



# Detour domination number of some path and cycle related graphs

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## Abstract

The detour distance  $D(u, v)$  between two vertices of a connected graph  $G$  is the length of a longest path between them. A set  $S$  of vertices of  $G$  is called a *detour dominating set* if every vertex of  $G$  is detour dominated by some vertex in  $S$ . A detour dominating set of minimum cardinality is a *minimum detour dominating set* and its cardinality is the *detour domination number*  $\gamma_D(G)$ . We have investigated detour domination number of larger graphs obtained from path and cycles by means of various graph operations.

## Keywords

Domination number, Detour distance, Detour domination number.

## AMS Subject Classification

05C69, 05C76, 05C12.

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## 1. Introduction

We begin with simple, finite, connected and undirected graph  $G$  with vertex set  $V(G)$  and edge set  $E(G)$ . For all standard terminology and notations we follow Harary [7] as well as Buckley and Harary [1] while the terms related to the theory of domination in graphs are used in the sense of Haynes *et al.* [8]. We will give brief summary of definitions which are useful for the present investigations.

**Definition 1.1.** A set  $S \subseteq V$  of vertices in a graph  $G = (V, E)$  is called a *dominating set* if every vertex  $v \in V$  is either an element of  $S$  or is adjacent to an element of  $S$ . A dominating set  $S$  is a *minimal dominating set* if no proper subset  $S' \subset S$  is a dominating set. The *domination number*  $\gamma(G)$  of a graph  $G$  is the minimum cardinality of a dominating set in graph  $G$ .

**Definition 1.2.** The distance  $d(u, v)$  between two vertices  $u$  and  $v$  in a connected graph  $G$  is the length of the shortest  $u - v$  path in  $G$ .

**Definition 1.3.** The detour distance  $D(u, v)$  between two vertices  $u$  and  $v$  in a connected graph  $G$  is the length of a longest  $u - v$  path in  $G$ .

The concept of detour distance was introduced by Chartrand *et al.* in [4, 5] while several results concerning detour distance and detour graphs are derived by Chartrand *et al.* [3]. Chartrand and Zhang [6] have also derived several results on detour distance, including connection of detour distance to domination, coloring and Hamiltonian properties of graphs. Let  $G$  be a nontrivial connected graph and for a vertex  $v$  in  $G$ , define

$$\bar{D}(v) = \min\{D(u, v) : u \in V(G) - \{v\}\}$$

A vertex  $u (u \neq v)$  is called a *detour neighbor* of  $v$  if  $D(u, v) = \bar{D}(v)$ . The set of all detour neighbors of  $v$  is denoted by  $N_D(v)$ . In graph  $G$  of Figure 1,  $N_D(v_1) = \{v_3\}$ ,  $N_D(v_2) = \{v_5\}$ ,  $N_D(v_3) = \{v_1, v_5\}$ ,  $N_D(v_4) = \{v_1, v_2, v_3, v_5\}$ ,  $N_D(v_5) = \{v_6\}$  and  $N_D(v_6) = \{v_5\}$ . If  $u$  is a detour neighbor of  $v$ , then  $v$  is not necessarily a detour neighbor of  $u$ . For example in Figure 1,  $v_5$  is a detour neighbor of  $v_2$  but  $v_2$  is not a detour neighbor of  $v_5$  and  $v_1, v_2, v_3$  and  $v_5$  are detour neighbors of  $v_4$  but  $v_4$  is not detour neighbor of any vertex from  $\{v_1, v_2, v_3, v_5\}$ .

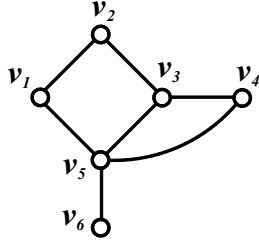


Figure 1. Illustrating detour neighbors

**Definition 1.4.** A vertex  $v$  is said to detour dominate a vertex  $u$  if  $u = v$  or  $u$  is detour neighbor of  $v$ .

A set  $S$  of vertices of  $G$  is called a detour dominating set if every vertex of  $G$  is detour dominated by some vertex in  $S$ . A detour dominating set of minimum cardinality is a minimum detour dominating set and its cardinality is the detour domination number  $\gamma_D(G)$ .

For the graph  $G$  in Figure 1,  $S = \{v_4\}$  is a detour dominating set of minimum cardinality. The concept of detour dominating set have been introduced in recent past by Chartrand *et al.* [2]. It is very interesting to investigate detour domination number of a graph as the detour domination numbers of very few graphs are known. Vaidya and Mehta [11] have derived detour domination number of degree splitting graph and helm graph while detour domination number of some cycle related graphs are discussed by Vaidya and Karkar [10]. Connected graph of order  $p$  with detour domination number  $p$  or  $p - 1$  is characterized by John and Arianayagam [9]. The problems to investigate detour domination number of larger graph (super graph) obtained from the given graph are challenging and interesting as well. We have explored such problems in the context of corona of two graphs.

## 2. Main Results

**Definition 2.1.** The middle graph  $M(G)$  of a graph  $G$  is the graph whose vertex set is  $V(G) \cup E(G)$  and in which two vertices are adjacent whenever either they are adjacent edges of  $G$  or one is a vertex of  $G$  and the other is an edge incident with it.

**Theorem 2.2.**  $\gamma_D(M(P_n)) = \left\lceil \frac{n}{2} \right\rceil + 1, n \geq 3$

*Proof.* Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices and  $e_1, e_2, e_3, \dots, e_{n-1}$  be the edges of path  $P_n$ . Then,  $V(M(P_n)) = \{v_1, v_2, v_3, \dots, v_n, e_1, e_2, e_3, \dots, e_{n-1}\}$ . The detour distance between any two vertices of graph is given below.

$$\begin{aligned} D(e_i, e_{i-1}) &= 2 \text{ for } 2 \leq i \leq n-1 \\ D(e_i, e_{i+1}) &= 2 \text{ for } 1 \leq i \leq n-2 \\ D(e_i, v_i) &= 2 \text{ for } 1 \leq i \leq n-1 \\ D(e_i, v_{i+1}) &= 2 \text{ for } 1 \leq i \leq n-2 \\ D(e_i, e_j) &> 2 \text{ for } 1 \leq i \leq n-1, 1 \leq j \leq n-1, j \neq i-1, i+1 \\ D(e_i, v_j) &> 2 \text{ for } 1 \leq i \leq n-1, 1 \leq j \leq n, j \neq i, i+1 \\ D(v_i, v_j) &> 2 \text{ for } 1 \leq i, j \leq n \\ D(v_i, e_j) &= 2 \text{ for } 2 \leq i \leq n-1, 1 \leq j \leq n-1, j = i-1, i \end{aligned}$$

$$D(v_i, e_j) > 2 \text{ for } 2 \leq i \leq n-1, 1 \leq j \leq n-1, j \neq i-1, i \\ D(v_1, e_1) = D(e_1, v_1) = D(v_n, e_{n-1}) = D(e_{n-1}, v_n) = 1$$

From the above pattern and definition of detour domination the vertices  $v_1$  and  $e_1$  detour dominate each other only while the vertices  $v_n$  and  $e_{n-1}$  detour dominate each other only. Therefore, either  $v_1$  or  $e_1$  must be in detour dominating set  $D$  as well as either  $v_n$  or  $e_{n-1}$  must be in detour dominating set  $D$ . Now the vertex  $v_i$  detour dominates only three vertices  $e_{i-1}, e_i$  and itself while  $e_i$  detour dominates five vertices  $e_{i-1}, e_{i+1}, v_i, v_{i+1}$  and itself. Thus, to obtain detour dominating set  $D$  of minimum cardinality, as  $e_i$  detour dominates its all neighbor, we should include  $e_2, e_4, \dots, e_{n-2}$  in  $D$ . So,  $D = \{e_1, e_2, e_4, \dots, e_{n-2}, e_{n-1}\}$  and  $D = \{v_1, e_2, e_4, \dots, e_{n-2}, v_n\}$  are detour dominating sets of minimum cardinality. Therefore, total  $\left\lceil \frac{n-2}{2} \right\rceil$  internal vertices of degree four from  $M(P_n)$  and either two end vertices  $v_1$  and  $v_n$  or two internal vertices of degree three  $e_1$  and  $e_{n-1}$  must be in  $D$ . Hence,

$$\gamma_D(M(P_n)) = \left\lceil \frac{n-2}{2} \right\rceil + 2 = \left\lceil \frac{n}{2} \right\rceil + 1$$

□

**Illustration 2.3.** For the graph  $M(P_6)$  in Figure 2,  $S = \{e_1, e_2, e_4, e_5\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(M(P_6)) = 4$ .

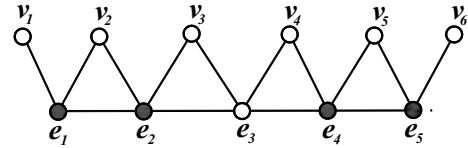


Figure 2

**Theorem 2.4.**  $\gamma_D(M(C_n)) = \begin{cases} n & n \text{ is odd} \\ \frac{n}{2} \text{ or } \frac{n}{2} + 1 & n \text{ is even.} \end{cases}$

*Proof.* Let  $v_1, v_2, v_3, \dots, v_n$  be the vertices and  $e_1, e_2, e_3, \dots, e_n$  be the edges of cycle  $C_n$ . Then,  $V(M(C_n)) = \{v_1, v_2, v_3, \dots, v_n, e_1, e_2, e_3, \dots, e_n\}$ .

**Case (i):**  $n$  is odd.

The detour distance between any two vertices of graph is given below.

$$\begin{aligned} D(v_i, e_j) &= n+1 \text{ for } 1 \leq i \leq \frac{n+1}{2}, j = i + \frac{n-1}{2} \\ D(v_i, e_j) &= n+1 \text{ for } \frac{n+1}{2} < i \leq n, j = i - \frac{n+1}{2} \\ D(v_i, e_j) &> n+1 \text{ otherwise} \\ D(v_i, v_j) &> n+1 \text{ for } 1 \leq i, j \leq n \\ D(e_i, e_j) &= n+1 \text{ for } 1 \leq i \leq \frac{n-1}{2}, j = i + \frac{n-1}{2}, i + \frac{n+1}{2} \\ D(e_{\frac{n+1}{2}}, e_1) &= D(e_{\frac{n+1}{2}}, e_n) = n+1 \\ D(e_i, e_j) &= n+1 \text{ for } \frac{n+1}{2} < i \leq n, j = i - \frac{n-1}{2}, i - \frac{n+1}{2} \end{aligned}$$



$$D(e_i, e_j) > n + 1 \text{ otherwise}$$

$$D(v_i, e_j) = D(e_j, v_i)$$

From the above pattern and definition of detour domination every  $v_i$  detour dominates only one vertex from  $\{e_1, e_2, e_3, \dots, e_n\}$  other than itself such that  $N_D[v_i] \cap N_D[v_j] = \emptyset$ . Now each  $e_i$  detour dominates two vertices from  $\{e_1, e_2, e_3, \dots, e_n\}$  other than itself and one vertex from  $\{v_1, v_2, v_3, \dots, v_n\}$  such that  $N_D[e_i] \cap N_D[e_j] \neq \emptyset$  but  $v_i \notin N_D[e_i] \cap N_D[e_j]$  for every  $v_i, 1 \leq i \leq n$ . Therefore, we need atleast  $n$  vertices to detour dominate all the vertices of the graph. Thus,  $\{e_1, e_2, e_3, \dots, e_n\}$  and  $\{v_1, v_2, v_3, \dots, v_n\}$  are detour dominating sets of minimum cardinality. Hence,  $\gamma_D(M(C_n)) = n$ .

**Case (ii):**  $n$  is even.

$$D(e_i, e_j) = n \text{ for } 1 \leq i \leq \frac{n}{2}, j = i + \frac{n}{2}$$

$$D(e_i, e_j) = n \text{ for } \frac{n}{2} < i \leq n, j = i - \frac{n}{2}$$

$$D(e_i, e_j) > n \text{ otherwise}$$

$$D(e_i, v_j) > n \text{ for } 1 \leq i, j \leq n$$

$$D(v_i, v_j) = n + 2 \text{ for } 1 \leq i \leq \frac{n}{2}, j = i + \frac{n}{2}$$

$$D(v_i, v_j) = n + 2 \text{ for } \frac{n}{2} < i \leq n, j = i - \frac{n}{2}$$

$$D(v_i, v_j) > n + 2 \text{ otherwise}$$

$$D(v_i, e_j) = n + 2 \text{ for } 1 \leq i \leq \frac{n}{2}, j = i + \frac{n}{2}, i + \frac{n}{2} - 1$$

$$D(v_i, e_j) = n + 2 \text{ for } \frac{n}{2} < i \leq n, j = i - \frac{n}{2}, i - \frac{n}{2} - 1$$

$$D(v_i, e_j) > n + 2 \text{ otherwise}$$

**Subcase (i):**  $\frac{n}{2}$  is odd.

From the above pattern and definition of detour domination every  $e_i$  detour dominates two vertices including itself while every  $v_i$  detour dominates four vertices including itself such that  $N_D[v_i] \cap N_D[v_j] = \emptyset$  where  $i$  and  $j$  both are even or odd together. But  $\cup N_D[v_i] = V(M(C_n))$  where  $i = 1, 3, 5, \dots, n-1$  or  $i = 2, 4, 6, \dots, n$ . Therefore,  $\{v_1, v_3, v_5, \dots, v_{n-1}\}$  or  $\{v_2, v_4, v_6, \dots, v_n\}$  are detour dominating set of minimum cardinality. Hence,  $\gamma_D(M(C_n)) = \frac{n}{2}$ .

**Subcase (ii):**  $\frac{n}{2}$  is even.

From the above pattern and definition of detour domination every  $e_i$  detour dominates two vertices including itself while every  $v_i$  detour dominates four vertices including itself such that  $N_D[v_i] \cap N_D[v_j] = \emptyset$  where  $i$  and  $j$  both are odd for  $1 \leq i, j \leq \frac{n}{2}$  and  $N_D[v_i] \cap N_D[v_j] = \emptyset$  where  $i$  and  $j$  both are even for  $\frac{n}{2} < i, j \leq n$ . Therefore, to obtain detour dominating set of minimum cardinality we must include  $v_1, v_3, \dots, v_{\frac{n}{2}-1}$  vertices from  $\{v_1, v_2, \dots, v_{\frac{n}{2}}\}$  and  $v_{\frac{n}{2}+2}, v_{\frac{n}{2}+4}, \dots, v_n$  from  $\{v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \dots, v_n\}$ . But to detour dominate the vertex  $e_n$  we must include the vertex  $v_{\frac{n}{2}}$  or  $v_{\frac{n}{2}+1}$  or the vertex itself in detour dominating set  $D$ . Thus,  $\{v_1, v_3, \dots, v_{\frac{n}{2}-1}, v_{\frac{n}{2}}, v_{\frac{n}{2}+2}, v_{\frac{n}{2}+4}, \dots, v_n\}$  is a detour dominating set of minimum cardinality. Hence,  $\gamma_D(M(C_n)) = \frac{n}{2} + 1$ . □

**Illustration 2.5.** For the graph  $M(C_5)$  in Figure 3,  $S = \{v_1, v_2,$

$v_3, v_4, v_5\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(M(C_5)) = 5$ .

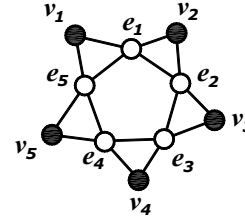


Figure 3

**Definition 2.6.** Let  $G$  and  $H$  be two graphs on  $n$  and  $m$  vertices, respectively. The corona of the graphs  $G$  and  $H$  denoted by  $G \circ H$  and is defined as the graph obtained by taking one copy of  $G$  and  $n$  copies of  $H$ , and then joining the  $i^{\text{th}}$  vertex of  $G$  to every vertex in the  $i^{\text{th}}$  copy of  $H$ .

**Theorem 2.7.**  $\gamma_D(P_n \circ P_m) = n$

*Proof.* Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$  and  $V(P_m) = \{u_1, u_2, \dots, u_m\}$ . In  $P_n \circ P_m$ , let's denote the vertices of  $i^{\text{th}}$  copy of the graph  $P_m$  by  $u_1^i, u_2^i, \dots, u_m^i$  for  $1 \leq i \leq n$ . The detour distance between any two vertices of graph is given below.

$$D(u_k^i, u_l^i) = m \text{ for } 1 \leq i \leq n, 1 \leq k, l \leq m$$

$$D(u_k^i, v_i) = m \text{ for } 1 \leq i \leq n, k = 1, m$$

$$D(u_k^i, v_i) < m \text{ for } 1 \leq i \leq n, 2 \leq k \leq m - 1$$

$$D(u_k^i, u_l^j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k, l \leq m, i \neq j$$

$$D(u_k^i, v_j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k \leq m, i \neq j$$

$$D(v_i, v_j) > 1 \text{ for } |i - j| > 1, 1 \leq i, j \leq n$$

$$D(v_i, u_k^l) > 1 \text{ for } 1 \leq i, l \leq n, 1 \leq k \leq m$$

$$D(v_i, v_{i+1}) = 1 \text{ for } 1 \leq i \leq n - 1$$

From the above pattern and definition of detour domination the vertices  $u_1^i$  and  $u_m^i$  detour dominate all the vertices of  $i^{\text{th}}$  copy of  $P_m$  as well as the vertex  $v_i$ . But any vertex of  $i^{\text{th}}$  copy of  $P_m$  can not detour dominate any vertex of  $j^{\text{th}}$  copy of  $P_m$ . Every  $v_i$  detour dominates its neighbors only. Therefore, it is enough to consider every  $u_1^i$  or every  $u_m^i, 1 \leq i \leq n$  in detour dominating set  $D$  to obtain detour dominating set of minimum cardinality. Hence,  $D = \{u_1^1, u_1^2, \dots, u_1^n\}$  and  $\{u_m^1, u_m^2, \dots, u_m^n\}$  become detour dominating set of minimum cardinality. Therefore, as there are  $n$  copies of  $P_m$  in  $P_n \circ P_m$  it is enough to consider  $n$  vertices in  $D$ . Hence,

$$\gamma_D(P_n \circ P_m) = n. \quad \square$$

**Illustration 2.8.** For the graph  $P_4 \circ P_3$  in Figure 4,  $S = \{u_1^1, u_1^2, u_1^3, u_1^4\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(P_4 \circ P_3) = 4$ .

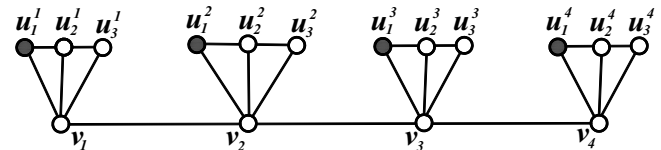


Figure 4

**observation 2.9.**  $\gamma_D(P_n \circ K_1) = n$

**Theorem 2.10.**  $\gamma_D(P_n \circ C_m) = n$

*Proof.* Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$  and  $V(C_m) = \{u_1, u_2, \dots, u_m\}$ . In  $P_n \circ C_m$ , let's denote the vertices of  $i^{th}$  copy of the graph  $P_m$  by  $u_1^i, u_2^i, \dots, u_m^i$  for  $1 \leq i \leq n$ . The detour distance between any two vertices of graph is given below.

$$D(u_k^i, u_l^i) = m \text{ for } 1 \leq i \leq n, 1 \leq k, l \leq m$$

$$D(u_k^i, v_i) = m \text{ for } 1 \leq i \leq n, 1 \leq k \leq m$$

$$D(u_k^i, u_l^j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k, l \leq m, i \neq j$$

$$D(u_k^i, v_j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k \leq m, i \neq j$$

$$D(v_i, v_j) > 1 \text{ for } |i - j| > 1, 1 \leq i, j \leq n$$

$$D(v_i, u_l^i) > 1 \text{ for } 1 \leq i, k \leq n, 1 \leq l \leq m$$

$$D(v_i, v_{i+1}) = 1 \text{ for } 1 \leq i \leq n - 1$$

From the above pattern and definition of detour domination the vertices  $u_k^i$ ,  $1 \leq i \leq n$ ,  $1 \leq k \leq m$  detour dominates all the vertices of  $i^{th}$  copy of  $C_m$  as well as the vertex  $v_i$ . But any vertex of  $i^{th}$  copy of  $C_m$  can not detour dominate any vertex of  $j^{th}$  copy of  $C_m$ . Every  $v_i$  detour dominates its neighbors only. Therefore, it is enough to consider any one vertex from each copy of  $C_m$  in detour dominating set to obtain detour dominating set of minimum cardinality.

$$\gamma_D(P_n \circ C_m) = n$$

□

**Illustration 2.11.** For the graph  $P_4 \circ C_3$  in Figure 5,  $S = \{u_1^1, u_2^1, u_3^1, u_4^1\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(P_4 \circ C_3) = 4$ .

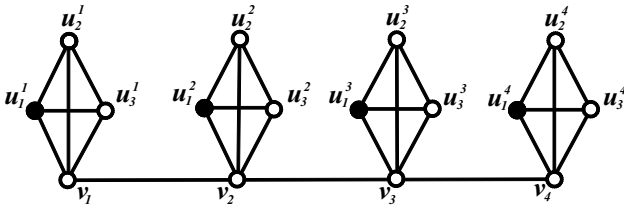


Figure 5

**Theorem 2.12.**  $\gamma_D(C_n \circ P_m) = n$

*Proof.* Let  $V(C_n) = \{v_1, v_2, \dots, v_n\}$  and  $V(P_m) = \{u_1, u_2, \dots, u_m\}$ . In  $C_n \circ P_m$ , let's denote the vertices of  $i^{th}$  copy of the graph  $P_m$  by  $u_1^i, u_2^i, \dots, u_m^i$  for  $1 \leq i \leq n$ . The detour distance between any two vertices of graph is given below.

$$D(u_k^i, u_l^i) = m \text{ for } 1 \leq i \leq n, 1 \leq k, l \leq m$$

$$D(u_k^i, v_i) = m \text{ for } 1 \leq i \leq n, k = 1, m$$

$$D(u_k^i, v_i) < m \text{ for } 1 \leq i \leq n, 2 \leq k \leq m - 1$$

$$D(u_k^i, u_l^j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k, l \leq m, i \neq j$$

$$D(u_k^i, v_j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k \leq m, i \neq j$$

$$D(v_i, v_j) = \left\lceil \frac{n}{2} \right\rceil \text{ for } 1 \leq i, j \leq n$$

$$D(v_i, u_k^i) = m \text{ for } 1 \leq i \leq n, k = 1, m$$

$$D(v_i, u_k^i) < m \text{ for } 1 \leq i \leq n, 2 \leq k \leq m - 1$$

$$D(v_i, u_k^j) > m \text{ for } 1 \leq i, k \leq m, 1 \leq j \leq n, i \neq j$$

From the above pattern and definition of detour domination the vertices  $u_1^i$  and  $u_m^i$  detour dominate all the vertices of  $i^{th}$  copy of  $P_m$  as well as the vertex  $v_i$ . But any vertex of  $i^{th}$  copy of  $P_m$  can not detour dominate any vertex of  $j^{th}$  copy of  $P_m$ . Every  $v_i$  can not detour dominate more than four vertices of  $C_n \circ P_m$ . Therefore, it is enough to consider every  $u_1^i$  or every  $u_m^i$  for  $1 \leq i \leq n$  in detour dominating set  $D$  to obtain detour dominating set of minimum cardinality. Hence,  $D = \{u_1^1, u_1^2, \dots, u_1^n\}$  and  $\{u_m^1, u_m^2, \dots, u_m^n\}$  become detour dominating set of minimum cardinality. Therefore, as there are  $n$  copies of  $P_m$  in  $C_n \circ P_m$  it is enough to consider  $n$  vertices in  $D$ .

$$\gamma_D(C_n \circ P_m) = n$$

□

**Illustration 2.13.** For the graph  $C_5 \circ P_2$  in Figure 6,  $S = \{u_1^1, u_2^1, u_3^1, u_4^1, u_5^1\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(C_5 \circ P_2) = 5$ .

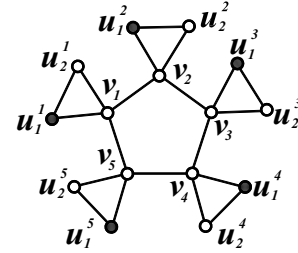


Figure 6

**Theorem 2.14.**  $\gamma_D(C_n \circ C_m) = n$

*Proof.* Let  $V(C_n) = \{v_1, v_2, \dots, v_n\}$  and  $V(C_m) = \{u_1, u_2, \dots, u_m\}$ . In  $C_n \circ C_m$ , let's denote the vertices of  $i^{th}$  copy of the graph  $C_m$  by  $u_1^i, u_2^i, \dots, u_m^i$  for  $1 \leq i \leq n$ . The detour distance between any two vertices of graph is given below.

$$D(u_k^i, u_l^i) = m \text{ for } 1 \leq i \leq n, 1 \leq k, l \leq m$$

$$D(u_k^i, v_i) = m \text{ for } 1 \leq i \leq n, 1 \leq k \leq m$$

$$D(u_k^i, u_l^j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k, l \leq m, i \neq j$$

$$D(u_k^i, v_j) > m \text{ for } 1 \leq i, j \leq n, 1 \leq k \leq m, i \neq j$$

$$D(v_i, v_j) = \left\lceil \frac{n}{2} \right\rceil \text{ for } 1 \leq i, j \leq n$$

$$D(v_i, u_k^i) = m \text{ for } 1 \leq i \leq n, 1 \leq k \leq m$$

$$D(v_i, u_k^j) > m \text{ for } 1 \leq i, k \leq m, 1 \leq j \leq n, i \neq j$$

From the above pattern and definition of detour domination the vertices  $u_k^i$  detour dominates all the vertices of  $i^{th}$  copy of  $C_m$  as well as the vertex  $v_i$ . But any vertex of  $i^{th}$  copy of  $C_m$  can not detour dominate any vertex of  $j^{th}$  copy of  $C_m$ . Every  $v_i$  can not detour dominate more than four vertices of  $C_n \circ C_m$ . Therefore, it is enough to consider any one vertex from each copy of  $C_m$  in detour dominating set to obtain detour dominating set of minimum cardinality.

$$\gamma_D(C_n \circ C_m) = n$$

□



**Illustration 2.15.** For the graph  $C_4 \circ C_3$  in Figure 7,  $S = \{u_1^1, u_1^2, u_1^3, u_1^4\}$  is a detour dominating set of minimum cardinality with  $\gamma_D(C_4 \circ C_3) = 4$ .

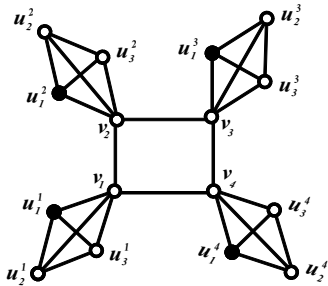


Figure 7

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### Conclusion

The concept of distance dominating set is well studied in various contexts. The present work is also a contribution in the same direction but the usual distance is replaced by detour distance in graphs. We have investigated detour domination number of some path and cycle related graphs.

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