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Nano semi $c(s)$ generalized continuous functions in nano topological spaces

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Abstract
The purpose of this paper is to introduce and study a new class of functions called nano semi c(s) generalized continuous functions in nano topological spaces. Some of the properties of nano semi c(s) - generalized continuous function are analyzed.

Keywords
$Nsc(s)g$-closed set, $Nsc(s)g$-continuous function.

AMS Subject Classification
54B05, 54C05.

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Contents

1. Introduction

The concept of continuity plays major role in general topology. Many authors have studied different types of continuity. M.Lellis Thivagar and Carmel Richard [4] introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of nano topological space are called nano open sets. He has defined nano closed sets, nano interior and nano closure of a set in nano topological space. He has also introduced a nano continuous function, nano open mappings, nano closed mappings and nano homeomorphisms in nano topological space.

In this paper we have introduced a new class of continuous functions called nano semi $c(s)$ generalized continuous functions and obtain some characterizations in terms of nano interior and nano closure in nano topological spaces.

Throughout this paper $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $(W,\tau_{R''}(Z))$ and are nano topological spaces with respect to X, where $X \subseteq U$, $Y \subseteq V$, $Z \subseteq W$. R, R^1 and R^{11} are an equivalence relations on U, V and W. U/R, V/R^1, W/R^{11} denotes the the family of equivalence classes by the equivalence relations R, R^1 and R^{11} respectively on U, V and W.

2. Preliminaries

Definition 2.1. *[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*
(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ *where $R(x)$ denotes the equivalence class determined by $x \in U$.*
(ii)The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x)\cap X \neq \phi\}$
(iii)The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X)$ - $L_R(X)$.

Proposition 2.2. *[4] If (U,R) is an approximation space and $X,Y \subseteq U$, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. *[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms*
(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Remark 2.4. *[4] If $\tau_R(X)$ is the nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[4] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then*
(i)The nano interior of A is defined as the union of all nano-open subsets of A is contained in A and is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano-open subset of A.
(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by $Ncl(A)$). That is, $Ncl(A))$ is the smallest nano-closed set containing A.

Definition 2.6. *A subset of a nano topolgical space $(U, \tau_R(X))$ is called*
a) Ng-closed [1] if $Ncl(A) \subseteq G$, whenever $H \subseteq G$ and G is nano open set in $(U, \tau_R(X))$.
b) Nsg-closed [2] if $Nscl(A) \subseteq G$, whenever $H \subseteq G$ and G is nano semi - open set in $(U, \tau_R(X))$.
c) Nt-set [3] if $Nint(A) = Nint(Ncl(A))$.
d) $Nc(s)$-set [7] if $A = G \cap F$, where G is Ng-open and F is Nt-set.
e) $Nsc(s)g$-closed [7] if $Nscl(A) \subseteq U$, whenever $A \subseteq U$ and U is $Nc(s)$-set in $(U, \tau_R(X))$.
f) Np-open [5] if $A \subseteq Nint(Ncl(A))$
g) Ns-open [5] if $A \subseteq Ncl(Nint(A))$
h) $N\alpha$-open [5] if $A \subseteq Nint(Ncl(Nint(A)))$
i) Nr-open [5] if $A = Nint(Ncl(A))$

Example 2.7. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\}$. Then $\tau_R(X) = \{\phi, U, \{c\}, \{b,c,d\}, \{a,c\}\}$ is a nano topology on U with respect to X and $\tau_{R^c}(X) = \{\phi, U, \{c\}, \{b,c,d\}, \{a,c\}\}$. A subset H of a nano topological space $(U, \tau_R(X))$ is called:*
(1) nano semi-closed: $\{\phi, U, \{a\}, \{c\}, \{a,c\}, \{b,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(2) nano α-closed : $\{\phi, U, \{c\}, \{a,c\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(3) nano g-closed : $\{\phi, U, \{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(4) nano sg-closed:$\{\phi, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(5) nano $g\alpha$-closed : $\{\phi, U, \{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(6) nano g^*- closed: $\{\phi, U, \{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(7) nano rgp-closed: $\{\phi, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(8) nano regular-closed: $\{\phi, U, \{a,c\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$
(9) nano $sc(s)g$-closed : $\{\phi, U, \{a\}, \{c\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$ in $(U, \tau_R(X))$

Definition 2.8. *[5] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be a nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano continuous on U if the inverse image of every nano open set in V is nano open in U.*

Definition 2.9. *[6] A subset $M_x \subset U$ is called a nano semi pre-neighbourhood ($N\beta$-nhd) of a point $x \in U$ iff there exists a $A \in N\beta O(U,X)$ such that $x \in A \subset M_x$ and a point x is called $N\beta$-nhd point of the set A.*

3. Nano semi $c(s)$ generalised continuous function

In this section we define and study the new class of function, namely nano semi $c(s)$ generalized continuous functions in nano topological spaces.

Definition 3.1. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be a nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano semi $c(s)$ generalized continuous (briefly $Nsc(s)g$-continuous) on U if the inverse image of every nano open set in V is nano semi $c(s)g$-open set in U.*

Example 3.2. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then the nano topology is $\tau_R(X) = \{\phi, U, \{a\}, \{a,b,d\}\{b,d\}\}$. Let $V = \{x,y,z,w\}$ with $V/R^1 = \{\{x,z\},\{y\},\{w\}\}$ and $Y = \{x,w\}$. Then $\tau_{R'}(Y) = \{\phi, V, \{w\}, \{x,z,w\}\{x,z\}\}$. $\tau^c_R(X) = \{\phi, U, \{c\}, \{b,c,d\}\{a,c\}\}$, $\tau^c_{R'}(Y) = \{\phi, V, \{y\}, \{x,y,z\}\{y,w\}\}$ are the complements of*

$\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=w$, $f(b)=x$, $f(c)=y$ and $f(d)=z$. Then $f^{-1}(\{w\})=\{a\}$, $f^{-1}(\{x,z,w\})=\{a,b,d\}$, $f^{-1}(\{x,z\})=\{b,d\}$ and $f^{-1}(V)=U$. That is the inverse image of every in nano open set in V is $Nsc(s)g$-open set in U. Therefore f is $Nsc(s)g$-continuous.

Theorem 3.3. *A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be $Nsc(s)g$-continuous iff the inverse image of every nano closed set in V is $Nsc(s)g$-closed set in U.*

Proof. Let f be $Nsc(s)g$-continuous and F be nano closed set in V. That is $V-F$ is nano open set in V. Since f is $Nsc(s)g$-continuous, $f^{-1}(V-f)$ is $Nsc(s)g$-open set in U. That is $f^{-1}(V)-f^{-1}(F)=U-f^{-1}(F)$ is $Nsc(s)g$-open set in U. Hence $f^{-1}(F)$ is $Nsc(s)g$-closed set in U, if f is $Nsc(s)g$-continuous on U. Conversely, let us assume that the inverse image of every nano closed set in V is $Nsc(s)g$-closed set in U. Let G be nano open set in V. Then $V-G$ is nano closed set in V. By our assumption $f^{-1}(V-G)$ is $Nsc(s)g$-closed set in U. That is $f^{-1}(V)-f^{-1}(G)=U-f^{-1}(G)$ is $Nsc(s)g$-closed set in U. Hence $f^{-1}(G)$ is $Nsc(s)g$-open set in U. That is the inverse image of every nano open set in V is $Nsc(s)g$-open set in U. That is f is $Nsc(s)g$-continuous on U. □

Theorem 3.4. *A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is $Nsc(s)g$-continuous iff $f(Ncl(A))\subseteq Ncl(f(A))$ for every subset A of U.*

Proof. Let f be $Nsc(s)g$-continuous and $A\subseteq U$. Then $f(A)\subseteq V$. Also $Ncl(f(A))$ is nano closed in V. Since f is $Nsc(s)g$-continuous, $f^{-1}(Ncl(f(A))$ is $Nsc(s)g$-closed set containing A. But every nano closed set is $Nsc(s)g$-closed set in U and is the smallest nano closed set containing A. Therefore $Ncl(A)\subseteq f^{-1}(Ncl(f(A)))$. (ie) $f(Ncl(A))\subseteq Ncl(f(A))$.
Conversely, let $f(Ncl(A))\subseteq Ncl(f(A))$ for every closed subset A of U. If F is nano closed set in V and since $f^{-1}(F)\subseteq U$ we have $f(Ncl(f^{-1}(F))\subseteq Ncl(f(f^{-1}(F))=Ncl(F)$. That is $Ncl(f^{-1}(F))\subseteq f^{-1}(Ncl(F))=f^{-1}(F)$, since F is nano closed set in V. Thus $Ncl(f^{-1}(F))\subseteq f^{-1}(F)$. But $f^{-1}(F)\subseteq Ncl(f^{-1}(F))$. That is $Ncl(f^{-1}(F))=f^{-1}(F)$. Therefore $f^{-1}(F)$ is nano closed in U. But every nano closed set is $Nsc(s)g$-closed set in U we have, $f^{-1}(F)$ is $Nsc(s)g$-closed set in V. Hence f is $Nsc(s)g$-continuous on U. □

Remark 3.5. *If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is $Nsc(s)g$-continuous, then $f(Ncl(A))$ is not necessarily equal to where $A\subseteq U$.*

For example, let $U=\{a,b,c,d\}$, $U/R=\{\{a\},\{d\},\{b,c\}\}$ and $X=\{a,c\}$. Then $\tau_R(X)=\{U,\phi,\{a\},\{a,b,c\},\{b,c\}\}$. Let $V=\{x,y,z,w\}$; $V/R'=\{\{x\},\{z\},\{y,w\}\}$ and $Y=\{y,w\}$ then $\tau_{R'}(Y)=\{V,\phi,\{y,w\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=x$, $f(b)=y$, $f(c)=w$ and $f(d)=z$. Then $\tau_R^c(X)=\{U,\phi,\{d\},\{a,d\},\{b,c,d\}\}$ and $\tau_{R'}^c(Y)=\{V,\phi,\{x,z\}\}$. Now $f^{-1}(\{x,z\})=\{a,d\}$. Therefore the inverse image of every nano closed in V is $Nsc(s)g$-closed set on U. Hence is $Nsc(s)g$-continuous on U. Let $A=\{c\}\subseteq U$. Then $f(Ncl(A))=f(\{b,c,d\})=\{y,w,z\}$. But $Ncl(f(A))=Ncl(\{w\})=V$. Thus $f(Ncl(A))\neq Ncl(f(A))$, even though f is $Nsc(s)g$-continuous. That is $f(Ncl(A))$ is not necessarily equal to $Ncl(A))$ where $A\subseteq U$ if f is $Nsc(s)g$-continuous on U.

Theorem 3.6. *Every nano continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano continuous function and A be nano closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is nano closed in $(U,\tau_R(X))$. Since every nano closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.7. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau_R^c(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau_{R'}^c(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=z$, $f(b)=x$, $f(c)=y$ and $f(d)=w$, then f is $Nsc(s)g$-continuous function. But not nano continuous function, since $f^{-1}(\{y,w\})=\{c,d\}$ is not nano closed in $(U,\tau_R(X))$.*

Theorem 3.8. *Every nano semi continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano semi continuous function and A be nano semi closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is nano semi closed in $(U,\tau_R(X))$. Since every nano semi closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.9. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau_R^c(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau_{R'}^c(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=z$, $f(b)=x$, $f(c)=y$ and $f(d)=w$, then f is $Nsc(s)g$-continuous function. But not nano semi continuous function, since $f^{-1}(\{y,w\})=\{c,d\}$ is not nano semi closed in $(U,\tau_R(X))$.*

Theorem 3.10. *Every nano generalized continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano generalized continuous function and A be Ng-closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is Ng-closed in $(U,\tau_R(X))$. Since every Ng-closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.11. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x\},\{z\},\{y,w\}\}$ and $Y=\{y,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{y,w\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{x,z\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=y$, $f(b)=x$, $f(c)=w$ and $f(d)=z$, then f is $Nsc(s)g$-continuous function. But not Ng-continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not Ng-closed in $(U,\tau_R(X))$.*

Theorem 3.12. *Every $Ng\alpha$-continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Ng\alpha$-continuous function and A be $Ng\alpha$-closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Ng\alpha$-closed in $(U,\tau_R(X))$. Since every $Ng\alpha$-closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.13. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x\},\{z\},\{y,w\}\}$ and $Y=\{y,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{y,w\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{x,z\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=y$, $f(b)=x$, $f(c)=w$ and $f(d)=z$, then f is $Nsc(s)g$-continuous function. But not $Ng\alpha$-continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not $Ng\alpha$-closed in $(U,\tau_R(X))$.*

Theorem 3.14. *Every Ng^*-continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a Ng^*-continuous function and A be Ng^*-closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is Ng^*-closed in $(U,\tau_R(X))$. Since every Ng^*-closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.15. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x\},\{z\},\{y,w\}\}$ and $Y=\{y,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{y,w\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{x,z\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=y$, $f(b)=x$, $f(c)=w$ and $f(d)=z$, then f is $Nsc(s)g$-continuous function. But not Ng^*-continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not Ng^*-closed in $(U,\tau_R(X))$.*

Theorem 3.16. *Every nano regular continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano regular continuous function and A be nano regular closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is nano regular closed in $(U,\tau_R(X))$. Since every nano regular closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.17. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=z$, $f(b)=x$, $f(c)=y$ and $f(d)=w$, then f is $Nsc(s)g$-continuous function. But not nano regular continuous function, since $f^{-1}(\{y,w\})=\{c,d\}$ is not nano regular closed in $(U,\tau_R(X))$.*

Theorem 3.18. *Every $N\alpha$-continuous function is $Nsc(s)g$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $N\alpha$-continuous function and A be $N\alpha$ closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $N\alpha$-closed in $(U,\tau_R(X))$. Since every $N\alpha$-closed is $Nsc(s)g$-closed set, $f^{-1}(A)$ is $Nsc(s)g$-closed set in $(U,\tau_R(X))$. Hence f is $Nsc(s)g$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.19. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by*

$f(a)=z$, $f(b)=x$, $f(c)=y$ and $f(d)=w$, then f is $Nsc(s)g$-continuous function. But not $N\alpha$-continuous function, since $f^{-1}(\{y,w\})=\{c,d\}$ is not $N\alpha$-closed in $(U,\tau_R(X))$.

Theorem 3.20. *Every $Nsc(s)g$-continuous function is Nsg-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Nsc(s)g$-continuous function and A be $Nsc(s)g$-closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Nsc(s)g$-closed in $(U,\tau_R(X))$. Since every $Nsc(s)g$-closed is Nsg-closed set, $f^{-1}(A)$ is Nsg-closed set in $(U,\tau_R(X))$. Hence f is Nsg-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.21. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=y$, $f(b)=w$, $f(c)=z$ and $f(d)=x$, then f is Nsg-continuous function. But not $Nsc(s)g$-continuous function, since $f^{-1}(\{w\})=\{b\}$ is not $Nsc(s)g$-closed in $(U,\tau_R(X))$.*

Theorem 3.22. *Every $Nsc(s)g$-continuous function is $Nrgb$-continuous function.*

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Nsc(s)g$-continuous function and A be $Nsc(s)g$-closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Nsc(s)g$-closed in $(U,\tau_R(X))$. Since every $Nsc(s)g$-closed is $Nrgb$-closed set, $f^{-1}(A)$ is $Nrgb$-closed set in $(U,\tau_R(X))$. Hence f is $Nrgb$-continuous function.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.23. *Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\},\{b,d\}\}$ is a nano topology on U. Let $V=\{x,y,w,z\}$ with $V/R'=\{\{x,z\},\{y\},\{w\}\}$ and $Y=\{x,w\}$ then $\tau_{R'}(Y)=\{\phi,V,\{w\},\{x,z,w\},\{x,z\}\}$ is a nano topology on V. Then $\tau^c_R(X)=\{U,\phi,\{c\},\{b,c,d\},\{a,c\}\}$ and $\tau^c_{R'}(Y)=\{V,\phi,\{y\},\{x,y,z\},\{y,w\}\}$ are the complement of $\tau_R(X)$ and $\tau_{R'}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=x$, $f(b)=z$, $f(c)=w$ and $f(d)=y$, then f is $Nrgb$-continuous function. But not $Nsc(s)g$-continuous function, since $f^{-1}(\{x,y,z\})=\{a,b,d\}$ and $f^{-1}(\{y\})=\{d\}$ is not $Nsc(s)g$-closed in $(U,\tau_R(X))$.*

Remark 3.24. *Composition of two $Nsc(s)g$-continuous function need not be a $Nsc(s)g$-continuous function.*

Example 3.25. *Let $U=V=W=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and $X=\{a,b\}$, $V/R^1=\{\{a,c\},\{b\},\{d\}\}$ and $Y=\{a,d\}$, $W/R^{11}=\{\{b\},\{d\},\{a,c\}\}$ and $Z=\{a,b\}$. Then the corresponding nano topologies of U, V and W are $\tau_R(X)=\{\phi,U,\{a\},\{a,b,d\}\{b,d\}\}$, $\tau_{R'}(Y)=\{\phi,V,\{d\},\{a,c,d\},\{a,c\}\}$ and $\tau_{R''}(Z)=\{\phi,W,\{b\},\{a,b,c\},\{a,c\}\}$ respectively. Define the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=d$, $f(b)=a$, $f(c)=b$ and $f(d)=c$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ identity function by $g(a)=a$, $g(b)=b$, $g(c)=c$, $g(d)=d$. Then f and g are $Nsc(s)g$-continuous function. But their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is not $Nsc(s)g$-continuous function, since the inverse image of the nano closed set is $\{a,c,d\}$ is $\{a,b,d\}$. But it is not a $Nsc(s)g$-closed set.*

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