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On nano semi $\widehat{g}$-closed sets in nano topological spaces

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Abstract
This paper focuses on $\mathcal{N}s\widehat{g}$-closed sets (nano semi $\widehat{g}$-closed sets) and $\mathcal{N}s\widehat{g}$-open sets (nano semi $\widehat{g}$-open sets) in nano topological spaces and certain properties are investigated. We also investigate and discussed their relationships with other forms of nano sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Keywords
$\mathcal{N}g$-closed sets, $\mathcal{N}s\widehat{g}$-closed sets and $\mathcal{N}s\widehat{g}$-open sets.

AMS Subject Classification
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Contents

1. Introduction

In 1970, Levine [10] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al [1] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2008, Jafari et.al [6] introduced $\widehat{g}$-closed sets in topological spaces.

The notion of nano topology was introduced by Lellis Thivagar [8] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari and Mythili Gnanapriya [2], introduced and studied the concept of Nano generalized-closed sets.

The structure of this manuscript is as follows. In section 2, we recall some fundamental definitions and results which are useful to prove our main results. In section 3 and 4, we define and study the notion of nano semi $\widehat{g}$-closed sets and nano semi $\widehat{g}$-open sets in nano topological spaces. We also discuss the concept of nano semi $\widehat{g}$-closed sets and discussed the relationships between the other existing nano sets.

2. Preliminaries

Definition 2.1. *[8] Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x.*

2. *The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$*

3. *The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by$B_R(X)$. That is, $B_R(X)$ = $U_R(X)$-$L_R(X)$*

Definition 2.2. *[8] If (U,R) is an approximation space and $X,Y \subseteq U$, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$.
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y)$
6. $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
9. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
10. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3. *[8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$*
2. *The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*
3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$*

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ is called the nano topological space.

Definition 2.4. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

1. *Nano semi-closed [8], if $\mathcal{N}int(\mathcal{N}cl(A)) \subseteq A$.*
2. *$\mathcal{N}g$-closed [2], if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.*
3. *$\mathcal{N}gs$-closed [3], if $\mathcal{N}scl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.*
4. *$\mathcal{N}gp$-closed [4], if $\mathcal{N}pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.*
5. *$\mathcal{N}gsp$-closed [11], if $\mathcal{N}spcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.*
6. *$\mathcal{N}\widehat{g}$-closed [7], if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open.*
7. *$\mathcal{N}\star_g$-closed [5], if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano $\widehat{g}$-open.*
8. *Nano $\#_{gs}$-closed [5], if $\mathcal{N}scl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano $\star_g$-open.*
9. *$\mathcal{N}sg$-closed [7], if $\mathcal{N}scl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open.*
10. *$\mathcal{N}\alpha g$-closed [9], if $\mathcal{N}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano-open.*
11. *A is nano locally closed [12], if $A = U \cap F$, where U is nano open , F is nano closed in U .*
12. *A is nano topological space (U, X) is extremely disconnected [13], if the nano closure of each nano open is nano open.*
13. *A is nano regular open [12], if $A = \mathcal{N}int(\mathcal{N}cl(A))$.*

3. Nano semi $\widehat{g}$-closed sets

In this section, we define and study the concept of nano semi $\widehat{g}$-closed (briefly, $\mathcal{N}\mathcal{S}\widehat{g}$-closed) sets in nano topological spaces and obtain some of its properties.

Definition 3.1. *A subset A of a nano topological space $(U, \tau_R(X))$ is called Nano semi $\widehat{g}$-closed set (briefly $\mathcal{N}s\widehat{g}$-closed set) if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano sg-open.*

Theorem 3.2. *Every nano closed set is a $\mathcal{N}s\widehat{g}$-closed but not conversely.*

Proof. Let A be a nano closed subset $(U, \tau_(x))$ and G is any nano sg-open. Since A is nano closed, $\mathcal{N}cl(A) = A \subseteq G$. That is $\mathcal{N}cl(A) \subseteq G$. Hence A is nano semi $\widehat{g}$-closed. ☐

Theorem 3.3. *Every $\mathcal{N}s\widehat{g}$-closed is a $\mathcal{N}\widehat{g}$-closed but not conversely.*

Proof. If A is $\mathcal{N}s\widehat{g}$-closed set in $(U, \tau_R(x))$ and G is any nano semi-open set containing A, $\mathcal{N}cl(A) = G \supseteq A$, sg-closed set. $G \supseteq A = \mathcal{N}cl(A)$ implies $G \supseteq \mathcal{N}cl(A)$. Hence A is $\mathcal{N}\widehat{g}$-closed. ☐

Theorem 3.4. *Every $\mathcal{N}s\widehat{g}$-closed is a $\mathcal{N}g$-closed but not conversely.*

Proof. If A is a $\mathcal{N}s\widehat{g}$-closed subset of $(U, \tau_R(x))$ and G is any open set containing A,since every nano open set is $\mathcal{N}sg$-open,we have $G \supseteq \mathcal{N}cl(A)$. Hence A is $\mathcal{N}g$-closed set. ☐

Example 3.5. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{a\},\{c\}, \{b,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\tau_R(X) = \{U, \phi, \{b,d\}, \{a\}, \{a,b,d\}\}$.*
It is clear that $\{b,c\},\{a,d\}$ is $\mathcal{N}s\widehat{g}$-closed set but it is not in nano closed .Its clear that $\{a,b,c\},\{a,c,d\}$ is in nano $\widehat{g}$ closed set but not in $\mathcal{N}s\widehat{g}$- closed . Its also clear that $\{a,b,c\},\{a,c,d\}$ is in nano g closed set but not in $\mathcal{N}s\widehat{g}$-closed.

Theorem 3.6. *Every $\mathscr{N}s\widehat{g}$-closed is a $\mathscr{N}\alpha g$-closed set.*

Proof. Let A is $\mathscr{N}s\widehat{g}$-closed subset of $\{U,\tau_R(X)\}$ and G is any nano open set containing A, since every nano open set is $\mathscr{N}sg$-open, we have $\alpha cl(A) \subseteq cl(A) \subseteq G$. Hence G is $\mathscr{N}s\widehat{g}$-closed set in $\{U,\tau_R(X)\}$ □

Example 3.7. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{a\},\{c\}, \{b,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\tau_R(X) = \{U,\phi,\{b,d\},\{a\},\{a,b,d\}\}$. Then $\mathscr{N}s\widehat{g}$-closed sets are $\{U,\phi,\{c\},\{a,c\},\{c,d\}\{b,c\}, \{b,c,d\}\}$ and $\mathscr{N}\alpha g$ closed set is $\{U,\phi,\{c\},\{a,c\},\{b,c,d\}\}$. It is clear that $\{b,c\},\{a,d\}$ is in $\mathscr{N}s\widehat{g}$-closed set but it is not in $\mathscr{N}\alpha g$ closed.*

Theorem 3.8. *Every $\mathscr{N}s\widehat{g}$-closed is $\mathscr{N}gs$ closed.*

Proof. If A is a $\mathscr{N}s\widehat{g}$ closed subset of $\{U,\tau_R(X)\}$ and G is any nano open set containing A, since every nano open set is $\mathscr{N}sg$ open, We have $\mathscr{N}scl(A) \subseteq \mathscr{N}cl(A) \subseteq G$. Hence G is $\mathscr{N}gs$ closed set. □

Theorem 3.9. *Every $\mathscr{N}s\widehat{g}$-closed is $\mathscr{N}gsp$ closed.*

Proof. If G is a $\mathscr{N}s\widehat{g}$-closed subset of $\{U,\tau_R(X)\}$ and A is any nano open set containing G, every nano open set is $\mathscr{N}sg$-open, we have $\mathscr{N}spcl(A) \subseteq cl(A) \subseteq G$. Hence A is $\mathscr{N}gsp$ closed . □

Theorem 3.10. *Every $\mathscr{N}s\widehat{g}$-closed set is $\mathscr{N}sg$ closed.*

Proof. If A is a $\mathscr{N}s\widehat{g}$-closed subset of $\{U,\tau_R(X)\}$ and G is any nano open set containing A, since every nano open set is $\mathscr{N}s$ open, We have $\mathscr{N}scl(A) \subseteq cl(A) \subseteq G$. Hence G is $\mathscr{N}sg$ closed . □

Example 3.11. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{c\},\{a,b,d\}\}$ and $X = \{b,c\}$. Then the nano topology, $\tau_R(X)$=$\{U,\phi,\{c\},\{a,b,d\}\}$.*
Then $\mathscr{N}s\widehat{g}$-closed sets are $\{U,\phi,\{c\},\{a,b,d\}\}$.
Here A = $\{a\}$ is $\mathscr{N}sg$- closed but not in $\mathscr{N}s\widehat{g}$ closed. And also A = $\{b,c\}$ is $\mathscr{N}gs$- closed but not in $\mathscr{N}s\widehat{g}$ closed.And also A = $\{a,d\}$ is $\mathscr{N}gsp$-closed but not in $\mathscr{N}s\widehat{g}$ closed.

Theorem 3.12. *Every $\mathscr{N}\alpha$ closed set is $\mathscr{N}s\widehat{g}$ closed.*

Proof. If A is any $\mathscr{N}\alpha$-closed set in and G is any nano sg-open set containing A then $G \supseteq A = \mathscr{N}\alpha cl(A)$. Hence A is $\mathscr{N}\mathscr{S}\widehat{g}$ closed. □

Example 3.13. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{a\},\{c\}, \{b,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $U,\tau_R(X)$= $\{U,\phi,\{b,d\}, \{a\},\{a,b,d\}\}$. Here $\{b,c\}\&\{a,d\}$ is in $\mathscr{N}s\widehat{g}$-closed set but it is not in $\mathscr{N}\alpha$ closed.*

4. Properties of nano semi $\widehat{g}$-closed sets

Lemma 4.1. *If F is a nano closed set of $(U,\tau_R(X))$. Then the following properties hold:*
(i) If A is nano semi- closed in $(U,\tau_R(X))$, then $A \cap F$ is semi-closed in $(U,\tau_R(X))$.
(ii) If A is nano sg-closed in $(U,\tau_R(X))$, then $A \cap F$ is sg-closed in $(U,\tau_R(X))$.

Corollary 4.2. *If A is a $\mathscr{N}s\widehat{g}$- closed set and F is a nano closed set, then $A \cap F$ is a $\mathscr{N}s\widehat{g}$- closed set.*

Proof. If G be a $\mathscr{N}sg$-open set of $(U,\tau_R(X))$ such that $A \cap F \subseteq G$. By lemma it is shows that $A \subseteq G \cup (U/F)$ and $G \cup (U/F)$ is $\mathscr{N}sg$-open in $(U,\tau_R(X))$. Since A is $\mathscr{N}s\widehat{g}$ -closed in $(U,\tau_R(X))$, we have $\mathscr{N}cl(A) \subseteq G \cup (U/F)$ and so $\mathscr{N}cl(A \cap F) \subseteq \mathscr{N}cl(A) \cap \mathscr{N}cl(F) = \mathscr{N}cl(A) \cap F \subseteq (G \cup (U/F) \cap F = G \cap F \subseteq G$. Therefore, $A \cap F$ is $\mathscr{N}s\widehat{g}$- closed in $(U,\tau_R(X))$. □

Proposition 4.3. *If A and B are $\mathscr{N}s\widehat{g}$- closed sets in $(U,\tau_R(X))$,then $A \cup B$ is $\mathscr{N}s\widehat{g}$ in $(U,\tau_R(X))$.*

Proof. If $A \cup B \subseteq G$ and $G$is $\mathscr{N}sg$ - open, then $A \subseteq G$ and $B \subseteq G$. Since A and B are $\mathscr{N}s\widehat{g}$- closed, $G \supseteq \mathscr{N}cl(A)$ and $G \supseteq \mathscr{N}cl(B)$ and hence $G \supseteq \mathscr{N}cl(A) \cup \mathscr{N}cl(B) = \mathscr{N}cl(A \cup B)$. Thus $(A \cup B)$ is $\mathscr{N}s\widehat{g}$- closed sets in $(U,\tau_R(X))$. □

Proposition 4.4. *If A and B are $\mathscr{N}s\widehat{g}$ - closed sets in $(U,\tau_R(X))$, then $A \cap B$ is $\mathscr{N}s\widehat{g}$ -closed in $(U,\tau_R(X))$.*

Proposition 4.5. *If a set A is $\mathscr{N}s\widehat{g}$- closed in $(U,\tau_R(X))$, then $\mathscr{N}cl(A) - A$ contains no non empty nano closed set in $(U,\tau_R(X))$.*

Proof. suppose that A is $\mathscr{N}s\widehat{g}$- closed. Let F be a closed subset of $\mathscr{N}cl(A) - A$. Then $A \subseteq F^c$. But A is $\mathscr{N}s\widehat{g}$ closed, therefore $\mathscr{N}cl(A) \subseteq F^c$. Consequently, $F \subseteq \mathscr{N}cl(A)^c$. We already have $F \subseteq \mathscr{N}cl(A)$. Thus $F \subseteq \mathscr{N}cl(A) \cap (\mathscr{N}cl(A))^c$ and F is empty.
The converse of these need not be true. □

Example 4.6. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{b\},\{c\},\{a,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\{\tau_R(X)\} = \{U,\phi,\{a,d\}, \{b\},\{a,b,d\}\}$. Then $\{\tau_R^c(X)\} = \{U,\phi,\{a,c,d\},\{c\},\{b,c\}\}$. If $A = \{ac\}$, then $\mathscr{N}cl(A) - A = \{d\}$ does not contain any nonempty closed set . But A is not $\mathscr{N}s\widehat{g}$ - closed in $(U,\tau_R(X))$.*

Proposition 4.7. *A set A is $\mathscr{N}s\widehat{g}$-closed if and only if $\mathscr{N}cl(A)$- A contains no nonempty $\mathscr{N}sg$ closed set.*

Proposition 4.8. *If A is $\mathscr{N}s\widehat{g}$- closed set and $A \subseteq B \subseteq \mathscr{N}cl(A)$, then B is $\mathscr{N}s\widehat{g}$- closed in $(U,\tau_R(X))$.*

Proof. Since $B \subseteq \mathscr{N}cl(A)$, we have $\mathscr{N}cl(B) \subseteq \mathscr{N}cl(A)$. Then $\mathscr{N}cl(B) - B \subseteq \mathscr{N}cl(A) - A$. Since $\mathscr{N}cl(A) - A$ has no nonempty $\mathscr{N}sg$- closed subsets, neither does $\mathscr{N}cl(B) - B$.Then B is $\mathscr{N}s\widehat{g}$ closed. □

Proposition 4.9. *Let $A \subseteq Y \subseteq U$ and suppose that A is $\mathcal{N}s\widehat{g}$-closed in $(U, \tau_R(X))$. Then A is $\mathcal{N}s\widehat{g}$- closed relative to Y.*

Proof. Let $A \subseteq Y \cap G$, where G is $\mathcal{N}sg$ - open in $(U, \tau_R(X))$. Then $A \subseteq G$ and hence $\mathcal{N}cl(A) \subseteq G$. This implies that $Y \cap \mathcal{N}cl(A) \subseteq Y \cap G$. Thus A is $\mathcal{N}s\widehat{g}$-closed relative to Y □

Proposition 4.10. *If A is a $\mathcal{N}sg$- open and $\mathcal{N}s\widehat{g}$- closed in $(U, \tau_R(X))$, then A is closed in $(U, \tau_R(X))$.*

Proof. Since A is $\mathcal{N}sg$ - open and $\mathcal{N}s\widehat{g}$- closed, $\mathcal{N}cl(A) \subseteq A$ and hence A is closed in $(U, \tau_R(X))$. □

Theorem 4.11. *Let $U, \tau_R(X)$ be extremely disconnected and A a nano semi-open subset of X . Then A is $\mathcal{N}s\widehat{g}$- closed if and only if it is $\mathcal{N}sg$- closed.*

Proof. It follows from the fact that if $U, \tau_R(X)$ is extremely disconnected and A is a nano semi open subset of X, then $\mathcal{N}scl(A) = \mathcal{N}cl(A)$. □

Theorem 4.12. *Let A be a locally nano closed set of $(U, \tau_R(X))$. Then A is nano closed if and only if A is $\mathcal{N}s\widehat{g}$- closed.*

Proof. $(i) \Rightarrow (ii)$. It is fact that every closed set is $\mathcal{N}s\widehat{g}$ - closed.
$(ii) \Rightarrow (i)$. We have, $A \cup (U - \mathcal{N}cl(A))$ is open in $(U, \tau_R(X))$, since A is locally nano closed.
Now $A \cup (X - \mathcal{N}cl(A))$ is sg- open set of $(U, \tau_R(X))$ such that $A \subseteq A \cup (U - \mathcal{N}cl(A))$. Since A is $\mathcal{N}s\widehat{g}$- closed, then $\mathcal{N}cl(A) \subseteq A \cup (U - \mathcal{N}cl(A))$. Thus, we have $\mathcal{N}cl(A) \subseteq A$ and hence A is a nano closed. □

Proposition 4.13. *For each $x \in U$, either $\{x\}$ is $\mathcal{N}sg$- closed or $\{x\}^c$ is $\mathcal{N}s\widehat{g}$- closed in $(U, \tau_R(X))$.*

Proof. suppose that $\{x\}$ is not $\mathcal{N}sg$- closed in $(U, \tau_R(X))$. Then $\{x\}^c$ is not $\mathcal{N}sg$-open and the only $\mathcal{N}sg$- open set containing $\{x\}^c$ is the space U itself .Therefore $cl(\{x\}^c) \subseteq U$ and so $\{x\}^c$ is $\mathcal{N}s\widehat{g}$- closed in $(U, \tau_R(X))$. □

Theorem 4.14. *Let A be a $\mathcal{N}s\widehat{g}$-closed set of a topological space $(U, \tau_R(X))$.Then,*
(i) $\mathcal{N}sint(A)$ is $\mathcal{N}s\widehat{g}$- closed.
(ii) If A is nano regular open, then $\mathcal{N}pint(A)$ and $\mathcal{N}scl(A)$ are also $\mathcal{N}s\widehat{g}$ -closed sets.
(iii) If A is regular closed, then $\mathcal{N}pcl(A)$ is also $\mathcal{N}s\widehat{g}$-closed set.

Proof. (i) Since $\mathcal{N}cl(\mathcal{N}int(A))$ is a nano closed set in $(U, \tau_R(X))$, $\mathcal{N}sint(A) = A \cap \mathcal{N}cl(\mathcal{N}int(A))$ is $\mathcal{N}s\widehat{g}$ in $(U, \tau_R(X))$.
(ii) Since A is nano regular open in U, $A = \mathcal{N}int(\mathcal{N}cl(A))$.Then $\mathcal{N}scl(A) = A \cup \mathcal{N}int(\mathcal{N}cl(A)) = A$. Thus $\mathcal{N}scl(A)$ is $\mathcal{N}s\widehat{g}$ in $(U, \tau_R(X))$. Since $\mathcal{N}pint(A) = A \cap \mathcal{N}int(\mathcal{N}cl(A)) = A$, $\mathcal{N}pint(A)$ is $\mathcal{N}s\widehat{g}$- closed.
(iii) Since A is nano regular closed in X, $A = \mathcal{N}cl(\mathcal{N}int(A))$. Then $\mathcal{N}pcl(A) = A \cup \mathcal{N}cl(\mathcal{N}int(A)) = A$ .Thus, $\mathcal{N}pcl(A)$ is $\mathcal{N}s\widehat{g}$ in $(U, \tau_R(X))$.
The converse of these need not to be true. □

Example 4.15. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{b\},\{c\}, \{a,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\tau_R(X) = \{U, \phi, \{a,d\}, \{b\}, \{a,b,d\}\}$.*
Then $\tau_R^c(X) = \{U, \phi, \{a,c,d\}, \{c\}, \{b,c\}\}$. and also $\mathcal{N}s\widehat{g}$ closed = $\{U, \phi, \{c\}, \{b,c\}, \{a,c,d\}\}$. If $A = \{c\}$, is not a $\mathcal{N}s\widehat{g}$ - closed set. However $\mathcal{N}sint(A) = \phi$ is $\mathcal{N}s\widehat{g}$ - closed

Example 4.16. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{b\},\{c\}, \{a,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\tau_R(X) = \{U, \phi, \{a,d\}, \{b\}, \{a,b,d\}\}$.*
Then $\tau_R^c(X) = \{U, \phi, \{a,c,d\}, \{c\}, \{b,c\}\}$. If $A = \{c\}$, is not nano regular open . However A is $\mathcal{N}s\widehat{g}$ - closed and $\mathcal{N}scl(A) = \{c\}$ is a $\mathcal{N}s\widehat{g}$ - closed and $\mathcal{N}pint(A) = \phi$ is also $\mathcal{N}s\widehat{g}$ -closed.

Example 4.17. *Let $U = \{a,b,c,d\}$, with $U \backslash R = \{\{b\},\{c\}, \{a,d\}\}$ and $X = \{a,b\}$. Then the nano topology, $\tau_R(X) = \{U, \phi, \{a,d\}, \{b\}, \{a,b,d\}\}$.*
Then $\tau_R^c(X) = \{U, \phi, \{a,c,d\}, \{c\}, \{b,c\}\}$. and also $\mathcal{N}s\widehat{g}$ closed = $\{U, \phi, \{c\}, \{b,c\}, \{a,c,d\}\}$. Then the set $A = \{b,c\}$, is not nano regular closed. However A is a $\mathcal{N}s\widehat{g}$ closed and $\mathcal{N}pcl(A) = \{b,c\}$ is $\mathcal{N}s\widehat{g}$-closed.

5. Nano semi $\widehat{g}$-open sets

In this section, we define and study the concept of nano semi $\widehat{g}$- open (briefly, $\mathcal{N}s\widehat{g}$- open) sets in nano topological spaces and obtain some of its properties.

Definition 5.1. *A subset A of $(U, \tau_R(X))$is called $\mathcal{N}s\widehat{g}$-open in X if A^c is $\mathcal{N}s\widehat{g}$- closed in $(U, \tau_R(X))$.*

Proposition 5.2. *For any nano topological space $(U, \tau_R(X))$, the following assertions hold:*
(i) Every nano open set is a $\mathcal{N}s\widehat{g}$-open set .
(ii) Every $\mathcal{N}s\widehat{g}$-open set is $\mathcal{N}\widehat{g}$-open set .
(iii) Every $\mathcal{N}s\widehat{g}$-open set is $\mathcal{N}g$-open set .
(iv) Every $\mathcal{N}s\widehat{g}$-open set is $\mathcal{N}gs$- open set .
(v) Every $\mathcal{N}s\widehat{g}$-open set is $\mathcal{N}gsp$- open set .
(vi) Every $\mathcal{N}\alpha g$ - open set is $\mathcal{N}s\widehat{g}$ - open set .
(vii) Every $\mathcal{N}s\widehat{g}$ - open set is $\mathcal{N}sg$- open set .
(viii) Every $\mathcal{N}\alpha$-open set is $\mathcal{N}s\widehat{g}$- open set .

Proof. Follows from 3.2, 3.3, 3.4, 3.6, 3.8, 3.9, 3.10, 3.12. □

Theorem 5.3. *A set A of U is $\mathcal{N}s\widehat{g}$ - open if and only if $F \subseteq \mathcal{N}int(A)$ whenever F is $\mathcal{N}sg$-closed and $F \subseteq A$.*

Proof. Suppose that $F \subseteq \mathcal{N}int(A)$ such that F is $\mathcal{N}sg$- closed and $F \subseteq A$. Let $A^c \subseteq G$ where G is $\mathcal{N}sg$-open. Then $(G)^c \subseteq A$ and G^c is $\mathcal{N}sg$- closed. Therefore $(G)^c \subseteq \mathcal{N}int(A)$. Since $G^c \subseteq \mathcal{N}int(A)$, we have $\mathcal{N}(int(A))^c \subseteq G$, i.e., $\mathcal{N}cl(A)^c \subseteq G$, Since $\mathcal{N}cl(A)^c = \mathcal{N}int(A)^c$. Thus A^c is $\mathcal{N}s\widehat{g}$. i.e.,A is $\mathcal{N}s\widehat{g}$-open.
Conversely, suppose that A is $\mathcal{N}s\widehat{g}$- open such that $F \subseteq A$ and F is $\mathcal{N}sg$-closed. Then F^c is $\mathcal{N}sg$-open and $A^c \subseteq F^c$. Therefore, $\mathcal{N}cl(A^c) \subseteq F^c$ by definition of $\mathcal{N}s\widehat{g}$ - closed and so $F \subseteq \mathcal{N}int(A)$, since $\mathcal{N}cl(A^c) = \mathcal{N}(int(A))^c$. □

Lemma 5.4. *For an $x \in X, x \in \mathcal{N}s\widehat{g}$ - $\mathcal{N}cl(A)$ if and only if $V \cap A \neq \phi$ for every $\mathcal{N}s\widehat{g}$ open set V containing x.*

Proof. Let $x \in \mathcal{N}s\widehat{g} - \mathcal{N}cl(A)$ for any $x \in X$. To prove $V \cap A \neq \phi$ for every $\mathcal{N}s\widehat{g}$ - open set V containing x. Prove the result by contradiction . Suppose there exists a $\mathcal{N}s\widehat{g}$ - open set V containing x such that $V \cap A \neq \phi$. Then $A \subseteq \{V\}^c and \{V\}^c$ is $\mathcal{N}s\widehat{g}$ closed. We have $\mathcal{N}s\widehat{g} - \mathcal{N}cl(A) \subseteq \{V\}^c$. This shows that $x \notin \mathcal{N}s\widehat{g} - \mathcal{N}cl(A)$ which is contradiction . Hence $V \cap A \neq \phi$ for every $\mathcal{N}s\widehat{g}$ - open set V containing x.
Conversely, let $V \cap A \neq \phi$ for every $\mathcal{N}s\widehat{g}$- open set V containing x. To prove $x \in \mathcal{N}s\widehat{g}$ - $\mathcal{N}cl(A)$. We prove the result by contradiction. Suppose $x \notin Ns\widehat{g}$ - $\mathcal{N}cl(A)$ .Then there exists a $\mathcal{N}s\widehat{g}$- closed set F containing A such that $x \notin F$. Then $x \in \{F\}^c$ and $\{F\}^c$ is $\mathcal{N}s\widehat{g}$- open. Also $\{F\}^c \cap A = \phi$, which is a contradiction to the hypothesis. Hence $x \in \mathcal{N}s\widehat{g} - \mathcal{N}cl(A)$. □

Definition 5.5. *For any $A \subseteq X$, $\mathcal{N}s\widehat{g}$- $\mathcal{N}int(A)$ is defined as the union of all $\mathcal{N}s\widehat{g}$ - nano open sets contained in A. i.e., $\mathcal{N}s\widehat{g}$ - $int(A) = \cup\{G : G \subseteq A$ and G is $\mathcal{N}s\widehat{g}$ - open$\}$.*

Lemma 5.6. *For any $A \subseteq X$, $int(A) \subseteq \mathcal{N}s\widehat{g}$ - $int(A) \subseteq A$.*

Proof. It follows from preposition 4.2(i) □

Proposition 5.7. *For any $A \subseteq X$, the following holds:*
(i) $\mathcal{N}s\widehat{g} - int(A)$ is the largest $\mathcal{N}s\widehat{g}$ - open set contained in A.
(ii) A is $\mathcal{N}s\widehat{g}$ -open if and only if $\mathcal{N}s\widehat{g} - int(A) = A$.

Proposition 5.8. *For any subsets A and B of $(U, \tau_R(X))$, the following holds:*
(i) $\mathcal{N}s\widehat{g}$ - $int(A \cap B) \subseteq \mathcal{N}s\widehat{g} - int(A) \cap \mathcal{N}s\widehat{g} - int(B)$.
(ii) $\mathcal{N}s\widehat{g}$ - $int(A \cup B) \supseteq \mathcal{N}s\widehat{g} - int(A) \cup \mathcal{N}s\widehat{g} - int(B)$.
(iii) If $A \subseteq B$, then $\mathcal{N}s\widehat{g}$- $int(A) \subseteq \mathcal{N}\widehat{\int\}}$ - $int(B)$.
(iv) $\mathcal{N}s\widehat{g}$ - $int(U) = U$ and $\mathcal{N}s\widehat{g}$ - $int(\phi) = \phi$.

Theorem 5.9. *Let A be any subset of X. Then,*
(i) $(\mathcal{N}s\widehat{g} - int(A))^c = \mathcal{N}s\widehat{g} - cl(A^c)$.
(ii)$\mathcal{N}s\widehat{g} - int(A) = (\mathcal{N}s\widehat{g} - cl(A^c))^c$.
(iii)$\mathcal{N}s\widehat{g} - cl(A) = (\mathcal{N}s\widehat{g} - int(A^c))^c$.

Proof. (i) Let $x \in \mathcal{N}s\widehat{g} - int(A))^c$. Then $x \notin \mathcal{N}s\widehat{g}int(A)$. That is, every $\mathcal{N}s\widehat{g}$ - open set U containing x is such that $U \nsubseteq A$. That is, every $\mathcal{N}s\widehat{g}$ - open set U containing x is such that $U \cap A^c \neq \phi$, $x \in \mathcal{N}s\widehat{g} - cl(A)^c$ and therefore $(\mathcal{N}s\widehat{g} - \mathcal{N}int(A))^c \subseteq \mathcal{N}s\widehat{g} - scl(A^c)$.
conversely, let $x \in \mathcal{N}s\widehat{g} - cl(A^c)$. Then by theorem , Every $\mathcal{N}s\widehat{g}$ -open set U containing x is such that $U \cap A^c \neq \phi$. That is, every $\mathcal{N}s\widehat{g}$ - open set U containing x is such that $U \nsubseteq A$. This implies by definition 5.5 , $x \notin \mathcal{N}s\widehat{g} - int(A)$. That is , $x \in (\mathcal{N}s\widehat{g} - int(A))^c$ and $\mathcal{N}s\widehat{g} - cl(A)^c \subseteq (\mathcal{N}s\widehat{g} - int(A))^c$. Thus $(\mathcal{N}s\widehat{g} - int(A))^c = \mathcal{N}s\widehat{g} - cl(A^c)$.
(ii) Follows by taking complements in (i).
(iii) Follows by replacing A by A^c in (i). □

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