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Nano generalized c^*-closed sets in nano topological spaces

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Abstract
In this paper, we have introduced a new class of closed set namely nano generalized c^*-closed sets in nano topological space.

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Nano generalized c^*-closed set.

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Contents

1. Introduction

Lellis Thivagar et al [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. Veerakumar introduced and investigated between closed sets and g^*-closed sets. Levine [4] introduced and investigated properties of generalized closed sets in topological spaces. The aim of this paper is to introduce and study properties of nano generalized c^*-closed sets in a nano topological spaces. Throughout this paper $(U,\tau_R(X))$ represent non-empty nano topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space $(U,\tau_R(X))$, $Ncl(A)$ and $Nint(A)$ denote the nano closure of A and nano interior of A respectively.

2. Preliminaries

We recall the following definitions:

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*
(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.
(ii)The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$
(iii)The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. *[5] If (U,R) is an approximation space and $X,Y \subseteq U$, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. *[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms*
(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as nano topological space. The elements of $\tau_R(X)$ are called as nano -open sets. The complement of the nano -open sets are called nano -closed sets.

Remark 2.4. *[6] If $\tau_R(X)$ is the nano topology on U with respect to X , then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[5] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then*
(i)The nano interior of A is defined as the union of all nano -open subsets of A is contained in A and is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest nano -open subset of A.
(ii)The nano closure of A is defined as the intersection of all nano -closed sets containing A and is denoted by $Ncl(A)$). That is, $Ncl(A)$) is the smallest nano -closed set containing A.

Definition 2.6. *[5] $(U, \tau_R(X))$ be a nano topological space and A then A is said to be*
i) nano pre open set if $A \subseteq Nint(Ncl(A))$
ii) nano semi open set if $A \subseteq Ncl(Nint(A))$
iii) nano-open set if $A \subseteq Nint(Ncl(Nint(A)))$
iv) nano regular open set if $A = Nint(Ncl(A))$
The complements of the above mentioned sets are called their respective closed sets.

Definition 2.7. *[1] A subset A of a nano topological space $(U, \tau_R(X))$ is called,*
i) nano generalized closed set if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open.
ii) Strongly nano g^*-closed set if $Ncl(Nint(A)) \subseteq U$ whenever $A \subseteq U$ and U is nano g-open.
iii) nano c^*-set if $S = G \cap F$ where G is nano g-open and F is nano α^*-set.
iv) nano α^*-set if $Nint(A) = Nint(Ncl(Nint(A)))$.

Definition 2.8. *[8] A subset $M_x \subset U$ is called a nano semi pre-neighbourhood ($N\beta$-nhd) of a point $x \in U$ iff there exists a $A \in N\beta O(U,X)$ such that $x \in A \subset M_x$ and a point x is called $N\beta$-nhd point of the set A.*

3. Nano generalized c^*-closed sets

In this section, we define and study the properties of nano generalized c^*-closed sets in nano topological spaces.

Definition 3.1. *A subset A of a space $(U, \tau_R(X))$ is called nano generalized c^*-closed set if $Ncl(A) \subseteq U$, whenever $A \subseteq U$ and U is nano c^*-set.*

Remark 3.2. *[1] Every nano closed set is a nano g-closed set.*

Theorem 3.3. *Every nano open set is nano c^*-set.*

Proof. Every nano open set is nano g-open set. But every nano c^*-set is the intersection of nano g-open set and nano α^*-set. So, by definition of nano c^*-set. The proof is immediate. The converse of the above theorem need not be true as seen from the following example. □

Example 3.4. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{b,d\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{d\}, \{b,c\}, \{b,c,d\}\}$. Here the set $\{a,b,c\}$ is nano c^*-set but not nano open set.*

Theorem 3.5. *Every nano g-closed set is a nano generalized c^*-closed set.*

Proof. Let A be a nano g-closed set of U and $A \subseteq V$, V is nano open in U. Every nano open set is nano c^*-set, V is nano c^*-set. Hence A is nano generalized c^*-closed set. The converse of the above theorem need not be true as seen from the following example. □

Example 3.6. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{b,d\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{d\}, \{b,c\}, \{b,c,d\}\}$. Here the set $\{a,c,d\}$ is nano generalized c^*-closed set but not nano g-closed set.*

Theorem 3.7. *Every nano generalized c^*-closed set is a Strongly nano g^*-closed set.*

Proof. Let A be a nano generalized c^*-closed set of U and $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano c^*-set. Then $A \subseteq Ncl(Nint(A)) \subseteq Ncl(A) \subseteq G$ whenever G is nano g-open. Thus A is Strongly nano g^*-closed set. The converse of the above theorem need not be true as seen from the following example. □

Example 3.8. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{a\}, \{b,d\}, \{a,b,d\}\}$. Here the set $\{a,b,c\}$ is Strongly nano g^*-set but not nano generalized c^*-closed set.*

Remark 3.9.
- *Every nano α-closed set is a nano generalized c^*-closed set.*
- *Every nano regular closed set is a nano generalized c^*-closed set.*

Remark 3.10. *The notion of nano generalized c^*-closed set is independent with nano pre closed set.*

Example 3.11. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{b,d\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{d\}, \{b,c\}, \{b,c,d\}\}$. Here the set $\{b\}$ is nano pre closed set but not a nano generalized c^*-closed set. The set $\{a,b,d\}$ is nano generalized c^*-closed set but not a nano pre closed set.*

Remark 3.12. *The intersection of two nano generalized c^*-closed sets in $(U,\tau_R(X))$ is also a nano generalized c^*-closed set in $(U,\tau_R(X))$ as seen from the following example.*

Example 3.13. *In the above Example 3.11. The nano generalized c^*-closed set $= \{\phi, U, \{a\},\{d\},\{a,b\},\{a,c\},\{a,d\}, \{a,b,c\},\{a,b,d\},\{a,c,d\}\}$. Here $\{a,b\}\cap\{a,d\} = \{a\}$ which is again a nano generalized c^*-closed set.*

Remark 3.14. *The union of two nano generalized c^*-closed sets in $(U,\tau_R(X))$ is also a nano generalized c^*-closed sets in $(U,\tau_R(X))$.*

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