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β-Weak necks of fuzzy automata

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Abstract

In this paper we introduce β-weak necks, β-weak directable, β-weak trap-directable fuzzy automata. We have shown that β-weak necks of fuzzy automaton exists then it is β-weak subautomaton and β-weak kernel. Consequently, we prove a fuzzy automaton is β-weak directable if and only if all pairs of states of D in S are β-weak mergeable and it is an extension of a β-weak strongly directable fuzzy automaton by a β-weak trap-directable fuzzy automaton.

Keywords

β-Weak necks, β-Weak directable, β-Weak trap-directable fuzzy automaton.

AMS Subject Classification

03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

Fuzzy concept is introduced whenever uncertainity occurs. Fuzzy sets are sets whose elements have degree of membership. Fuzzy set was introduced by Zadeh in 1965 [12] which is an extension of classical notion set. Fuzzy set generalize the classical set, since the indicator functions of classical sets are special cases of the membership function of fuzzy set, if the later only take values 0 or 1. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise.

Automata are the prime example of general computational systems over discrete spaces. The incorporation of fuzzy logic into automata theory resulted in fuzzy automata which can handle continuous spaces. Moreover, they able to model uncertainty which inherent in many applications. Fuzzy set ideas have been applied to wide range of scientific areas. W. Z. Wee [11] applied the ideas of fuzzy in automata and language theory. E.S. Santos [9] proposed fuzzy automata as a model of pattern recognition and control systems.

K. S. Fu and R. W. McLaren (1965) worked in applications of stochastic automata as a model of learning systems [4]. The syntactic approach to pattern recognition was examined by K. S. Fu (1982) using formal deterministic and stochastic languages [5]. Friedrich Steimann and Klaus-Peter Adlassnig (1994) dealt with applications of fuzzy automata in the field of Clinical Monitoring [10]. J. N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and languages in their book (2002) [8].

T. Petkovic et al. [1] discussed directable automata, monogenically directable, generalized directable using necks. T. Petkovic et al.[3] introduce and studied trap-directable, trapped automata and other related automata. Also, we refer the survey paper Directable automata and their generalizations were investigated by S. Bogdanovic et al [2]. Further the necks of fuzzy automata were studied and discussed in [6]. In this paper we introduce β-weak necks, β-weak directable, β-weak trap-directable fuzzy automata. Subsequently we discuss their structural characterizations and their properties using β-weak necks. Also, we prove β-weak directable fuzzy automaton is an extension of a β-weak strongly directable fuzzy automaton by a β-weak trap-directable fuzzy automaton.

2. Preliminaries

Definition 2.1. [8] *A fuzzy automaton $S = (D,\ I,\ \psi)$, where,*

D - set of states $\{d_0,\ d_1,\ d_2, ...,\ d_n\}$,

I - alphabets (or) input symbols,

ψ - function from $D \times I \times D \to [0,1]$,

The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $t \in I^*$ is denoted by $|t|$.

Definition 2.2. [8] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. The extended transition function is defined by $\psi^* : D \times I^* \times D \to [0,1]$ and is given by*

$$\psi^*(d_i,\ \lambda,\ d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases}$$

$\psi^*(d_i, tt', d_j) = \vee_{q_r \in D}\{\psi^*(d_i, t, d_r) \wedge \psi(d_r, t', d_j)\}, t \in I^*, t' \in I$.

Definition 2.3. [6] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $D' \subseteq D$. Let ψ' is the restriction of ψ and let $S' = (D',\ I,\ \psi')$. The fuzzy automaton S' is called a subautomaton of S if*
(i) $\psi' : D' \times I \times D' \to [0,1]$ and
(ii) For any $d_i \in D'$ and $\psi'(d_i, t, d_j) > 0$ for some $t \in I^*$, then $d_j \in D'$.

Definition 2.4. [8] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is said to be strongly connected if for every $d_i,\ d_j \in D$, there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$. Equivalently, S is strongly connected if it has no proper subautomaton.*

Definition 2.5. [8] *A relation R on a set D is said to be equivalence relation if it is reflexive, symmetric and transitive.*

Definition 2.6. [7] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. An equivalence relation R on D in S is called congruence relation if $\forall d_i, d_j \in D$ and $t \in I, d_i R d_j$ implies that, then there exists $d_l, d_k \in D$ such that $\psi(d_i, a, d_l) > 0, \psi(d_j, a, d_k) > 0$ and $d_l R d_k$.*

Definition 2.7. [6] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $S' = (D',\ I,\ \psi')$ be a subautomaton of S. A relation $R_{S'}$ on S is defined as follows. For any $d_i, d_j \in D$, we say that $(d_i, d_j) \in R_{S'}$ if and only if either $d_i = d_j$ or $d_i, d_j \in D'$.*

This relation is clearly an equivalence relation and it is also congruence. This relation is called Rees congruence relation on D in S determined by S'. A fuzzy automaton S/S' is called Rees factor fuzzy automaton determined by the relation $R_{S'}$ and it is defined as $S/S' = (\ \overline{D},\ I,\ \psi_{S/S'})$, where $\overline{D} = \{\ [d_i]\ /\ d_i \in D\}$ and $\psi_{S/S'} : \overline{D} \times I \times \overline{D} \to [0,1]$.

Definition 2.8. [6] *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a neck of S if there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$ for every $d_i \in D$.*
In that case d_j is also called t-neck of S and the word t is called a directing word of S.
If S has a directing word, then we say that S is a directable fuzzy automaton.

Remark 2.9. *In this paper we consider only determinstic fuzzy automaton.*

3. β-Weak Necks of Fuzzy Automata

Definition 3.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If S is said to be β-weak fuzzy automaton then $\{\psi(d_i, t', d_j) < \beta\} > 0, \forall t' \in I, \beta =$ Fixed value in [0,1].*

Definition 3.2. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton and let $d_i \in D$. The β-weak subautomaton of S generated by d_i is denoted by $\langle d_i \rangle$. It is given by*
$\langle d_i \rangle = \{\{\ d_j \mid \psi^*(d_i,\ t,\ d_j) < \beta\} > 0,\ t \in I^*$,
$\beta =$ Fixed value in $[0,1]\}$. If it exists, then it is called the β-weak least subautomaton of S containing d_i.

Definition 3.3. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. For any non-empty $D' \subseteq D$, the β-Weak subautomaton of S generated by D' is denoted by $\langle D' \rangle$ and is given by*
$\langle D' \rangle = \{\{\ d_j \mid \psi^*(d_i,\ t,\ d_j) < \beta\} > 0,\ d_i \in D',\ t \in I^*\}$. It is called the β-weak least subautomaton of S containing D'. The β-weak least subautomaton of a fuzzy automaton S if it exists is called the β-weak kernel of S.

Definition 3.4. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a β-weak neck of S if there exists $t \in I^*$ such that $\{\psi^*(d_i,\ t,\ d_j) < \beta\} > 0, \beta \in [0,1]$ for every $d_i \in D$. In that case d_j is also called t-β-weak neck of S and the word t is called a β-weak directing word of S.*
If S has a β-weak directing word, then we say that S is a β-weak directable fuzzy automaton.

Remark 3.5. *1) The set of all β-weak necks of a fuzzy automaton S is denoted by $\beta WN(S)$.*

2) The set of all β-weak directing words of a fuzzy automaton S is denoted by $\beta WDW(S)$.

3) A fuzzy automaton S is called strongly β-weak directable if $D = \beta WN(S)$.

Definition 3.6. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a β-weak trap of S if $\{\psi^*(d_j,\ t,\ d_j) < \beta\} > 0, \forall t \in I^*$.*

If S has exactly one β-weak trap, then S is called one β-weak trap fuzzy automaton. The set of all β-weak traps of a fuzzy automaton S is denoted by $\beta WTR(S)$.

A fuzzy automaton S is called a β-weak trapped fuzzy automaton, for each $d_i \in D$,if there exists a word $t \in I^*$ such that $\{\psi^*(d_i,\ t,\ d_j) < \beta\} > 0,\ \ d_j \in \beta WTR(S)$.

Definition 3.7. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If S has a single β-weak neck, then S is called a β-weak trap-directable fuzzy automaton.*

Definition 3.8. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_i \in D$ is called β-weak reversible if for every* *word $t' \in I^*$, there exists a word $t \in I^*$ such that $\{\psi^*(d_i,\ t't,\ d_i) < \beta\} > 0$ and the set of all β-weak reversible states of S is called the β-weak reversible part of S. It is denoted by $\beta WR(S)$. If it is non-empty, $\beta WR(S)$ is a β-Weak subautomaton of S.*

Definition 3.9. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A fuzzy automaton is called a direct sum of its β-weak subautomata S_α, $\alpha \in Y$, if $S = \cup_{\alpha \in Y} S_\alpha$ and $S_\alpha \cap S_\gamma = \phi$, for every $\alpha,\ \gamma \in Y$ such that $\alpha \neq \gamma$.*

Definition 3.10. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $d_i, d_j \in D$ are said to be β-weak mergeable if there exists a word $t \in I^*$ and $d_k \in D$ such that $\{\psi^*(d_i,t,d_k) < \beta\} > 0 \Leftrightarrow \{\psi^(d_j,t,d_k) < \beta\} > 0$.*

4. Properties of β-Weak Necks of Fuzzy Automata

Theorem 4.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $\beta WN(S) \neq \phi$, then $\beta WN(S)$ is a β-weak subautomaton of S.*

Proof. Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $d_j \in \beta WN(S)$ and $t' \in I^*$. Assume that d_j is a t-β-weak neck of S, for some $t \in I^*$. Then for each $d_i \in D$ Now, we have
$\{\psi^*(d_i,\ tt',\ d_k) < \beta\} > 0 =$
$\wedge_{d_j \in D}\{\psi^*(d_i,\ t,\ d_j), \psi^*(d_j,\ t',\ d_k)\} < \beta,\ \beta > 0$, it means that d_k is a tt'-β-weak neck of S and hence, $d_k \in \beta WN(S)$. Therefore, $\beta WN(S)$ is a subautomaton of S. □

Theorem 4.2. *Let $S = (D,\ I,\ \psi)$ be β-weak directable fuzzy automaton. Then $\beta WN(M)$ is the β-weak kernel of S and $\beta WN(S) = \beta WR(S)$.*

Proof. Let $S = (D,\ I,\ \psi)$ be a β-weak directable fuzzy automaton. Let $d_j \in \beta WN(S)$ and $d_i \in D$. Then $\{\psi^*(d_i,\ t,\ d_j) < \beta\} > 0$, for every $t \in \beta WDW(S)$ and hence $d_j \in \langle d_i \rangle$. Therefore, $\beta WN(S) \subseteq \langle d_i \rangle$, for every $d_i \in D$. This means that $\beta WN(S)$ is a β-weak subautomaton contained in every other β-weak subautomaton of S. Thus, $\beta WN(S)$ is the β-weak kernel of S.
On the other hand, $d_j \in \beta WR(S)$. Then for every $t' \in I^*$, there exists $t \in I^*$ such that $\{\psi^*(d_j,\ t't,\ d_j) < \beta\} > 0$. Consider t as a β-weak directing word.
$\{\psi^*(d_j,\ t't,\ d_j) < \beta\} > 0 =$
$\{\wedge_{d_i \in D}\{\psi^*(d_j,\ t',\ d_i), \psi^*(d_i,\ t,\ q_j)\}\} < \beta,\ \beta > 0$
$\Longrightarrow \{\psi^*(d_i,\ t,\ d_j) < \beta\} > 0$, for every $d_i \in D$.
$\Longrightarrow d_j \in \beta WN(S)$
$\Longrightarrow \beta WR(S) \subseteq \beta WN(S)$.
Let $d_j \in \beta WN(S)$ and let $t' \in I^*$. Then $\{\psi^*(d_i,\ t,\ d_j) < \beta\} > 0$, for every $d_i \in D$. Now,
$\{\psi^*(d_j,\ t',\ d_k) < \beta\} > 0$ for some $d_k \in D$ and $\{\psi^*(d_k,\ t,\ d_j) < \beta\} > 0$
$\Longrightarrow \{\psi^*(d_j,\ t't,\ d_j) < \beta\} > 0$
$\Longrightarrow d_j \in \beta WR(S)$
$\Longrightarrow \beta WN(S) \subseteq \beta WR(S)$. Therefore, $\beta WN(S) = \beta WR(S)$. □

Theorem 4.3. *A fuzzy automaton $S = (D,\ I,\ \psi)$ is β-weak strongly directable fuzzy automaton if and only if it is strongly connected and β-weak directable.*

Proof. Let $S = (D,\ I,\ \psi)$ be a strongly β-weak directable fuzzy automaton. it is β-weak directable. Now we will prove that it is strongly connected, it is enough to show that for any $d_i, d_j \in D$, there exists $t \in I^*$ such that $\{\psi^*(d_i,t,d_j) < \beta\} > 0$. Since $d_j \in \beta WN(S)$ [$\beta WN(S) = D$], $\{\psi^*(d_k,t,d_j) < \beta\} > 0$, for every $d_k \in S$.
Therefore, $\{\psi^*(d_i,t,d_j) < \beta\} > 0$. Thus, S is strongly connected.
Conversely, let S be strongly connected and β-weak directable. Then $\beta WN(S) \neq \phi$ and by Theorem 4.1, $\beta WN(S)$ is β-weak subautomaton of S. But since S is strongly connected, there is no proper subautomaton. Hence, $D = \beta WN(S)$. Thus, S is β-weak strongly directable. □

Theorem 4.4. *A fuzzy automaton $S = (D,\ I,\ \psi)$ is β-weak directable fuzzy automaton if and only if all pairs of states of D are β-weak mergeable.*

Proof. Let $S = (D,\ I,\ \psi)$ β-weak directable fuzzy automaton. Thus there exists a β-weak directing word $t \in I^*$ and $d_j \in D$ such that $\{\psi^*(d_i,\ t,\ d_k) < \beta\} > 0 \forall d_i \in Q$. Let $d_k, d_l \in D$. Then $\{\psi^*(d_k,\ t,\ d_j) < \beta\} > 0 \Leftrightarrow \{\psi^*(d_l,\ t,\ d_j) < \beta\} > 0$. Thus, d_k and d_l are β-weak mergeable.
Conversely, Assume that S is not a β-weak directable fuzzy automaton. Then there exists $t_1 \in I^*$ such that $\{\psi^*(d_i,\ t_1,\ d_k) < \beta\} > 0$ and $\{\psi^*(d_j,\ t_1,\ d_l) < \beta\} > 0$ for $d_i's, d_j's \in D$. Now consider the states d_k and d_l. By hypothesis, the states d_k and $d_l$are β-weak mergeable.
Then there exists a word $t_2 \in I^*$ and $d_m \in D$ such that
$\{\psi^*(d_k,\ t_2,\ d_m) < \beta\} > 0 \Leftrightarrow \{\psi^*(d_l,\ t_2,\ d_m) < \beta\} > 0$. Now, $\{\psi^*(d_i,\ t_1t_2,\ d_m) < \beta\} > 0, \forall d_i \in D$, which is contradiction to our assumption. Hence, S is a β-weak directable fuzzy automaton □

Theorem 4.5. *A fuzzy automaton $S = (D,\ I,\ \psi)$ is β-weak directable if and only if it is an extension of a β-weak strongly directable fuzzy automaton S' by a β-weak trap-directable fuzzy automaton S''.*
(i) $\beta WDW(S'').\beta WDW(S') \subseteq \beta WDW(S) \subseteq \beta WDW(S'') \cap \beta WDW(S')$;
(ii) $\beta WN(S) = S'$.

Proof. Let S be β-weak directable fuzzy automaton. Then $\beta WN(S)$ is non-empty and by Theorem 4.1, $\beta WN(S)$ is a β-weak subautomaton of S.
The Rees factor fuzzy automaton $S/\beta WN(S)$ is also β-weak directable.
Further, by Rees factor, $S/\beta WN(S)$ is a β-weak trap-directable fuzzy automaton and hence, S is an extension of a strongly β-weak directable fuzzy automaton $\beta WN(S)$ by a β-weak trap-directable fuzzy automaton $S/\beta WN(S)$.
Conversely, let S be an extension of strongly β-weak directable fuzzy automaton S' by a β-weak trap-directable fuzzy

automaton S''. Let $t \in \beta WDW(S'')$ and $t' \in \beta WDW(S')$. Then for all $d_i,\ d_j \in D$ we have that $\{\psi^*(d_i,\ t,\ d_k) < \beta\} > 0$, $\{\psi^*(d_j,\ t,\ d_k) < \beta\} > 0$, where $d_k \in S'$.
Hence, $\psi^*(d_i, tt', d_m) = \{\wedge\{\psi^*(d_i, t, d_k), \psi^*(d_k, t', d_m)\}\} < \beta,\ \beta > 0$
Thus, $tt' \in \beta WDW(S)$ and hence, S is a β-weak directable fuzzy automaton.
If $t \in \beta WDW(S'')$ and $t' \in \beta WDW(S')$, then $tt' \in \beta WDW(S)$.
Therefore, $\beta WDW(S'').\beta WDW(S') \subseteq \beta WDW(S)$.
Let $t \in \beta WDW(S)$. Since S is an extension of a strongly β-weak directable fuzzy automaton S' by a β-weak trap-directable fuzzy automaton S''.
Therefore, t is a β-weak directing word of S' and S''.
Hence, $\beta WDW(S) \subseteq \beta WDW(S') \cap \beta WDW(S'')$.
Thus, (i) holds.
By Theorem 4.2, $\beta WN(S)$ is the β-weak kernel of S, so $\beta WN(S) \subseteq S'$.
Conversely, assume that $d_j \in S'$. Since S' is strongly β-weak directable, we conclude that there exists $t' \in \beta WDW(S')$ such that $\{\psi^*(d_i,\ t',\ d_j) < \beta\} > 0$, for every $d_i \in S'$. Hence, for every $d_i \in D$ and $t \in \beta WDW(S'')$, $\{\psi^*(d_i,\ t,\ d_l) < \beta\} > 0$, where $d_l \in S'$.
Now,$\{\psi^*(d_i,\ tt',\ d_j) < \beta\} > 0 =$
$\{\wedge_{d_l\ \in\ S'}\{\psi^*(d_i,\ t,\ q_l), \psi^*(d_l,\ t',\ d_j)\} < \beta\} > 0$. Therefore, $d_j \in \beta WN(S)$ and hence, $\beta WN(S) = S'$.

□

5. Conclusion

We introduce β-weak necks, β-weak directable, β-weak trap-directable fuzzy automata. We show that β-weak necks of fuzzy automata exists then it is β-weak subautomata,β-weak kernel. Consequently we prove a fuzzy automaton is β-weak directable if and only if it is an extension of a β-weak strongly directable fuzzy automaton by a β-weak trap-directable fuzzy automaton.

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