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Improved makespan of the branch and bound solution for a fuzzy flow-shop scheduling problem using the maximization operator

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Abstract
In practical situations, the processing times are not known exactly i.e., they are not crisp. They lie in an interval. A fuzzy number is essentially a generalized interval which can represent these processing times naturally. In the literature, Triangular, trapezoidal and octagonal fuzzy numbers are used in to solve fuzzy flow-shop scheduling problems with the objective of minimizing the makespan using the branch and bound algorithm of Ignall and Scharge which is modified to fuzzy scenario. The fuzzy makespan and fuzzy mean flow times are then calculated for making decisions using fuzzy addition and fuzzy subtraction. While calculating the waiting time and completion times of a job on a machine,fuzzy subtraction leads to negative processing times which are not realistic and hence they are neglected for the evaluation of the makespan. In this paper, the makespan is calculated using the fuzzy maximization operator which in turn improves the makespan in comparison with fuzzy subtraction.

Keywords
Flow-shop scheduling, Branch and bound, Octagonal fuzzy numbers, Ranking methods,Fuzzy maximization.

AMS Subject Classification
90C70,97M40.

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Contents

1. Introduction

Scheduling problems occurring in real life applications generally are flow-shop scheduling problems. Each job has the same routing through machines and the sequence of operations is fixed in a flow-shop.Branch- and- bound algorithms are the mostly used optimal technique to solve such type of problems excluding heuristic methods[8]. Branch and bound technique is an Integer programming solving technique and it was first applied to scheduling problems by Lomnicki,Ignall and Scharge in 1965. Several branch and bound technique are developed by Brooks and White in 1965, Brown and Lomnicki in 1966, McMahon and Burton in 1967,Bestwig and Hasting in 1976,Potts in 1980, Carlier and Pinson in 1989, Applegate and Cook in 1991 and Brucker, Jurisch and Sievers in 1994.

2. Fuzzy flow-shop scheduling

In the most studies concerned with the scheduling problems, processing times were taken as certain and fixed value.

But in the real world application, information is often ambiguous, vague and imprecise. Several techniques are proposed for managing uncertainty. To solve vague situations in real problems, the first systemic approach related to fuzzy set theory was successfully applied in many areas such as in scheduling problems. In recent studies, scheduling problems were fuzzified by using the concept of fuzzy due date and processing times. Dumitru and Luban(1982) investigated the application of fuzzy mathematical programming model on the problem of the production scheduling. Especially from the beginning of the 1990s fuzzy logic applications on the scheduling problems are increased. Han et al.(1994);Ishibuchi et al.(1994 a,b);Ishii et al.(1992) and Murata et al.(1997) used fuzzy due-date in their studies.Adamapoulos and Pappis (1996); Kuroda and Wang(1996);Hong et al.(1995); Hong and Chuang (1996); Ishibuchi et al.(1995), McCahon and Lee[7](1990,1992); Izzettin Temiz[4] et al.(1994);Murata et al.(1996), Stanfield et al.(1996) fuzzified scheduling problems by using fuzzy processing times. Moreover Cheng et al.(1994); Dubois et al.(1995);Ishii and Tada (1995); Watanabe et al.(1992) used fuzzy precedence relations in scheduling problems. Ambika G,Uthra G[1] had applied the branch and bound technique to TFNs; Vinoba V and Selvamalar N had used octagonal fuzzy processing times to branch and bound algorithm[10], CDS algorithm[9], Johnson's algorithm[11] and NEH algorithm[12] to minimize the makespan of the fuzzy flowshop problem.

3. Fuzzy branch and bound algorithm for N jobs and 3 workstations

Ignall and Scharge's branch and bound algorithm[3] for a general three machine flow-shop problem is considered. The problem is represented as a tree where each node has a possibility to emanate into a partial sequence. To determine the best partial sequence node , the lower bounds (LB) of all the partial sequences are calculated and the node with the lowest lower bound is chosen. The procedure is continued till the least lower bound is found.After obtaining an order where all the jobs are scheduled, the nodes having the upper lower bounds than the completion time of this schedule are fathomed. The tree is fathomed when no more branching is possible. To fuzzify the algorithm ,the fuzzy processing times are used and their lower bounds are also expressed as octagonal fuzzy numbers and a comparison of these fuzzy numbers are done by finding the measure found in Malini[6].Only makespan ($\tilde{M}$) and mean flowtime ($M\tilde{F}T$) are used as the performance criteria in this work , while the symbol '$\sim$' indicates fuzzy. In the computation process , addition, subtraction and maximum operations[5] are fuzzy operations.

The fuzzy lower bounds on the fuzzy makespan of all schedules beginning with the sequence S_r are calculated using

$$L\tilde{B}(S_r) = \max \begin{cases} C\tilde{T}1(S_r)(+)\underset{S'_r}{(+)}\tilde{p}_{i1}(+)\underset{S'_r}{\min}(\tilde{p}_{i2}(+)\tilde{p}_{i3}), \\ C\tilde{T}2(S_r)(+)\underset{S'_r}{(+)}\tilde{p}_{i2}(+)\underset{S'_r}{\min}(\tilde{p}_{i3}), \\ C\tilde{T}3(S_r)(+)\underset{S'_r}{(+)}\tilde{p}_{i3} \end{cases} \tag{3.1}$$

Where
$\tilde{p}_{ij}$ is the fuzzy processing time of the i^{th} job in the j^{th} machine.
S'_r is the set of $(n-r)$ jobs yet to be assigned to the machines.
$C\tilde{T}K(S_r)$ is the fuzzy completion time of the last job in the sequence S_r at the machine K.
$(+)$ refers to fuzzy addition.
After fuzzy lower bound values for nodes are calculated , branching is done from the lowest bound to form new nodes for all unscheduled jobs. This process is continued till all the jobs are scheduled.

3.1 An illustrative example

An illustrative example using octagonal processing times is the following four jobs, three machines scheduling problem:

Throughout this illustration, we have taken the value of $k = 0.5$ for octagonal fuzzy numbers. Using the equation(3.1),the first level of fuzzy lower bounds are calculated as:

For machine 1,
$[L\tilde{B}(1)]_1 = (12,13,14,15,17,18,19,20)$
$(+)(34,37,40,43,49,52,55,58)$
$(+)\min[(13,15,17,19,23,25,27,29),$
$(27,29,31,33,37,39,41,43),$
$(26,28,30,32,36,38,40,42)]$
For Machine 2,
$[L\tilde{B}(1)]_2 = (26,28,30,32,36,38,40,42)$
$(+)(33,36,39,42,48,51,54,57)$
$(+)\min[(7,8,9,10,12,13,14,15),$
$(11,12,13,14,16,17,18,19),$
$(15,16,17,18,20,21,22,23)]$
For machine3,
$[L\tilde{B}(1)]_3 = (34,37,40,43,49,52,55,58)$
$(+)(33,36,39,42,48,51,54,57)$
To find the minimum of the fuzzy numbers, the generalized mean value(GMV) of these numbers are calculated and the fuzzy number with the smallest GMV is considered the smallest. If two fuzzy numbers have the same GMV's , to break the tie ,the spread is calculated for each fuzzy number as given in [2] and the number with the smaller spread is adjudged

Job	Machine1	Machine2	Machine3
1	(12,13,14,15,17,18,19,20)	(14,15,16,17,19,20,21,22)	(8,9,10,11,13,14,15,16)
2	(10,11,12,13,15,16,17,18)	(6,7,8,9,11,12,13,14)	(7,8,9,10,12,13,14,15)
3	(9,10,11,12,14,15,16,17)	(16,17,18,19,21,22,23,24)	(11,12,13,14,16,17,18,19)
4	(15,16,17,18,20,21,22,23)	(11,12,13,14,16,17,18,19)	(15,16,17,18,20,21,22,23)

smallest.

$$\begin{aligned} \tilde{LB}(1) &= \max\{[\tilde{LB}(1)]_1, [\tilde{LB}(1)]_2, [\tilde{LB}(1)]_3 \\ &= \max\{(59,65,71,77,89,95,101,107), \\ &\quad (66,72,78,84,96,102,108,114), \\ &\quad (67,73,79,85,97,103,109,115) \\ &= (67,73,79,85,97,103,109,115) \end{aligned}$$

Since $\max(83,90,91) = 91$ using the GMV of the fuzzy numbers.Proceeding as above,the sequence which gives the lowest bound for the entire makespan is 3-4-1-2 with the fuzzy lower bound (66, 72, 78, 84, 96, 102, 108, 114)...3-4-1-2 is the optimum schedule. The entire tree network is illustrated in figure1.

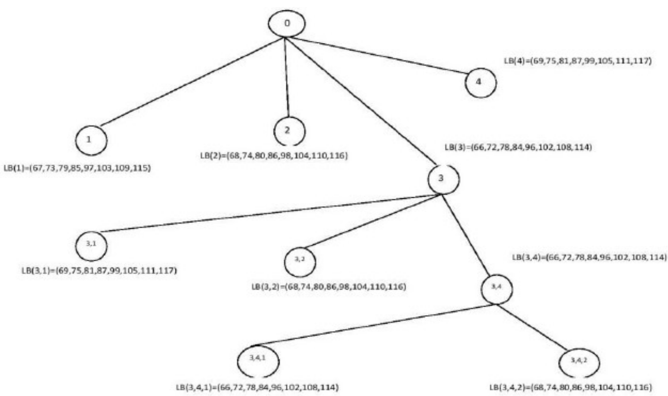

Figure 1. Fuzzy branch and bound solution for 4 job, 3 machine example

Since the times are fuzzy, the fuzzy makespan must be calculated as the maximum of the job completion times $\tilde{C}_{i3}$ or $\tilde{M} = m\tilde{a}x_i \tilde{C}_{i3}$ where each $\tilde{C}_{i3}$ in 3-workstations case, is calculated as
$\tilde{C}_{i3} = \overset{3}{\underset{j=1}{(+)}} q_{ij}(+)p_{ij}$.
For fuzzy waiting time for workstation 3 , use $\tilde{q}_{i3} = \tilde{C}_{k3}(-)\tilde{C}_{i2}$, here (-) refers to fuzzy subtraction.
The fuzzy makespan of the sequence is:
$\therefore \tilde{M} = \underset{i}{max} \tilde{C}_{i3} = (61,65,71,77,139,201,281,343)$
$M\tilde{F}T = \frac{\left(\overset{4}{\underset{i=1}{(+)}} \tilde{C}_{ij}\right)}{4}$
$= (51.25, 55.25, 59.75, 64.25, 91.75, 132.50, 156.75, 184.75)$

The fuzzy waiting, processing and completion times for each of the job sequences 3,4,1,2 are calculated and listed in Table1.

3.2 Improvisation of makespan using maximization operator

While calculating the waiting time and completion times of a job on a machine,fuzzy subtraction leads to negative processing times which are not realistic and hence the negative portion is neglected for the evaluation of the makespan. Hence we arrive at fuzzy numbers which are not octagonal. This will cause subsequent waiting time fuzzy numbers to degenerate into many-pieced membership functions,and make subsequent calculations unwieldly.This may have an effect on the mean flow time and the makespan which in turn affects the optimality of the solution.

In the process of improving the makespan , the fuzzy completion time for job i at machine j, $\tilde{C}_{ij}$,is calculated using fuzzy maximization operator as

$$\tilde{C}_{ij} = m\tilde{a}x\{\tilde{C}_{i-1,j}; \tilde{C}_{i,j-1}\}(+)\tilde{p}_{ij} \tag{3.2}$$

assuming job $i-1$ precedes job i in the sequence.

Completion time due to fuzzy maximization operator are given in Table 2 for the illustrative problem.
In this case, The fuzzy makespan of the sequence is:
$\tilde{M} = \underset{i}{max} \tilde{C}_{i3} = (61,65,71,77,83,95,101,107,113)$
$M\tilde{F}T = \frac{\left(\overset{4}{\underset{i=1}{(+)}} \tilde{C}_{ij}\right)}{4} = (51.5, 56, 60.5, 65, 74, 78.5, 83, 87.5)$

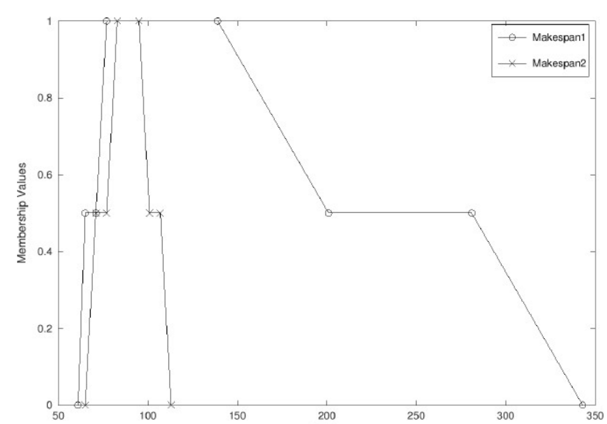

Figure 2. Makespans based on Fuzzy subtraction and Maximization

Table 1. Fuzzy Parameters of 3 machines

Job	$\tilde{q}_{i1}$	$\tilde{p}_{i1}$	$\tilde{C}_{i1}$	$\tilde{q}_{i2}$	$\tilde{p}_{i2}$	$\tilde{C}_{i2}$	$\tilde{q}_{i3}$	$\tilde{p}_{i3}$	$\tilde{C}_{i3}$
3	0	(9,10,11,12,14,15,16,17)	(9,10,11,12,14,15,16,17)	0	(16,17,18,19,21,22,23,24)	(25,27,29,31,35,37,39,41)	0	(11,12,13,14,16,17,18,19)	(36,39,42,45,51,54,57,60)
4	(9,10,11,12,14,15,16,17)	(15,16,17,18,20,21,22,23)	(24,26,28,30,34,36,38,40)	(0,0,0,0,5,9,13,17)	(11,12,13,14,16,17,18,19)	(35,38,41,44,55,62,69,76)	(0,0,0,0,7,13,19,25)	(15,16,17,18,20,21,22,23)	(50,54,58,62,82,96,110,124)
1	(24,26,28,30,34,36,38,40)	(12,13,14,15,17,18,19,20)	(36,39,42,45,51,54,57,60)	(0,0,0,0,10,20,30,40)	(14,15,16,17,19,20,21,22)	(50,54,58,62,80,94,108,122)	(0,0,0,0,2,20,56,74)	(8,9,10,11,13,14,15,16)	(58,63,68,73,95,128,179,212)
2	(36,39,42,45,51,54,57,60)	(10,11,12,13,15,16,17,18)	(46,50,54,58,66,70,74,78)	(0,0,0,0,22,40,58,76)	(6,7,8,9,11,12,13,14)	(52,57,62,67,99,122,145,168)	(0,0,0,0,28,66,122,160)	(7,8,9,10,12,13,14,15)	(61,65,71,77,139,201,281,343)

Table 2. Completion time due to fuzzy maximization operator

Parameter	Job			
	3	4	1	2
$\tilde{p}_{i1}$	(9,10,11,12,14,15,16,17)	(15,16,17,18,20,21,22,23)	(12,13,14,15,17,18,19,20)	(10,11,12,13,15,16,17,18)
$\tilde{C}_{i1}$	(9,10,11,12,14,15,16,17)	(24,26,28,30,34,36,38,40)	(36,39,42,45,51,54,57,60)	(46,50,54,58,66,70,74,78)
$\tilde{p}_{i2}$	(16,17,18,19,21,22,23,24)	(11,12,13,14,16,17,18,19)	(14,15,16,17,19,20,21,22)	(6,7,8,9,11,12,13,14)
$\tilde{C}_{i2}$	(25,27,29,31,35,37,39,41)	(36,39,42,45,51,54,57,60)	(50,54,58,62,70,74,78,82)	(56,61,66,71,81,86,91,96)
$\tilde{p}_{i3}$	(11,12,13,14,16,17,18,19)	(15,16,17,18,20,21,22,23)	(8,9,10,11,13,14,15,16)	(7,8,9,10,12,13,14,15)
$\tilde{C}_{i3}$	(36,39,42,45,51,54,57,60)	(47,51,55,59,67,71,75,79)	(58,63,68,73,83,88,93,98)	(65,71,77,83,95,101,107,113)

Conclusion

The flow-shop scheduling algorithm of Ignall and Scharge is modified to accept octagonal fuzzy numbers as job processing times. The resultant job sequences are non-fuzzy but the makespan and mean flow time are fuzzy. By keeping the fuzziness throughout, the decision maker can have a intact information. We have illustrated the problem with $k = 0.5$, but the algorithm works for any value of k. When the problem is solve deterministically with the means of the fuzzy numbers as the deterministic inputs, the results are identical i.e., we get the identical sequence 3-4-1-2 with a makespan of 90 units which is the mode of the makespan in the fuzzy case.Through this work, it is possible to say that maximization operator is producing better makespans than the fuzzy subtraction since the GMV of the makespan in fuzzy subtraction case is 90 which is the same as the deterministic problem while the

GMV of the makespan in fuzzy maximization operator case is only 89.

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