Remark 2.5. *From Example 2.2, it is clear that the complement of a binary soft locally closed set need not be binary soft locally closed.*

Theorem 2.6. *For a binary soft subset (L, ρ) of (U_1, U_2, η, ρ), the following statements are equivalent:*

1. *(L, ρ) is binary soft locally closed.*
2. *$(L, \rho) = (O, \rho) \cap \overline{\overline{(L, \rho)}}$ for some $(O, \rho) \in \eta$.*
3. *$\overline{\overline{(L, \rho)}} \setminus (L, \rho)$ is binary soft closed.*
4. *$(L, \rho) \cup \left(\overline{\overline{(L, \rho)}}\right)'$ is binary soft open.*

Proof. $(1) \Rightarrow (2)$
Let $(L, \rho) = (O, \rho) \cap (C, \rho)$ for some (O, ρ), $(C, \rho)' \in \eta$.
Since $(L, \rho) \subseteq (O, \rho)$ and $(L, \rho) \subseteq \overline{\overline{(L, \rho)}}$, we have $(L, \rho) \subseteq (O, \rho) \cap \overline{\overline{(L, \rho)}}$.
Again, $\overline{\overline{(L, \rho)}} \subseteq \overline{\overline{(C, \rho)}} = (C, \rho)$. Therefore, $(O, \rho) \cap \overline{\overline{(L, \rho)}} \subseteq (O, \rho) \cap (C, \rho) = (L, \rho)$.
Hence, $(L, \rho) = (O, \rho) \cap \overline{\overline{(L, \rho)}}$.
$(2) \Rightarrow (1)$
$\overline{\overline{(L, \rho)}}$ is binary soft closed. Therefore, $(O, \rho) \cap \overline{\overline{(L, \rho)}} = (L, \rho)$ is binary soft locally closed.
$(2) \Rightarrow (3)$
$\overline{\overline{(L, \rho)}} \setminus (L, \rho) = \overline{\overline{(L, \rho)}} \setminus \left[(O, \rho) \cap \overline{\overline{(L, \rho)}}\right] = \left[\overline{\overline{(L, \rho)}} \setminus (O, \rho)\right] \cup \tilde{\tilde{\emptyset}} = \overline{\overline{(L, \rho)}} \cap (O, \rho)'$.
Therefore, $\overline{\overline{(L, \rho)}} \setminus (L, \rho)$ is binary soft closed.
$(3) \Rightarrow (2)$
Let $(O, \rho) = \left[\overline{\overline{(L, \rho)}} \setminus (L, \rho)\right]'$. Therefore, $(O, \rho) \in \eta$.
Now,
$(O, \rho) \cap \overline{\overline{(L, \rho)}} = \left[\overline{\overline{(L, \rho)}} \setminus (L, \rho)\right]' \cap \overline{\overline{(L, \rho)}} = \left[\overline{\overline{(L, \rho)}} \cap (L, \rho)'\right]' \cap \overline{\overline{(L, \rho)}} = \left[\overline{\overline{(L, \rho)}}' \cup (L, \rho)\right] \cap \overline{\overline{(L, \rho)}} = \tilde{\tilde{\emptyset}} \cup \left[\overline{\overline{(L, \rho)}} \cap (L, \rho)\right] = (L, \rho)$.
$(3) \Rightarrow (4)$
Let $(C, \rho) = \overline{\overline{(L, \rho)}} \setminus (L, \rho)$. Therefore, $(C, \rho)' \in \eta$.
That is, $(C, \rho)' = \left[\overline{\overline{(L, \rho)}} \setminus (L, \rho)\right]' = \left[\overline{\overline{(L, \rho)}} \cap (L, \rho)'\right]' = (L, \rho) \cup \left(\overline{\overline{(L, \rho)}}\right)'$ is binary soft open.
$(4) \Rightarrow (3)$
Let $(O, \rho) = (L, \rho) \cup \left(\overline{\overline{(L, \rho)}}\right)'$ be binary soft open.
Then, $(O, \rho)' = \left[(L, \rho) \cup \left(\overline{\overline{(L, \rho)}}\right)'\right]' = \overline{\overline{(L, \rho)}} \cap (L, \rho)' = \overline{\overline{(L, \rho)}} \setminus (L, \rho)$ is binary soft closed.

■

Remark 2.7. *The family $BSLC(U_1, U_2)$ is closed under finite intersection.*

Remark 2.8. *Union of two binary soft locally closed sets need not be binary soft locally closed.*

In Example 2.2,
$\{(\varrho_1, (\{a_1\}, \{b_1\})), (\varrho_2, (\{a_1\}, \{b_1\}))\}, \{(\varrho_1, (\{a_1\}, \{b_2\})), (\varrho_2, (\{a_1\}, \{b_2\}))\} \in BSLC(U_1, U_2)$. But $\{(\varrho_1, (\{a_1\}, \{b_1, b_2\})), (\varrho_2, (\{a_1\}, \{b_1, b_2\}))\} \notin BSLC(U_1, U_2)$.

Theorem 2.9. *If (A, ρ), $(B, \rho) \in BSLC(U_1, U_2)$ are binary soft separated, then, $(A, \rho) \cup (B, \rho) \in BSLC(U_1, U_2)$.*

Proof. Since, $(A,\rho),(B,\rho)\in BSLC(U_1,U_2)$, there exist $(U,\rho),(V,\rho)\in\eta$ such that
$(A,\rho)=(U,\rho)\cap\overline{\overline{(A,\rho)}}$ and $(B,\rho)=(V,\rho)\cap\overline{\overline{(B,\rho)}}$.
Therefore, $(A,\rho)\cup(B,\rho)=[(U,\rho)\cup(V,\rho)]\cap\overline{\overline{[(A,\rho)\cup(B,\rho)]}}\in BSLC(U_1,U_2)$. ■

Theorem 2.10. *In a binary soft topological space (U_1,U_2,η,ρ), let $(A,\rho)\in BSLC(U_1,U_2)$. If $(B,\rho)\subseteq(A,\rho)$ and $(B,\rho)\in BSLC(A,\eta_A,\rho)$, then, $(B,\rho)\in BSLC(U_1,U_2)$.*

Proof. Since $(B,\rho)\in BSLC(A,\eta_A,\rho)$, $(B,\rho)=(^AO,\rho)\cap(^AC,\rho)$ for some $(^AO,\rho),(^AC,\rho)'\in\eta_A$, where $(O,\rho)\cap(C,\rho)'\in\eta$.
Therefore, $(B,\rho)=[(A,\rho)\cap(O,\rho)]\cap[(A,\rho)\cap(C,\rho)]=(A,\rho)\cap[(O,\rho)\cap(C,\rho)]$.
By Remark 2.7, $(B,\rho)\in BSLC(U_1,U_2)$. ■

Theorem 2.11. *If $(A,\rho)\subseteq(B,\rho)$ in (U_1,U_2,η,ρ) and $(B,\rho)\in BSLC(U_1,U_2)$, then there exist $(U,\rho)\in BSLC(U_1,U_2)$ such that $(A,\rho)\subseteq(U,\rho)\subseteq(B,\rho)$.*

Proof. $(B,\rho)=(O,\rho)\cap\overline{\overline{(B,\rho)}}$ for some $(O,\rho)\in\tau$. Since $(A,\rho)\subseteq(B,\rho)\subseteq(O,\rho)$ and $(A,\rho)\subseteq\overline{\overline{(A,\rho)}}$, we get $(A,\rho)\subseteq(O,\rho)\cap\overline{\overline{(A,\rho)}}=(U,\rho)$, say. Thus, $(U,\rho)\in BSLC(U_1,U_2)$ and $(A,\rho)\subseteq(U,\rho)\subseteq(B,\rho)$. ■

Definition 2.12. *A binary soft topological space (U_1,U_2,η,ρ) is binary soft submaximal if and only if every binary soft subset over U_1, U_2 is binary soft locally closed.*

Definition 2.13. *A binary soft subset (A,ρ) of a binary soft topological space (U_1,U_2,η,ρ) is said to be*

1. *binary soft semi-open if $(A,\rho)\subseteq\overline{\overline{((A,\rho)^{\circledcirc})}}$.*
2. *binary soft pre-open if $(A,\rho)\subseteq\left(\overline{\overline{(A,\rho)}}\right)^{\circledcirc}$ [6].*
3. *binary soft α-open if $(A,\rho)\subseteq\left(\overline{\overline{((A,\rho)^{\circledcirc})}}\right)^{\circledcirc}$.*
4. *binary soft β-open or binary soft semi-pre-open if $(A,\rho)\subseteq\overline{\overline{\left(\overline{\overline{(A,\rho)}}\right)^{\circledcirc}}}$.*

The respective complements of above binary soft sets are known as binary soft semi-closed, binary soft pre-closed [6], binary soft α-closed and binary soft β-closed or binary soft semi-pre-closed sets.

Definition 2.14. *A binary soft function $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ is said to be a binary soft semi-continuous (binary soft pre-continuous, binary soft α-continuous, binary soft β-continuous respectively) if the binary soft inverse image of any binary soft open set in (V_1,V_2,η,ω) is binary soft semi-open (binary soft pre-open, binary soft α-open, binary soft β-open respectively).*

Theorem 2.15. *For a binary soft subset (A,ρ) of (U_1,U_2,τ,ρ), the following statements are equivalent:*

1. *$(A,\rho)\in BSO(U_1,U_2)$.*
2. *$(A,\rho)\in BSLC(U_1,U_2)$ and binary soft α-open.*
3. *$(A,\rho)\in BSLC(U_1,U_2)$ and binary soft pre-open.*

Proof. $(1)\Rightarrow(2)$ and $(2)\Rightarrow(3)$ are obvious.
$(3)\Rightarrow(1)$
$(A,\rho)\subseteq\left(\overline{\overline{(A,\rho)}}\right)^{\circledcirc}$ and $(A,\rho)=(U,\rho)\cap\overline{\overline{(A,\rho)}}$ for some $(U,\rho)\in BSO(U_1,U_2)$.
Now, $(A,\rho)\subseteq(U,\rho)\cap\left(\overline{\overline{(A,\rho)}}\right)^{\circledcirc}=(U,\rho)^{\circledcirc}\cap\left(\overline{\overline{(A,\rho)}}\right)^{\circledcirc}=\left((U,\rho)\cap\overline{\overline{(A,\rho)}}\right)^{\circledcirc}=(A,\rho)^{\circledcirc}$.
Therefore, $(A,\rho)\in BSO(U_1,U_2)$. ■

3. Binary Soft Locally Closed Continuous Maps

Definition 3.1. *A binary soft function $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ is said to be a binary soft locally closed continuous (briefly, BSLC-continuous) map if $f^{-1}(V,\omega)\in BSLC(U_1,U_2)$ for every $(V,\omega)\in BSO(V_1,V_2)$.*

Definition 3.2. *A binary soft function $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ is said to be a binary soft locally closed irresolute (briefly, BSLC-irresolute) map if $f^{-1}(V,\omega)\in BSLC(U_1,U_2)$ for every $(V,\omega)\in BSLC(V_1,V_2)$.*

Theorem 3.3. *Every binary soft continuous function is BSLC-irresolute, but not conversely.*

Proof. Let $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ be binary soft continuous. Then, for any $(V,\omega)\in BSO(V_1,V_2)\subseteq BSLC(V_1,V_2)$, $f^{-1}(V,\omega)\in BSO(U_1,U_2)\subseteq BSLC(U_1,U_2)$. Thus, f is BSLC-irresolute.

Example 3.4. *Let $U_1=\{a_1,a_2\}$, $U_2=\{b_1,b_2\}$, $\rho=\{1,2\}$ and $V_1=\{x_1,x_2\}$, $V_2=\{y_1,y_2\}$, $\omega=\{i,ii\}$ with $\tau=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\rho}},\{(1,(\{a_1\},\{b_1\})),(2,(\{a_1,a_2\},\{b_2\}))\}\}$ and $\eta=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\omega}},\{(i,(\{x_1\},\{y_2\})),(ii,(\emptyset,\{y_1\}))\}\}$.*
Define $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ as $u_1:U_1\to V_1$, $u_2:U_2\to V_2$ and $p:\rho\to\omega$ so that
$u_1(a_1)=x_2$, $u_1(a_2)=x_1$, $u_2(b_1)=y_1$, $u_2(b_2)=y_2$, $p(1)=i$, $p(2)=ii$.
Then, $BSLC(U_1,U_2)=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\rho}},\{(1,(\{a_1\},\{b_1\})),(2,(\{a_1,a_2\},\{b_2\}))\},\{(1,(\{a_2\},\{b_2\})),(2,(\emptyset,\{b_1\}))\}\}$ and $BSLC(V_1,V_2)=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\omega}},\{(i,(\{x_1\},\{y_2\})),(ii,(\emptyset,\{y_1\}))\},\{(i,(\{x_2\},\{y_1\})),(ii,(\{x_1,x_2\},\{y_2\}))\}\}$.
Now, f is BSLC-irresolute but not binary soft continuous.

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Theorem 3.5. *Every BSLC-irresolute map is BSLC-continuous, but not conversely.*

Proof. Let $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ be BSLC-irresolute. Then, for any $(V,\omega)\in BSO(V_1,V_2)\subseteq BSLC(V_1,V_2)$, $f^{-1}(V,\omega)\in BSLC(U_1,U_2)$. Thus, f is BSLC-continuous.

Example 3.6. *Let $U_1=\{a_1,a_2\}$, $U_2=\{b_1,b_2\}$, $\rho=\{1,2\}$ and $V_1=\{x_1,x_2\}$, $V_2=\{y_1,y_2,y_3\}$, $\omega=\{i,ii\}$ with*
$\tau=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\rho}},\{(1,(\{a_1\},\{b_1\})),(2,(\{a_1\},\{b_1\}))\},\{(1,(\{a_2\},\{b_1\})),(2,(\{a_2\},\{b_1\}))\},$
$\{(1,(\{a_1,a_2\},\{b_1\})),(2,(\{a_1,a_2\},\{b_1\}))\},\{(1,(\emptyset,\{b_1\})),(2,(\emptyset,\{b_1\}))\}\}$ *and*
$\eta=\{\widetilde{\widetilde{\emptyset}},\widetilde{\widetilde{\omega}},\{(i,(\{x_1\},\emptyset)),(ii,(\{x_1\},\emptyset))\}\}$.
Define $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ as $u_1:U_1\to V_1$, $u_2:U_2\to V_2$ and $p:\rho\to\omega$ so that
$u_1(a_1)=x_1$, $u_1(a_2)=x_2$, $u_2(b_1)=y_1$, $u_2(b_2)=y_2$, $p(1)=i$, $p(2)=ii$.
Now, f is BSLC-continuous but not BSLC-irresolute.
Because, $\{(i,(\{x_2\},\{y_1,y_2\})),\ (ii,(\{x_2\},\{y_1,y_2\}))\}\in BSLC(V_1,V_2)$ but f^{-1} $(\{(i,(\{x_2\},\{y_1,y_2\})),(ii,(\{x_2\},\{y_1,y_2\}))\})=\{(1,(\{a_2\},\{b_1,b_2\})),(2,(\{a_2\},\{b_1,b_2\}))\}\notin BSLC(U_1,U_2)$

■

Theorem 3.7. *A binary soft topological space (U_1,U_2,τ,ρ) is binary soft submaximal if and only if every binary soft function $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$ is BSLC-continuous, where (V_1,V_2,η,ω) is any binary soft topological space.*

Proof. Consider any $(V,\omega)\in BSO(V_1,V_2)$. Then, for any $f:(U_1,U_2,\tau,\rho)\to(V_1,V_2,\eta,\omega)$, $f^{-1}(V,\omega)\in BSLC(U_1,U_2)$ as (U_1,U_2,τ,ρ) is binary soft submaximal. Therefore, f is BSLC-continuous.
Conversely, take any binary soft subset (A,ρ) over U_1, U_2. Then, there exist some $(V,\omega)\in BSO(V_1,V_2)$ and BSLC-continuous function f so that $f^{-1}(V,\omega)=(A,\rho)\in BSLC(U_1,U_2)$. Hence, (U_1,U_2,τ,ρ) is binary soft submaximal. ■

Theorem 3.8. *Let $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ and $g : (V_1, V_2, \eta, \omega) \to (W_1, W_2, \mu, \sigma)$ be two binary soft functions. Then,*

1. *$g \circ f$ is BSLC-irresolute if both f and g are BSLC-irresolute.*
2. *$g \circ f$ is BSLC-continuous if f is BSLC-irresolute and g is BSLC-continuous.*

Proof. (1) Let f and g are BSLC-irresolute and $(W, \sigma) \in BSLC(W_1, W_2)$. Since g is BSLC-irresolute, $g^{-1}(W, \sigma) \in BSLC(V_1, V_2)$. Further, since f is BSLC-irresolute, $f^{-1}(g^{-1}(W, \sigma)) = (g \circ f)^{-1}(W, \sigma) \in BSLC(U_1, U_2)$. Hence, $g \circ f$ is BSLC-irresolute.

(2) Let f be BSLC-irresolute, g is BSLC-continuous and $(W, \sigma) \in BSO(W_1, W_2)$. Since g is BSLC-continuous, $g^{-1}(W, \sigma) \in BSLC(V_1, V_2)$. Further, since f is BSLC-irresolute, $f^{-1}(g^{-1}(W, \sigma)) = (g \circ f)^{-1}(W, \sigma) \in BSLC(U_1, U_2)$. Hence, $g \circ f$ is BSLC-continuous. ■

Proposition 3.9. *Let $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ be a binary soft function. Then f is binary soft continuous if and only if*

1. *f is both BSLC-continuous and binary soft α-continuous.*
2. *f is both BSLC-continuous and binary soft pre-continuous.*

Definition 3.10. *If $(A, \rho) \in BSLC(U_1, U_2)$ in (U_1, U_2, τ, ρ) then $(A, \rho)'$ is called a binary soft contra locally closed set.*

The family of all binary soft contra locally closed sets is denoted by $BSLC'(U_1, U_2)$.

Remark 3.11. *If $(B, \rho) \in BSLC'(U_1, U_2)$, then there exist some (G, ρ), $(H, \rho)' \in BSO(U_1, U_2)$ so that $(B, \rho) = (G, \rho) \cup (H, \rho)$.*

Example 3.12. *In Example 2.2,*

$$\begin{aligned} BSLC'(U_1, U_2) = \{&\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}}, \{(\varrho_1, (\{a_1\}, \{b_1\})), (\varrho_2, (\{a_1\}, \{b_1\}))\}, \{(\varrho_1, (\{a_2\}, \{b_1\})), (\varrho_2, (\{a_2\}, \{b_1\}))\}, \\ &\{(\varrho_1, (\{a_1, a_2\}, \{b_1\})), (\varrho_2, (\{a_1, a_2\}, \{b_1\}))\}, \{(\varrho_1, (\emptyset, \{b_1\})), (\varrho_2, (\emptyset, \{b_1\}))\}, \\ &\{(\varrho_1, (\{a_2\}, \{b_2\})), (\varrho_2, (\{a_2\}, \{b_2\}))\}, \{(\varrho_1, (\{a_1\}, \{b_2\})), (\varrho_2, (\{a_1\}, \{b_2\}))\}, \\ &\{(\varrho_1, (\emptyset, \{b_2\})), (\varrho_2, (\emptyset, \{b_2\}))\}\{(\varrho_1, (\{a_1, a_2\}, \{b_2\})), (\varrho_2, (\{a_1, a_2\}, \{b_2\}))\}, \\ &\{(\varrho_1, (\{a_1\}, \{b_1, b_2\})), (\varrho_2, (\{a_1\}, \{b_1, b_2\}))\}, \{(\varrho_1, (\{a_2\}, \{b_1, b_2\})), (\varrho_2, (\{a_2\}, \{b_1, b_2\}))\}, \\ &\{(\varrho_1, (\emptyset, \{b_1, b_2\})), (\varrho_2, (\emptyset, \{b_1, b_2\}))\}\}. \end{aligned}$$

Remark 3.13. *Every binary soft open and binary soft closed sets are binary soft contra locally closed sets.*

Theorem 3.14. *A binary soft set $(A, \rho) \in BSLC'(U_1, U_2)$ if and only if $(A, \rho) = (A, \rho)^{\circledcirc} \cup (C, \rho)$ for some binary soft closed set (C, ρ).*

Proof. We have, $(A, \rho) = (O, \rho) \cup (C, \rho)$ for some (O, ρ), $(C, \rho)' \in BSO(U_1, U_2)$. Since, $(C, \rho) \subseteq (A, \rho)$ and $(A, \rho)^{\circledcirc} \subseteq (A, \rho)$, we get $(A, \rho)^{\circledcirc} \cup (C, \rho) \subseteq (A, \rho)$.
Also, $(O, \rho) \subseteq (A, \rho)^{\circledcirc}$ as $(O, \rho) \subseteq (A, \rho)$. Now, $(O, \rho) \cup (C, \rho) = (A, \rho) \subseteq (A, \rho)^{\circledcirc} \cup (C, \rho)$.
Hence, $(A, \rho) = (A, \rho)^{\circledcirc} \cup (C, \rho)$. ■

Theorem 3.15. *If a binary soft contra locally closed set is binary soft pre-closed, then it is binary soft closed.*

Proof. We have, $(A, \rho) = (A, \rho)^{\circledcirc} \cup (C, \rho)$ for some binary soft closed set (C, ρ) as well as $\overline{\overline{(A, \rho)^{\circledcirc}}} \subseteq (A, \rho)$. Now, $\overline{\overline{(C, \rho)}} \cup \overline{\overline{(A, \rho)^{\circledcirc}}} \subseteq (A, \rho)$, since (C, ρ) is binary soft closed. Therefore, $\overline{\overline{(C, \rho) \cup (A, \rho)^{\circledcirc}}} = \overline{\overline{(A, \rho)}} \subseteq (A, \rho)$. Hence, (A, ρ) is binary soft closed. ■

Remark 3.16. *The family $BSLC'(U_1, U_2)$ is closed under union.*

Remark 3.17. *Intersection of two binary soft contra locally closed sets need not be binary soft contra locally closed.*

In Example 2.2,
$\{(\varrho_1, (\{a_1\}, \{b_1\})), (\varrho_2, (\{a_1\}, \{b_1\}))\}, \{(\varrho_1, (\{a_1\}, \{b_2\})), (\varrho_2, (\{a_1\}, \{b_2\}))\} \in BSLC'(U_1, U_2)$. But $\{(\varrho_1, (\{a_1\}, \emptyset)), (\varrho_2, (\{a_1\}, \emptyset))\} \notin BSLC'(U_1, U_2)$.

Definition 3.18. *A binary soft function $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ is called a binary soft contra locally closed continuous (briefly, BScoLC-continuous) function if for any $(V, \omega) \in BSO(V_1, V_2)$, $f^{-1}(V, \omega) \in BSLC'(U_1, U_2)$.*

Definition 3.19. *A binary soft function $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ is called a binary soft contra locally closed irresolute (briefly, BScoLC-irresolute) function if for any $(V, \omega) \in BSLC(V_1, V_2)$, $f^{-1}(V, \omega) \in BSLC'(U_1, U_2)$.*

Theorem 3.20. *Every binary soft continuous function is BScoLC-irresolute.*

Theorem 3.21. *Every BScoLC-irresolute function is BScoLC-continuous.*

Remark 3.22. *The binary soft functions BSLC-continuous and BScoLC-continuous are independent.*

Example 3.23. *In Example 3.6, f is BSLC-continuous but not BScoLC-continuous.*
Further, consider $U_1 = \{a_1, a_2\}$, $U_2 = \{b_1, b_2\}$, $\rho = \{1, 2\}$ and
$V_1 = \{x_1, x_2\}$, $V_2 = \{y_1, y_2\}$, $\omega = \{i, ii\}$ with

$$\tau = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}}, \{(1, (\{a_1\}, \{b_1\})), (2, (\{a_1\}, \{b_1\}))\}, \{(1, (\{a_2\}, \{b_1\})), (2, (\{a_2\}, \{b_1\}))\}, \{(1, (\{a_1, a_2\}, \{b_1\})), (2, (\{a_1, a_2\}, \{b_1\}))\}, \{(1, (\emptyset, \{b_1\})), (2, (\emptyset, \{b_1\}))\}\}$$

and $\eta = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\omega}}, \{(i, (\{x_1\}, \{y_1, y_2\})), (ii, (\{x_1\}, \{y_1, y_2\}))\}\}$.
Define $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ as $u_1 : U_1 \to V_1$, $u_2 : U_2 \to V_2$ and $p : \rho \to \omega$ so that $u_1(a_1) = x_1$, $u_1(a_2) = x_2$, $u_2(b_1) = y_1$, $u_2(b_2) = y_1$, $p(1) = i$, $p(2) = ii$.
Now, f is BScoLC-continuous but not BSLC-continuous.

Remark 3.24. *The binary soft functions BSLC-irresolute and BScoLC-irresolute are independent.*

Example 3.25. *Let $U_1 = \{a_1, a_2\}$, $U_2 = \{b_1, b_2\}$, $\rho = \{1, 2\}$;*
$V_1 = \{x_1, x_2\}$, $V_2 = \{y_1, y_2\}$, $\omega = \{i, ii\}$ and
$W_1 = \{c_1, c_2\}$, $W_2 = \{d_1, d_2\}$, $\sigma = \{i, ii\}$ with

$$\tau = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\rho}}, \{(1, (\{a_1\}, \{b_1\})), (2, (\{a_1\}, \{b_1\}))\}, \{(1, (\{a_2\}, \{b_1\})), (2, (\{a_2\}, \{b_1\}))\}, \{(1, (\{a_1, a_2\}, \{b_1\})), (2, (\{a_1, a_2\}, \{b_1\}))\}, \{(1, (\emptyset, \{b_1\})), (2, (\emptyset, \{b_1\}))\}\};$$

$\eta = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\omega}}, \{(i, (\{x_1\}, \emptyset)), (ii, (\{x_1\}, \emptyset))\}\}$ *and*

$\mu = \{\widetilde{\widetilde{\emptyset}}, \widetilde{\widetilde{\sigma}}, \{(i, (\{c_1\}, \{d_1, d_2\})), (ii, (\{c_1\}, \{d_1, d_2\}))\}\}$.
Define $f : (U_1, U_2, \tau, \rho) \to (V_1, V_2, \eta, \omega)$ as $u_1 : U_1 \to V_1$, $u_2 : U_2 \to V_2$ and $p : \rho \to \omega$ so that $u_1(a_1) = x_2$, $u_1(a_2) = x_1$, $u_2(b_1) = y_1$, $u_2(b_2) = y_2$, $p(1) = ii$, $p(2) = i$.
Then, f is BSLC-irresolute but not BScoLC-irresolute.
Define $g : (U_1, U_2, \tau, \rho) \to (W_1, W_2, \mu, \sigma)$ as $u_1 : U_1 \to W_1$, $u_2 : U_2 \to W_2$ and $p : \rho \to \sigma$ so that $u_1(a_1) = c_1$, $u_1(a_2) = c_2$, $u_2(b_1) = d_2$, $u_2(b_2) = d_1$, $p(1) = i$, $p(2) = ii$.
Then, f is BScoLC-irresolute but not BSLC-irresolute.

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