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Hamiltonian laceability in the shadow distance graph of path graphs

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Abstract
A connected graph G is termed hamiltonian-t-laceable (t*-laceable) if there exists in it a hamiltonian path between every pair (at least one pair) of distinct vertices u and v with the property $d(u,v)=t, 1 \leq t \leq diam(G)$, where t is a positive integer. In this paper, we establish laceability properties in the edge tolerant shadow distance graph of the path graph P_n with distance set $D_s=\{1,2k\}$.

Keywords
Hamiltonian laceable, hamiltonian-t-laceable, hamiltonian-t^*-laceable, shadow graph, shadow distance graph.

AMS Subject Classification
05C45, 05C99

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1. Introduction

Let G is a finite, simple, connected and undirected graph. Let u and v be two vertices in G. The distance between u and v denoted by $d(u,v)$ is the length of a shortest path in G. G is hamiltonian laceable if there exists in it a hamiltonian path for every pair of vertices at an odd distance. G is hamiltonian-t-laceable (t*-laceable) if there exists in it a hamiltonian path between every pair (at least one pair) of vertices u and v with the property $d(u,v)=t, 1 \leq t \leq diam(G)$, where t is a positive integer. Throughout this paper, P_n denotes the path graph on n vertices.

Laceability in brick products of even cycles was explored by Alspach et.al. in [1]. A characterization for a 1-connected graph to be hamiltonian-t-laceable for $t=1,2$ and 3 was given in [7] and this was extended to $t=4$ and 5 by Thimmaraju and Murali in [9]. Leena Shenoy [8] studied hamiltonian laceability properties in product graphs involving cycles and paths. More results in the laceability properties of product graphs can be found in [5], [6] and [4].

Definition 1.1. *The shadow graph of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' of G' to the neighbors of the corresponding vertex u'' of G''.*

The shadow graph of G is denoted by $D_2(G)$.

In figure 1, the shadow graph of the wheel graph $W_{1,5}$ is illustrated.

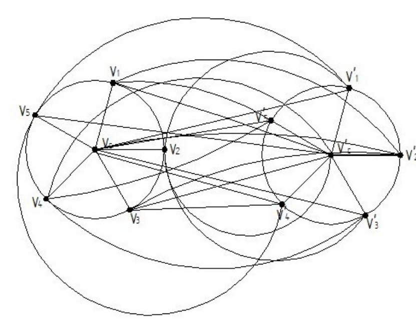

Figure 1. The graph of $D_2(W_{1,5})$

Definition 1.2. *A graph G^* is k-edge fault tolerant with respect to a graph G if the graph obtained by removing any k edges from G^* contains G, where k is a positive integer.*

Definition 1.3. *Let P be a path between the vertices v_i to v_j in a graph G and let P' be a path between the vertices v_j and v_k. Then, the path $P \cup P'$ is the path obtained by extending the path P from v_i to v_j to v_k through the common vertex v_j (i.e. if $P : v_i....v_j$ and $P' : v_j....v_k$ then $P \cup P' : v_i....v_j.....v_k$)*

2. Distance graph and shadow distance graph

Definition 2.1. *For a graph G, Let D_s be the set of all distances between distinct pairs of its vertices and let D_s (called the distance set) be a subset of D. The distance graph [11] of G, denoted by $D(G,D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G,D_s)$ whenever $d(u,v) \in D_s$.*

By definition, if $D_s = \{1\}$, then $D(G,D_s) \cong G$, $D(G,D_s)$ is a complete graph and $D(G,\{\})$ is a completely disconnected graph.

In [9], the authors have shown that distance graph of the path graph of even order is hamiltonian-1*-laceable and 2*-laceable with the distance set $D_s = \{1,2k\}$. Leena Shenoy et.al. in [8], have shown that the pairs of vertices (a_1,a_k) and (a_1,a_{2k}) are attainable (in the sense that there exists a hamiltonian path) in the distance graph of the path graph of order $n > 5$ with distance set $S = \{1,k,2k\}$.

Definition 2.2. *The shadow distance graph [10] of G, denoted by $D_{sd}(G,d)$ is constructed from G with the following conditions:*

1. *consider two copies of G say G itself and G'*
2. *if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u' \in V(G')$ (second copy)*
3. *the vertex set of $D_{sd}(G,d)$ is $V(G) \cup V(G')$*
4. *the edge set of $D_{sd}(G,d)$ is $E(G) \cup E(G') \cup E_d$ where E_d is the set of all edges between distinct vertices $u \in V(G)$ and $v' \in V(G')$ that satisfy the condition $d(u,v) \in D_s$ in G.*

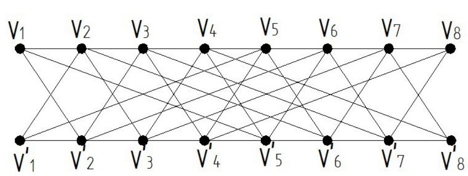

Figure 2. The shadow distance graph $D_{Sd}(P_8,(1,4))$

3. Terminologies

We use the following terminologies in our results.

- $xP[n] = x(x+1)(x+2)......(x+n-1)$.
- $xP^{-1}[n] = x(x-1)(x-2)......(x-n+1)$.
- $xZ[n] = x(x'+1)(x+2)(x'+3).........(x+n-2)(x'+n-1) or (x+n-1)$ (moving from one vertex to another vertex in the adjacent level *i.e.,* moving from left to right).
- $xZ^{-1}[n] = x(x'-1)(x-2)(x'-3).........(x-n+2)(x'-n+1) or (x-n+1)$ (moving from one vertex to another vertex in the adjacent level *i.e.,* moving from right to left).
- $xJ = v_i \to v_j$ and $xJ^{-1} = v'_j \to v_i$ where $i \leq n-2k$ and j is taken under modulo n.

4. Results

Theorem 4.1. *If $n \geq 4$ is even, the 1-edge fault tolerant graph $H = D_{sd}(P_n,(1,2k))$ is Hamiltonian-1*- laceable.*

Proof. Consider two copies of P_n, say P_n and P'_n. Let $v_1,v_2,v_3,.....,v_n$ and $v'_1,v'_2,v'_3......,v'_n$ be the vertices of P_n and P'_n respectively.

Clearly, H has $2n$ vertices and $4(n+1)$ edges.

Now in H, $d(v_1,v'_{2k+1}) = 1$ and the path

$P : v_1\left[P(2)Z^{-1}(2)P(2)Z(2)\right]^k P(n-2k)(v'_n,v_n)P^{-1}(n-2k)$ v'_{2k+1} in the 1-edge fault tolerant graph H^* is a hamiltonian path between the vertices v_1 and v'_{2k+1}.

Hence the proof. □

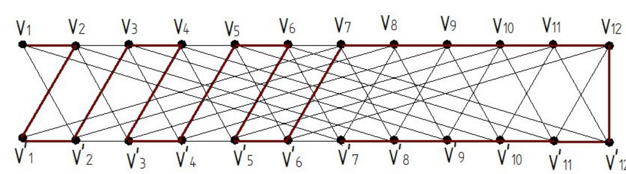

Figure 3. Hamiltonian path in the graph $D_{sd}\{P_12,(1,6)\}$ with $d(v_1,v'_7) = 1$

Theorem 4.2. *If $n \geq 4$ is even, the 1-edge fault tolerant graph $H = D_{sd}(P_n,(1,2k))$ is Hamiltonian-2*- laceable.*

Proof. The vertex set of H is as in theorem 4.1.

In H, $d(v_1,v_{2k+2}) = 2$ and the path $P : v_1\left[Jp^{-1}(2k+1)Z(2)P(2k)Z(2)P(n-2k-1)\right](v'_n,v_n)P^{-1}(n-2k-1)v_{2k+2}$ in the 1-edge fault tolerant graph H^* is a hamiltonian path between the vertices v_1 and v'_{2k+2}.

Hence the proof. □

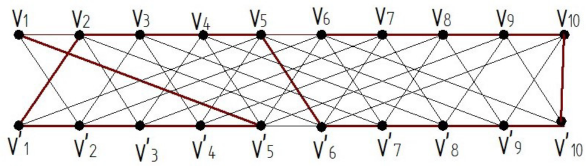

Figure 4. Hamiltonian path in the graph $D_{sd}\{P_{10},(1,4)\}$ with $d(v_1,v_6)=2$

Theorem 4.3. *If $n \geq 6$ is even, the 1-edge fault tolerant graph $H = D_{sd}(P_n,(1,2k))$ is Hamiltonian-3^*- laceable.*

Proof. The vertex set of H is as in theorem 4.1.

In H, $d(v_1,v_{2k+1})=3$ and the path $P: v_1\left[P(2)Z^{-1}(2)P(2)Z(2)\right]^{k-1}(P(2)Z^{-1}(2))P(n-2k+2)$ $(v'_n,v_n)P^{-1}(n-2k)v_{2k+1}$ in the 1-edge fault tolerant graph H^* is a hamiltonian path between the vertices v_1 and v_{2k+1}.

Hence the proof. □

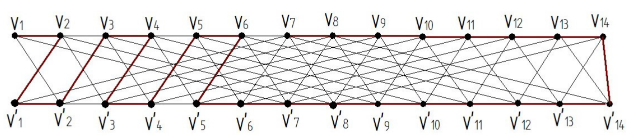

Figure 5. Hamiltonian path in the graph $D_{Sd}\{P_{14},(1,6)\}$ with $d(v_1,v_7)=3$

Theorem 4.4. *For even $n > j$ and odd $t \geq 5$, then 1-edge fault tolerant graph $H = D_{sd}(P_n,(1,2k))$ is Hamiltonian-t^*-laceable where $j = \frac{n+2k+t+1}{2}$.*

Proof. The vertex set of H is as in theorem 4.1.

In H, $d(v_i,v_j)=t$ and the path $P: v_1\left[P(2)Z^{-1}(2)P(2)Z(2)\right]^{k+q}\left[P(2)Z^{-1}(2)\right]P(4)\left(v'_n \cup v_n\right)$ $P^{-1}(2)v_j$ where $q = \frac{t-3}{2}$ and $k = \frac{n-t+1}{2}$ in the 1-edge fault tolerant graph H^* is a hamiltonian path between the vertices v_i and v_j.

Hence the proof. □

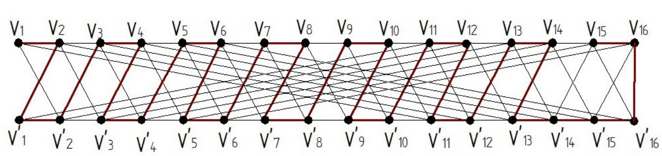

Figure 6. Hamiltonian path in the graph $D_{Sd}\{P_{16},(1,10)\}$ with $d(v_1,v_{15})=5$

Theorem 4.5. *For even $n \geq 2k+t$ and $t \geq 4$, the graph $H = D_{sd}(P_n,(1,2k))$ is Hamiltonian-t^*-laceable.*

Proof. The vertex set of H is as in theorem 4.1.

In H, $d(v_1,v_n)=t$ and the path $P: v_1\left[P(2)Z^{-1}(2)P(2)Z(2)\right]^{k+q}(Z(2)P^{-1}(2)Z(2))v_n$ where $q = \frac{t-2}{2}$. and $k = \frac{n-t}{2}$ in the 1-edge fault tolerant graph H^* is a hamiltonian path between the vertices v_1 and v_n.

Hence the proof. □

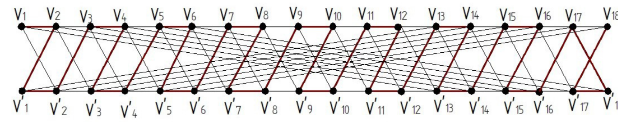

Figure 7. Hamiltonian path in the graph $D_{sd}\{P_{18},(1,12)\}$ with $d(v_1,v_{18})=6$

5. Conclusion

In this paper, hamiltonian laceability properties of the 1-edge fault tolerant shadow distance graphs associated with the path graph P_n is studied. We have shown that for $n \geq 5$, this graph is hamiltonian-t^*-laceable for all t, and, for all $n = 2k+t$, this graph is hamiltonian-t^*-laceable.

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