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A study on intuitionistic (λ, μ)- fuzzy subrings of a ring

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Abstract
In this paper, we made an attempt to study the algebraic nature of intuitionistic (λ, μ)-fuzzy subring of a rings.

Keywords
Fuzzy set, (λ, μ)-fuzzy subring, (λ, μ)-anti-fuzzy subring, intuitionistic fuzzy set, intuitionistic (λ, μ)-fuzzy subring.

AMS Subject Classification
03F55, 06D72, 08A72.

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Contents

1. Introduction

After the introduction of fuzzy sets by L.A.Zadeh [18], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov [3, 4], as a generalization of the notion of fuzzy set. The concepts of (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals was introduced by Bingxue Yao [17]. In this paper, we introduce the concept of intuitionistic (λ, μ)-fuzzy subrings of a ring and established some results. Throughout this article, we will always assume that $0 \leq \lambda < \mu \leq 1$.

2. Preliminaries

Definition 2.1. *Let X be a non–empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.*

Definition 2.2. *The union of two fuzzy subsets A and B of a set X, denoted by $(A \cup B)(x) = \max\{A(x), B(x)\}$, for all $x \in X$.*

Definition 2.3. *The intersection of two fuzzy subsets A and B of a set X, denoted by $(A \cap B)(x) = \min\{A(x), B(x)\}$, for all $x \in X$.*

Definition 2.4. *Let R be a subring. A fuzzy subset A of R is said to be a (λ, μ)-fuzzy subring of R if it satisfies the following conditions:*

(i) $A(x+y) \vee \lambda \geq \min\{A(x), A(y)\} \wedge \mu$,

(ii) $A(-x) \vee \lambda \geq A(x) \wedge \mu$,

(iii) $A(xy) \vee \lambda \geq \min\{A(x), A(y)\} \wedge \mu$, *for all x and y in R*

Definition 2.5. *Let R be a subring. A fuzzy subset A of R is said to be an (λ, μ) anti-fuzzy subring of R if it satisfies the following conditions:*

(i) $A(x+y) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$,

(ii) $A(-x) \wedge \mu \leq A(x) \vee \lambda$,

(iii) $A(xy) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$, *for all x and y in R.*

Definition 2.6. *[3, 4] An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Example 2.7. *Let $X = \{a, b, c\}$ be a set. Then $A = \{\langle a, 0.62, 0.24\rangle, \langle b, 0.34, 0.61\rangle, \langle c, 0.28, 0.44\rangle\}$ is an intuitionistic fuzzy subset of X.*

Definition 2.8. *If Ai s a intuitionistic fuzzy subset of X, then the complement of A, denoted A^c is the intuitionistic fuzzy set of X, given by $A^c(x) = \{< x, \nu_A(x), \mu_A(x) > /x \in X\}$, for all $x \in X$.*

Example 2.9. *Let $A = \{\langle a, 0.8, 0.1\rangle, \langle b, 0.7, 0.2\rangle, \langle c, 0.5, 0.3\rangle\}$ is a fuzzy subset of $X = \{a, b, c\}$. The complement of A is $A^c = \{\langle a, 0.1, 0.8\rangle, \langle b, 0.2, 0.7\rangle, \langle c, 0.3, 0.5\rangle\}$.*

Definition 2.10. *Let A and B be any two intuitionistic (λ,μ)-fuzzy subsets of a set X. We define the following operations:*

(i) $A \cap B$

$$= \{\langle x, \min\{\mu_A(x), \mu_B(x)\} \wedge \mu, \max\{\nu_A(x), \nu_B(x)\} \vee \lambda\rangle\},$$

for all $x \in X$.

(ii) $A \cup B$

$$= \{\langle x, \max\{\mu_A(x), \mu_B(x)\} \vee \lambda, \min\{\nu_A(x), \nu_B(x)\} \wedge \mu\rangle\},$$

for all $x \in X$.

(iii) $\Box A = \{\langle x, \mu_A(x) \vee \lambda, (1-\mu_A(x)) \wedge \mu\rangle /x \in X\}$, *for all x in X.*

(iv) $\diamond A = \{\langle x, (1-\nu_A(x)) \vee \lambda, \nu_A(x) \wedge \mu\rangle /x \in X\}$, *for all x in X.*

Definition 2.11. *Let R be a subring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic (λ,μ)-fuzzy subring of R if it satisfies the following conditions:*

(i) $\mu_A(x-y) \vee \lambda \geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$,

(ii) $\mu_A(xy) \vee \lambda \geq \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$,

(iii) $\nu_A(x-y) \wedge \mu \leq \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$,

(iv) $\nu_A(xy) \wedge \mu \leq \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$, *for all x and y in R.*

Definition 2.12. *Let A and B be intuitionistic (λ,μ)-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by $A \times B$, is defined as $A \times B$*

$$= \{\langle (x,y), \mu_{A\times B}(x,y) \vee \lambda, \nu_{A\times B}(x,y) \wedge \mu\rangle /$$

for all x in G and y in H$\}$, where

$$\mu_{A\times B}(x,y) \vee \lambda = \min\{\mu_A(x), \mu_B(y)\} \wedge \mu$$

and $\nu_{A\times B}(x,y) \wedge \mu = \max\{\nu_A(x), \nu_B(y)\} \vee \lambda$.

Definition 2.13. *Let A be an intuitionistic (λ,μ)-fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is a intuitionistic fuzzy relation on A is V given by $\mu_V(x,y) \vee \lambda = \min\{\mu_A(x), \mu_A(y)\} \wedge \mu$ and $\nu_V(x,y) \wedge \mu = \max\{\nu_A(x), \nu_A(y)\} \vee \lambda$, for all x and y in S.*

Definition 2.14. *Let $(R,+,\cdot)$and $(R',+,\cdot)$ be any two subrings. Let $f: R \to R'$ be any function and A be an intuitionistic fuzzy subring in R, V be an intuitionistic fuzzy subring in $f(R) = R'$, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R'. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.*

Definition 2.15. *Let A be an intuitionistic (λ,μ)-fuzzy subring of a ring $(R,+,\cdot)$ and a in R. Then the pseudo intuitionistic (λ,μ)-fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) \vee \lambda = p(a)\mu_A(x) \vee \lambda$ and $((a\nu_A)^p)(x) \wedge \mu = p(a)\nu_A(x) \wedge \mu$, for every x in R and for some p in P.*

3. Properties of intuitionistic (λ,μ)-fuzzy subrings of a ring

Theorem 3.1. *Intersection of any two intuitionistic (λ,μ)-fuzzy subring of a ring R is a intuitionistic (λ,μ)-fuzzy subring of R.*

Proof. Let A and B be any two intuitionistic (λ,μ)-fuzzy subrings of a ring R and x and$y$ in R. Let $A = \{x, \mu_A(x), \nu_A(x))/x \in R\}$and $B = \{(x, \mu_B(x), \nu_B(x))/x \in R\}$ and also let $C = A \cap B = \{(x, \mu_C(x), \nu_C(x))/x \in R\}$, where $\min\{\mu_A(x), \mu_B(x)\} \wedge \mu = \mu_C(x)$ and $\max\{\nu_A(x), \nu_B(x)\} \vee \lambda = \nu_C(x)$. Now,

$$\begin{aligned}
&\mu_C(x-y) \vee \lambda \\
&= \min\{\mu_A(x-y), \mu_B(x-y)\} \vee \lambda \\
&\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\} \wedge \mu \\
&= \min\{\min\{\mu_A(x), \mu_B(x)\} \wedge \mu, \min\{\mu_A(y), \mu_B(y)\} \wedge \mu\} \\
&= \min\{\mu_C(x), \nu_C(y)\} \wedge \mu.
\end{aligned}$$

Therefore, $\mu_C(x-y) \vee \lambda \geq \min\{\mu_C(x), \nu_C(y)\} \wedge \mu$, for all x and y in R. And,

$$\begin{aligned}
&\mu_C(xy) \vee \lambda \\
&= \min\{\mu_A(xy), \mu_B(xy)\} \vee \lambda \\
&\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\} \wedge \mu \\
&= \min\{\min\{\mu_A(x), \mu_B(x)\} \wedge \mu, \min\{\mu_A(y), \mu_B(y)\} \wedge \mu\} \\
&= \min\{\mu_C(x), \mu_C(y)\} \wedge \mu.
\end{aligned}$$

Therefore, $\mu_C(xy) \vee \lambda \geq \min\{\mu_C(x), \mu_C(y)\} \wedge \mu$, for all x and y in R. Now,

$$\begin{aligned}
&\nu_C(x-y) \wedge \mu \\
&= \max\{\nu_A(x-y), \nu_B(x-y)\} \wedge \mu \\
&\leq \max\{\max\{\nu_A(x), \nu_A(y)\}, \max\{\nu_B(x), \nu_B(y)\}\} \vee \lambda \\
&= \max\{\max\{\nu_A(x), \nu_B(x)\} \vee \lambda, \max\{\nu_A(y), \nu_B(y)\} \vee \lambda\} \\
&= \max\{\nu_C(x), \nu_C(y)\} \vee \lambda.
\end{aligned}$$

Therefore, $\nu_C(x-y) \wedge \mu \leq \max\{\nu_C(x), \nu_C(y)\} \vee \lambda$, for all x and y in R. And,

$$\begin{aligned}
&\nu_C(xy) \wedge \mu \\
&= \max\{\nu_A(xy), \nu_B(xy)\} \wedge \mu \\
&\leq \max\{\max\{\nu_A(x), \nu_A(y)\}, \max\{\nu_B(x), \nu_B(y)\}\} \vee \lambda \\
&= \max\{\max\{\nu_A(x), \nu_B(x)\} \vee \lambda, \max\{\nu_A(y), \nu_B(y)\} \vee \lambda\} \\
&= \max\{\nu_C(x), \nu_C(y)\} \vee \lambda.
\end{aligned}$$

Therefore, $\nu_C(xy)\wedge\mu \le \max\{\nu_C(x),\nu_C(y)\}\vee\lambda$, for all x and y in R. Therefore C is an intuitionistic (λ,μ)-fuzzy subring of R. Hence the intersection of any two intuitionistic (λ,μ)-fuzzy subrings of a ring R is an intuitionistic (λ,μ)-fuzzy subring of R. □

Theorem 3.2. *The intersection of a family of intuitionistic (λ,μ)-fuzzy subrings of ring R is an intuitionistic (λ,μ)-fuzzy subring of R.*

Proof. Let $\{V_i : i\in I\}$ be a family of intuitionistic (λ,μ)-fuzzy subrings of a ring R and let $A=\cap_{i\in I}V_i$. Let x and y in R. Then,

$$\begin{aligned}
&\mu(x-y)\vee\lambda\\
&=\inf_{i\in I}\mu_{v_i}(x-y)\vee\lambda\\
&\ge\inf_{i\in I}\min\{\mu_{V_i}(x),\mu_{V_i}(y)\}\wedge\mu\\
&=\min\{\inf_{i\in I}\mu_{V_i}(x),\inf_{i\in I}\mu_{V_i}(y)\}\wedge\mu\\
&=\min\{\mu_A(x),\mu_A(y)\}\wedge\mu.
\end{aligned}$$

Therefore, $\mu_A(x-y)\vee\lambda\ge\min\{\mu_A(x),\mu_A(y)\}\wedge\mu$, for all x and y in R. And,

$$\begin{aligned}
&\mu_A(xy)\vee\lambda\\
&=\inf_{i\in I}\mu_{V_i}(xy)\vee\lambda\\
&\ge\inf_{i\in I}\min\{\mu_{V_i}(x),\mu_{V_i}(y)\}\wedge\mu\\
&=\min\{\inf_{i\in I}\mu_{V_i}(x),\inf_{i\in I}\mu_{V_i}(y)\}\wedge\mu\\
&=\min\{\mu_A(x),\mu_A(y)\}\wedge\mu.
\end{aligned}$$

Therefore, $\mu_A(xy)\vee\lambda\ge\min\{\mu_A(x),\mu_A(y)\}\wedge\mu$, for all x and y in R. Now,

$$\begin{aligned}
&\nu_A(x-y)\wedge\mu\\
&=\sup_{i\in I}\nu_{V_i}(x-y)\wedge\mu\\
&\le\sup_{i\in I}\max\{\nu_{V_i}(x),\nu_{V_i}(y)\}\vee\lambda\\
&=\max\{\sup_{i\in I}\nu_{V_i}(x),\sup_{i\in I}\nu_{V_i}(y)\}\vee\lambda\\
&=\max\{\nu_A(x),\nu_A(y)\}\vee\lambda.
\end{aligned}$$

Therefore, $\nu_A(x-y)\wedge\nu\le\max\{\nu_A(x),\nu_A(y)\}\vee\lambda$, for all x and y in R. And,

$$\begin{aligned}
&\nu_A(xy)\wedge\mu\\
&=\sup_{i\in I}\nu_{V_i}(xy)\wedge\mu\\
&\le\sup_{i\in I}\max\{\nu_{V_i}(x),\nu_{V_i}(y)\}\vee\lambda\\
&=\max\{\sup_{i\in I}\nu_{V_i}(x),\sup_{i\in I}\nu_{V_i}(y)\}\vee\lambda\\
&=\max\{\nu_A(x),\nu_A(y)\}\vee\lambda.
\end{aligned}$$

Therefore, $\nu_A(xy\wedge\mu\le\max\{\nu_A(x),\nu_A(y)\}\vee\lambda$, for all x and y in R. That is, A is an intuitionistic (λ,μ)-fuzzy subring of a ring R. Hence, the intersection of a family of intuitionistic (λ,μ)-fuzzy subrings of R is an intuitionistic (λ,μ)-fuzzy subring of R. □

Theorem 3.3. *If A and B are any two intuitionistic (λ,μ)-fuzzy subrings of the rings R_1 and R_2 respectively, then $A\times B$ is an intuitionistic (λ,μ)-fuzzy subring of $R_1\times R_2$.*

Proof. Let A and B be two intuitionistic (λ,μ)-fuzzy subrings of the rings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1,y_1) and (x_2,y_2) are in $R_1\times R_2$. Now,

$$\begin{aligned}
&\mu_{A\times B}[(x_1,y_1)-(x_2,y_2)]\vee\lambda\\
&=\mu_{A\times B}(x_1-x_2,y_1-y_2)\vee\lambda\\
&=\min\{\mu_A(x_1-x_2),\mu_B(y_1-y_2)\}\vee\lambda,\\
&\ge\min\{\min\{\mu_A(x_1),\mu_A(x_2)\}\\
&\quad\wedge\mu,\min\{\mu_B(y_1),\mu_B(y_2)\}\wedge\mu\}\\
&=\min\{\min\{\mu_A(x_1),\mu_B(y_1)\}\\
&\quad\wedge\mu,\min\{\mu_A(x_2),\mu_B(y_2)\}\wedge\mu\}\\
&=\min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}\wedge\mu.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mu_{A\times B}[(x_1,y_1)-(x_2,y_2)]\vee\lambda\\
&\ge\min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}\wedge\mu.
\end{aligned}$$

Also,

$$\begin{aligned}
&\mu_{A\times B}[(x_1,y_1)(x_2,y_2)]\vee\lambda\\
&=\mu_{A\times B}(x_1x_2,y_1y_2)\vee\lambda\\
&=\min\{\mu_A(x_1x_2),\mu_B(y_1y_2)\}\vee\lambda\\
&\ge\min\{\min\{\mu_A(x_1),\mu_A(x_2)\}\\
&\quad\wedge\mu,\min\{\mu_B(y_1),\mu_B(y_2)\}\wedge\mu\}\\
&=\min\{\min\{\mu_A(x_1),\mu_B(y_1)\}\\
&\quad\wedge\mu,\min\{\mu_A(x_2),\mu_B(y_2)\}\wedge\mu\}\\
&=\min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}\wedge\mu.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mu_{A\times B}[(x_1,y_1)(x_2,y_2)]\vee\lambda\\
&\ge\min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}\wedge\mu.
\end{aligned}$$

Now,

$$\begin{aligned}
&\nu_{A\times B}[(x_1,y_1)-(x_2,y_2)]\wedge\mu\\
&=\nu_{A\times B}(x_1-x_2,y_1-y_2)\wedge\mu\\
&=\max\{\nu_A(x_1-x_2),\nu_B(y_1-y_2)\}\wedge\mu\\
&\le\max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda,\\
&\quad\max\{\nu_B(y_1),\nu_B(y_2)\}\vee\lambda\\
&=\max\{\max\{\nu_A(x_1),\nu_B(y_1)\}\vee\lambda,\\
&\quad\max\{\nu_A(x_2),\nu_B(y_2)\}\vee\lambda\}\\
&=\max\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}\vee\lambda.
\end{aligned}$$

Therefore,

$$\nu_{A\times B}[(x_1,y_1)-(x_2,y_2)]\wedge\mu \\ \leq \max\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}\vee\lambda.$$

Also,

$$\nu_{A\times B}[(x_1,y_1)(x_2,y_2)]\wedge\mu \\ = \nu_{A\times B}(x_1x_2,y_1y_2)\wedge\mu \\ = \max\{\nu_A(x_1x_2),\nu_B(y_1y_2)\}\wedge\mu \\ \leq \max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda, \\ \max\{\nu_B(y_1),\nu_B(y_2)\}\vee\lambda\} \\ = \max\{\max\{\mu_A(x_1),\nu_B(y_1)\}\vee\lambda, \\ \max\{\nu_A(x_2),\nu_B(y_2)\}\vee\lambda \\ = \max\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}\vee\lambda.$$

Therefore,

$$\nu_{A\times B}[(x_1,y_1)(x_2,y_2)]\wedge\mu \\ \leq \max\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}\vee\lambda.$$

Hence $A\times B$ is an intuitionistic (λ,μ)-fuzzy subring of ring of $R_1\times R_2$. □

Theorem 3.4. *Let A be an intuitionistic (λ,μ)-fuzzy subset of a subring R and V be the strongest intuitionistic (λ,μ)-fuzzy relation of R. Then A is an intuitionistic (λ,μ)-fuzzy subring of R if and only if V is an intuitionistic (λ,μ)-fuzzy subring of $R\times R$.*

Proof. Suppose that A is an intuitionistic (λ,μ)-fuzzy subring of a ring R. Then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in $R\times R$. We have,

$$\mu_V(x-y)\vee\lambda \\ = \mu_V[(x_1,x_2)-(y_1,y_2)]\vee\lambda \\ = \mu_V(x_1-y_1,x_2-y_2)\vee\lambda \\ = \min\{\mu_A(x_1-y_1),\mu_A(x_2-y_2)\}\vee\lambda \\ \geq \min\{\min\{\mu_A(x_1),\mu_A(y_1)\}, \\ \min\{\mu_A(x_2),\mu_A(y_2)\}\}\wedge\mu \\ = \min\{\min\{\mu_A(x_1),\mu_A(x_2)\wedge\mu, \\ \min\{\mu_A(y_1),\mu_A(y_2)\}\wedge\mu\} \\ = \min\{\mu_V(x_1,x_2),\mu_V(y_1,y_2)\}\wedge\mu \\ = \min\{\mu_V(x),\mu_V(y)\}\wedge\mu.$$

Therefore,

$$\mu_V(x-y)\vee\lambda \\ \geq \min\{\mu_V(x),\mu_V(y)\}\wedge\mu,$$

for all x and y in $R\times R$. And,

$$\mu_V(xy)\vee\lambda \\ = \mu_V[(x_1,x_2)(y_1,y_2)]\vee\lambda \\ = \mu_V(x_1y_1,x_2y_2)\vee\lambda \\ = \min\{\mu_A(x_1y_1),\mu_A(x_2y_2)\}\vee\lambda \\ \geq \min\{\min\{\mu_A(x_1),\mu_A(y_1)\},\min\{\mu_A(x_2),\mu_A(y_2)\}\}\wedge\mu \\ = \min\{\min\{\mu_A(x_1),\mu_A(x_2)\}\wedge\mu,\min\{\mu_A(y_1),\mu_A(y_2)\}\wedge\mu\} \\ = \min\{\mu_V(x_1,x_2),\mu_V(y_1,y_2)\}\wedge\mu \\ = \min\{\mu_V(x),\mu_V(y)\}\wedge\mu.$$

Therefore,

$$\mu_V(xy)\vee\lambda \geq \min\{\mu_V(x),\mu_V(y)\}\wedge\mu,$$

for all x and y in $R\times R$. We have,

$$\nu_V(x-y)\wedge\mu \\ = \nu_V[(x_1,x_2)-(y_1,y_2)]\wedge\mu \\ = \nu_V(x_1-y_1,x_2-y_2)\wedge\mu \\ = \max\{\nu_A(x_1-y_1),\nu_A(x_2-y_2)\}\wedge\mu \\ \leq \max\{\max\{\nu_A(x_1),\nu_A(y_1)\},\max\{\nu_A(x_2),\nu_A(y_2)\}\}\vee\lambda \\ = \max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda,\max\{\nu_A(y_1),\nu_A(y_2)\}\vee\lambda\} \\ = \max\{\nu_V(x_1,x_2),\nu_V(y_1,y_2)\}\vee\lambda \\ = \max\{\nu_V(x),\nu_V(y)\}\vee\lambda.$$

Therefore,

$$\nu_V(x-y)\wedge\mu \leq \max\{\nu_V(x),\nu_V(y)\}\vee\lambda,$$

for all x and y in $R\times R$. And,

$$\nu_V(xy)\wedge\mu \\ = \nu_V[(x_1,x_2)(y_1,y_2)]\wedge\mu \\ = \nu_V(x_1y_1,x_2y_2)\wedge\mu \\ = \max\{\nu_A(x_1y_1),\nu_A(x_2y_2)\} \\ \leq \max\{\max\{\nu_A(x_1),\nu_A(y_1)\},\max\{\nu_A(x_2),\nu_A(y_2)\}\}\vee\lambda \\ = \max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda,\max\{\nu_A(y_1),\nu_A(y_2)\}\vee\lambda\} \\ = \max\{\nu_V(x_1,x_2),\nu_V(y_1,y_2)\}\vee\lambda \\ = \max\{\nu_V(x),\nu_V(y)\}\vee\lambda.$$

Therefore,

$$\nu_V(xy)\wedge\mu \leq \max\{\nu_V(x),\nu_V(y)\}\vee\lambda,$$

for all x and y in $R\times R$. This proves that V is an intuitionistic (λ,μ)-fuzzy subring of $R\times R$.

Conversely assume that V is an intuitionistic (λ,μ)-fuzzy subring of $R\times R$, then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in $R\times R$, we have

$$\min\{\mu_A(x_1-y_1),\mu_A(x_2-y_2)\}\vee\lambda \\ = \mu_V(x_1-y_1,x_2-y_2)\vee\lambda \\ = \mu_V[(x_1,x_2)-(y_1,y_2)]\vee\lambda \\ = \mu_V(x-y)\vee\lambda \\ \geq \min\{\mu_V(x),\mu_V(y)\}\wedge\mu \\ = \min\{\mu_V(x_1,x_2),\mu_V(y_1,y_2)\}\wedge\mu \\ = \min\{\min\{\mu_A(x_1),\mu_A(x_2)\}\wedge\mu, \\ \min\{\mu_A(y_1),\mu_A(y_2)\}\wedge\mu\}.$$

If

$$\begin{aligned}&\mu_A(x_1-y_1)\vee\lambda\\ &\leq \mu_A(x_2-y_2)\vee\lambda,\mu_A(x_1)\vee\lambda\\ &\leq \mu_A(x_2)\vee\lambda,\mu_A(y_1)\vee\lambda\\ &\leq \mu_A(y_2)\vee\lambda,\end{aligned}$$

we get,

$$\mu_A(x_1-y_1)\vee\lambda \geq \min\{\mu_A(x_1),\mu_A(y_1)\}\wedge\mu,$$

for all x_1 and y_1 in R. And,

$$\begin{aligned}&\min\{\mu_A(x_1y_1),\mu_A(x_2y_2)\}\vee\lambda\\ &= \mu_V(x_1y_1,x_2y_2)\vee\lambda\\ &= \mu_V[(x_1,x_2)(y_1,y_2)]\vee\lambda\\ &= \mu_V(xy)\vee\lambda\\ &\geq \min\{\mu_V(x),\mu_V(y)\}\wedge\mu\\ &= \min\{\mu_V(x_1,x_2),\mu_V(y_1,y_2)\}\wedge\mu\\ &= \min\{\min\{\mu_A(x_1),\mu_A(x_2)\}\wedge\mu,\\ &\quad \min\{\mu_A(y_1),\mu_A(y_2)\}\wedge\mu\}.\end{aligned}$$

If

$$\begin{aligned}&\mu_A(x_1y_1)\wedge\mu\\ &\leq \mu_A(x_2y_2)\wedge\mu,\mu_A(x_1)\wedge\mu\\ &\leq \mu_A(x_2)\wedge\mu,\mu_A(y_1)\wedge\mu\\ &\leq \mu_A(y_2)\wedge\mu,\end{aligned}$$

we get

$$\mu_A(x_1y_1)\vee\lambda \geq \min\{\mu_A(x_1),\mu_A(y_1)\}\wedge\mu,$$

for all x_1 and $y_1$in R. We have

$$\begin{aligned}&\max\{\nu_A(x_1-y_1),\nu_A(x_2-y_2)\}\wedge\mu\\ &= \nu_V(x_1-y_1,x_2-y_2)\wedge\mu\\ &= \nu_V[(x_1,x_2)-(y_1,y_2)]\wedge\mu\\ &= \nu_V(x-y)\wedge\mu\\ &\leq \max\{\nu_V(x),\nu_V(y)\}\vee\lambda\\ &= \max\{\nu_V(x_1,x_2),\nu_V(y_1,y_2)\}\vee\lambda\\ &= \max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda,\\ &\quad \max\{\nu_A(y_1),\nu_A(y_2)\}\vee\lambda\}.\end{aligned}$$

If

$$\begin{aligned}&\nu_A(x_1-y_1)\wedge\mu\\ &\geq \nu_A(x_2-y_2)\wedge\mu,\nu_A(x_1)\wedge\mu\\ &\geq \nu_A(x_2)\wedge\mu,\nu_A(y_1)\wedge\mu\\ &\geq \nu_A(y_2)\wedge\mu,\end{aligned}$$

we get,

$$\nu_A(x_1-y_1)\wedge\mu \leq \max\{\nu_A(x_1),\nu_A(y_1)\}\vee\lambda,$$

for all x_1 and y_1 in R. And,

$$\begin{aligned}&\max\{\nu_A(x_1y_1),\nu_A(x_2y_2)\}\wedge\mu\\ &= \nu_V(x_1y_1,x_2y_2)\wedge\mu\\ &= \nu_V[(x_1,x_2)(y_1,y_2)]\wedge\mu\\ &= \nu_V(xy)\wedge\mu\\ &\leq \max\{\nu_V(x),\nu_V(y)\}\vee\lambda\\ &= \max\{\nu_V(x_1,x_2),\nu_V(y_1,y_2)\}\vee\lambda\\ &= \max\{\max\{\nu_A(x_1),\nu_A(x_2)\}\vee\lambda,\\ &\quad \max\{\nu_A(y_1),\nu_A(y_2)\}\vee\lambda\}.\end{aligned}$$

If

$$\begin{aligned}&\nu_A(x_1y_1)\wedge\mu\\ &\geq \nu_A(x_2y_2)\wedge\mu,\nu_A(x_1)\wedge\mu\\ &\geq \nu_A(x_2)\wedge\mu,\nu_A(y_1)\wedge\mu\\ &\geq \nu_A(y_2)\wedge\mu,\end{aligned}$$

we get

$$\nu_A(x_1y_1)\wedge\mu \leq \max\{\nu_A(x_1),\nu_A(y_1)\}\vee\lambda,$$

for all x_1 and y_1 in R. Therefore A is an intuitionistic (λ,μ)-fuzzy subring of R. □

Theorem 3.5. *If A is an intuitionistic (λ,μ)- fuzzy subring of a ring $(R,+,\cdot)$, then $\Box A$ is an intuitionistic (λ,μ)- fuzzy subring of R.*

Proof. Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring R. Consider $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle\}$, for all x in R, we take $\Box A = B = \{\langle x,\mu_B(x),\nu_B(x)\rangle\}$, where

$$\mu_B(x)=\mu_A(x),\nu_B(x)=1-\mu_A(x).$$

Clearly, $\mu_B(x-y)\vee\lambda \geq \min\{\mu_B(x),\mu_B(y)\}\wedge\mu$, for all x and y in R and $\mu_B(xy)\vee\lambda \geq \min\{\mu_B(x),\mu_B(y)\}\wedge\mu$, for all x and y in R. Since A is an intuitionistic (λ,μ)- fuzzy subring of R, we have $\mu_A(x-y)\vee\lambda \geq \min\{\mu_A(x),\mu_A(y)\}\wedge\mu$, for all x and y in R, which implies that

$$(1-\nu_B(x-y))\vee\lambda \geq \min\{(1-\nu_B(x)),(1-\nu_B(y))\}\wedge\mu,$$

which implies that

$$\begin{aligned}&\nu_B(x-y)\wedge\mu\\ &\leq \{1-\min\{(1-\nu_B(x)),(1-\nu_B(y))\}\}\vee\lambda\\ &= \max\{\nu_B(x),\nu_B(y)\}\vee\lambda.\end{aligned}$$

Therefore,

$$\nu_B(x-y)\wedge\mu \leq \max\{\nu_B(x),\nu_B(y)\}\vee\lambda,$$

for all x and y in R. And

$$\mu_A(xy)\vee\lambda \geq \min\{\mu_A(x),\mu_A(y)\}\wedge\mu,$$

for all x and y in R, which implies that

$$(1-\nu_B(xy))\vee\lambda \geq \min\{(1-\nu_B(x)),(1-\nu_B(y))\}\wedge\mu$$

which implies that

$$\begin{aligned}&\nu_B(xy)\wedge\mu\\&\leq\{1–\min\{(1–\nu_B(x)),(1–\nu_B(y))\}\}\vee\lambda\\&=\max\{\nu_B(x),\nu_B(y)\}\vee\lambda.\end{aligned}$$

Therefore,

$$\nu_B(xy)\wedge\mu\leq\max\{\nu_B(x),\nu_B(y)\}\vee\lambda,$$

for all x and y in R. Hence $B=\Box A$ is an intuitionistic (λ,μ)-fuzzy subring of a ring R. □

Remark 3.6. *The converse of the above theorem is not true. It is shown by the following example:*

Consider the subring $Z_5=\{0,1,2,3,4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A=\{\langle 0,0.8,0.2\rangle,\langle 1,0.6,0.1\rangle,\langle 2,0.6,0.3\rangle,\langle 3,0.5,0.3\rangle,$ $\langle 4,0.5,0.4\rangle\}$ is not an intuitionistic (0.3,0.7)-fuzzy subring of Z_5, but $\Box A=\{\langle 0,0.8,0.2\rangle,\langle 1,0.6,0.4\rangle,\langle 2,0.6,0.4\rangle,\langle 3,0.5,0.5\rangle,$ $\langle 4,0.5,0.5\rangle\}$ is an intuitionistic (0.3 , 0.7)-fuzzy subring of Z_5.

Theorem 3.7. *If A is an intuitionistic (λ,μ)- fuzzy subring of a ring $(R,+,\cdot)$, then $\diamond A$ is an intuitionistic (λ,μ)- fuzzy subring of R.*

Proof. Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring R. That is $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle\}$, for all x in R. Let $\diamond A=B=\{\langle x,\mu_B(x),\nu_B(x)\rangle\}$, where $\mu_B(x)=1-\nu_A(x),\nu_B(x)=\nu_A(x)$. Clearly, $\nu_B(x-y)\wedge\mu\leq\max\{\nu_B(x),\nu_B(y)\}\vee\lambda$, for all x and y in R and $\nu_B(xy)\wedge\mu\leq\max\{\nu_B(x),\nu_B(y)\}\vee\lambda$, for all x and y in R. Since A is an intuitionistic $(\lambda,\mu)-$ fuzzy subring of R, we have $\nu_A(x-y)\wedge\mu\leq\max\{\nu_A(x),\nu_A(y)\}\vee\lambda$, for all x and y in R, which implies that

$$\{1–\mu_B(x-y)\}\wedge\mu\leq\max\{(1–\mu_B(x)),(1–\mu_B(y))\}\vee\lambda,$$

which implies that

$$\begin{aligned}&\mu_B(x-y)\vee\lambda\\&\geq\{1–\max\{(1-\mu_B(x)),(1-\mu_B(y))\}\wedge\mu\}\\&=\min\{\mu_B(x),\mu_B(y)\}\wedge\mu.\end{aligned}$$

Therefore,

$$\mu_B(x-y)\vee\lambda\geq\min\{\mu_B(x),\mu_B(y)\}\wedge\mu,$$

for all x and y in R. And

$$\nu_A(xy)\wedge\mu\leq\max\{\nu_A(x),\nu_A(y)\}\vee\lambda,$$

for all x and y in R, which implies that

$$\begin{aligned}&\{1–\mu_B(xy)\}\wedge\mu\\&\leq\max\{(1-\mu_B(x)),(1–\mu_B(y))\}\vee\lambda,\end{aligned}$$

which implies that

$$\begin{aligned}&\mu_B(xy)\vee\lambda\\&\geq\{1–\max\{(1–\mu_B(x)),(1–\mu_B(y))\}\wedge\mu\\&=\min\{\mu_B(x),\mu_B(y)\}\wedge\mu.\end{aligned}$$

Therefore,

$$\mu_B(xy)\vee\lambda\geq\min\{\mu_B(x),\mu_B(y)\}\wedge\mu,$$

for all x and y in R. Hence $B=\diamond A$ is an intuitionistic (λ,μ)-fuzzy subring of a ring R. □

Remark 3.8. *The converse of the above theorem is not true. It is shown by the following example:*

Consider the subring $Z_5=\{0,1,2,3,4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A=\{\langle 0,0.4,0.2\rangle,\langle 1,0.5,0.3\rangle,\langle 2,0.6,0.4\rangle,\langle 3,0.5,0.3\rangle,\langle 4,0.6,0.4\rangle\}$ is not an intuitionistic (0.3,0.8)-fuzzy subring of Z_5, but $\diamond A=\{\langle 0,0.8,0.2\rangle,\langle 1,0.7,0.3\rangle,\langle 2,0.6,0.4\rangle,\langle 3,0.7,0.3\rangle,\langle 4,0.6,0.4\rangle\}$ is an intuitionistic (0.3 , 0.8)-fuzzy subring of Z_5.

In the following Theorem $\circ$ is the composition operation of functions:

Theorem 3.9. *Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring H and f is an isomorphism from a subring R onto H. Then $A\circ f$ is an intuitionistic (λ,μ)- fuzzy subring of R.*

Proof. Let x and y in R and A be an intuitionistic (λ,μ)-fuzzy subring of a ring H. Then we have,

$$\begin{aligned}&(\mu_A\circ f)(x-y)\vee\lambda\\&=\mu_A(f(x-y))\vee\lambda\\&=\mu_A(f(x)-f(y))\vee\lambda\\&\geq\min\{\mu_A(f(x)),\mu_A(f(y))\}\wedge\mu\\&\geq\min\{(\mu_A\circ f)(x)\wedge\mu,(\mu_A\circ f)(y)\wedge\mu\},\end{aligned}$$

which implies that

$$\begin{aligned}&(\mu_A\circ f)(x-y)\vee\lambda\\&\geq\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\wedge\mu\}.\end{aligned}$$

And

$$\begin{aligned}&(\mu_A\circ f)(xy)\vee\lambda\\&=\mu_A(f(xy))\vee\lambda\\&=\mu_A(f(x)f(y))\vee\lambda\\&\geq\min\{\mu_A(f(x)),\mu_A(f(y))\}\wedge\mu\\&\geq\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu,\end{aligned}$$

which implies that

$$\begin{aligned}&(\mu_A\circ f)(xy)\vee\lambda\\&\geq\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu.\end{aligned}$$

Then we have,

$$\begin{aligned}&(\nu_A \circ f)(x-y)\wedge\mu\\&=\nu_A(f(x-y))\wedge\mu\\&=\nu_A(f(x)-f(y))\wedge\mu\\&\le\max\{\nu_A(f(x)),\nu_A(f(y))\}\vee\lambda\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda,\end{aligned}$$

which implies that

$$\begin{aligned}&(\nu_A \circ f)(x-y)\wedge\mu\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda.\end{aligned}$$

And

$$\begin{aligned}&(\nu_A \circ f)(xy)\wedge\mu\\&=\nu_A(f(xy))\wedge\mu\\&=\nu_A(f(x)f(y))\wedge\mu\\&\le\max\{\nu_A(f(x)),\nu_A(f(y))\}\vee\lambda\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda,\end{aligned}$$

which implies that

$$\begin{aligned}&(\nu_A \circ f)(xy)\wedge\mu\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda.\end{aligned}$$

Therefore $(A\circ f)$ is an intuitionistic (λ,μ)- fuzzy subring of a ring R. □

Theorem 3.10. *Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring H and f is an anti-isomorphism from a subring R onto H. Then $A\circ f$ is an intuitionistic (λ,μ)- fuzzy subring of R.*

Proof. Let x and y in R and A be an intuitionistic (λ,μ)-fuzzy subring of a ring H. Then we have,

$$\begin{aligned}&(\mu_A \circ f)(x-y)\vee\lambda\\&=\mu_A(f(x-y))\vee\lambda\\&=\mu_A(f(y)-f(x))\vee\lambda\\&\ge\min\{\mu_A(f(x)),\mu_A(f(y))\}\wedge\mu\\&\ge\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu,\end{aligned}$$

which implies that

$$\begin{aligned}&(\mu_A \circ f)(x-y)\vee\lambda\\&\ge\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu.\end{aligned}$$

And

$$\begin{aligned}&(\mu_A \circ f)(xy)\vee\lambda\\&=\mu_A(f(xy))\vee\lambda\\&=\mu_A(f(y)f(x))\vee\lambda\\&\ge\min\{\mu_A(f(x)),\mu_A(f(y))\}\wedge\mu\\&\ge\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu,\end{aligned}$$

which implies that

$$\begin{aligned}&(\mu_A \circ f)(xy)\vee\lambda\\&\ge\min\{(\mu_A\circ f)(x),(\mu_A\circ f)(y)\}\wedge\mu.\end{aligned}$$

Then we have,

$$\begin{aligned}&(\nu_A \circ f)(x-y)\wedge\mu\\&=\nu_A(f(x-y))\wedge\mu\\&=\nu_A(f(y)-f(x))\wedge\mu\\&\le\max\{\nu_A(f(x)),\nu_A(f(y))\}\vee\lambda\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda,\end{aligned}$$

which implies that

$$\begin{aligned}&(\nu_A \circ f)(x-y)\wedge\mu\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda.\end{aligned}$$

And

$$\begin{aligned}&(\nu_A \circ f)(xy)\wedge\mu\\&=\nu_A(f(xy))\wedge\mu\\&=\nu_A(f(y)f(x))\wedge\mu\\&\le\max\{\nu_A(f(x)),\nu_A(f(y))\}\vee\lambda\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda,\end{aligned}$$

which implies that

$$\begin{aligned}&(\nu_A \circ f)(xy)\wedge\mu\\&\le\max\{(\nu_A\circ f)(x),(\nu_A\circ f)(y)\}\vee\lambda.\end{aligned}$$

Therefore $A\circ f$ is an intuitionistic (λ,μ)-fuzzy subring of a ring R. □

Theorem 3.11. *Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring $(R,+,\cdot)$, then the pseudo intuitionistic (λ,μ)- fuzzy coset $(aA)^p$ is an intuitionistic (λ,μ)- fuzzy subring of a ring R, for every a in R.*

Proof. Let A be an intuitionistic (λ,μ)- fuzzy subring of a ring R. For every x and y in R, we have,

$$\begin{aligned}&((a\mu_A)^p)(x-y)\vee\lambda\\&=p(a)\mu_A(x-y)\vee\lambda\\&\ge p(a)\min\{(\mu_A(x),\mu_A(y)\}\wedge\mu\\&=\min\{p(a)\mu_A(x),p(a)\mu_A(y)\}\wedge\mu\\&=\min\{((a\mu_A)^p)(x),((a\mu_A)^p)(y)\}\wedge\mu.\end{aligned}$$

Therefore,

$$\begin{aligned}&((a\mu_A)^p)(x-y)\vee\lambda\\&\ge\min\{((a\mu_A)^p)(x),((a\mu_A)^p)(y)\}\wedge\mu.\end{aligned}$$

Now,

$$((a\mu_A)^p)(xy)\vee\lambda$$
$$= p(a)\mu_A(xy)\vee\lambda$$
$$\geq p(a)\min\{\mu_A(x),\mu_A(y)\}\wedge\mu$$
$$= \min\{p(a)\mu_A(x),p(a)\mu_A(y)\}\wedge\mu$$
$$= \min\{((a\mu_A)^p)(x),((a\mu_A)^p)(y)\}\wedge\mu.$$

Therefore,

$$((a\mu_A)^p)(xy)\vee\lambda$$
$$\geq \min\{((a\mu_A)^p)(x),((a\mu_A)^p)(y)\}\wedge\mu.$$

For every x and y in R, we have,

$$((a\nu_A)^p)(x-y)\wedge\mu$$
$$= p(a)\mu_A(x-y)\wedge\mu$$
$$\leq p(a)\max\{(\nu_A(x),\nu_A(y)\}\vee\lambda$$
$$= \max\{p(a)\nu_A(x),p(a)\nu_A(y)\}\vee\lambda$$
$$= \max\{((a\nu_A)^p)(x),((a\nu_A)^p)(y)\}\vee\lambda.$$

Therefore,

$$((a\nu_A)^p)(x-y)\wedge\mu$$
$$\leq \max\{((a\nu_A)^p)(x),((a\nu_A)^p)(y)\}\vee\lambda.$$

Now,

$$((a\nu_A)p)(xy)\wedge\mu$$
$$= p(a)\nu_A(xy)\wedge\mu$$
$$\leq p(a)\max\{\nu_A(x),\nu_A(y)\}\vee\lambda$$
$$= \max\{p(a)\nu_A(x),p(a)\nu_A(y)\}\vee\lambda$$
$$= \max\{((a\nu_A)^p)(x),((a\nu_A)^p)(y)\}\vee\lambda.$$

Therefore,

$$((a\nu_A)^p)(xy)\wedge\mu$$
$$\leq \max\{((a\nu_A)^p)(x),((a\nu_A)^p)(y)\}\vee\lambda.$$

Hence $(aA)^p$ is an intuitionistic (λ,μ)-fuzzy subring of a ring R. □

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