



# Operation approaches on decompositions of $\gamma$ -continuous function

E. Hatir<sup>1\*</sup>**Abstract**

In this paper, we introduce the notions of  $\alpha^* - \gamma - set$ ,  $t - \gamma - set$ ,  $s - \gamma - set$ ,  $\beta^* - \gamma - set$ ,  $C_\gamma - continuity$ ,  $B_\gamma - continuity$ ,  $S_\gamma - continuity$  and  $\beta_\gamma - continuity$ . Thus we have decompositions of  $\gamma - continuity$ .

**Keywords**

$\alpha - \gamma - open$ ,  $semi - \gamma - open$ ,  $pre - \gamma - open$ ,  $\beta - \gamma - open$ .

**AMS Subject Classification**

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## 1. Introduction

In [13], Kasahara unified several known characterizations of compactness, nearly compact spaces and H-closed spaces by introducing a certain operation on a topology. By using operation Jankovic [14] investigated functions with closed graphs. Ogata [7] defined the concept of  $\gamma - open$  sets with an operation  $\gamma$  in the manner of Kasahara [13] and introduced some new separation axioms of topological spaces. In [11], the authors introduced and investigated the notions of  $\alpha - \gamma - open$  sets. In [5, 6] the authors introduced and investigated the notions of  $semi - \gamma - open$  set,  $pre - \gamma - open$  set and  $\beta - \gamma - open$  set. A decomposition of  $\gamma - continuity$  is a pair of properties of functions between topological spaces with an operation  $\gamma$  each of which is weaker than  $\gamma - continuity$ , and which are together equivalent to  $\gamma - continuity$ . One member of the pair is a  $\gamma - continuity$  dual of the other. In this paper, we introduce the notions of  $\alpha^* - \gamma - set$ ,  $t - \gamma - set$ ,  $s - \gamma - set$ ,  $\beta - \gamma - set$ ,  $C_\gamma - continuity$ ,  $B_\gamma - continuity$ ,  $S_\gamma - continuity$ ,  $\beta_\gamma - continuity$ . Thus we have decompositions of

$\gamma - continuity$ .

## 2. Preliminaries

Let  $(X, \tau)$  be a topological space. Let  $\gamma$  be an operation on  $\tau$ , that is,  $\gamma$  is a function from  $\tau$  into the power set  $\wp(X)$  of  $X$  such that  $V \subset \gamma(V)$  for any  $V \in \tau$  where  $\gamma(V)$  denotes the value of  $\gamma$  at  $V$ . This operation denoted by  $\gamma: \tau \rightarrow \wp(X)$ . Let us take a topological space  $(X, \tau)$  and  $W \subset X$  with an operation  $\gamma$  on  $\tau$ . Then  $W$  is called  $\gamma - open$  [7] if for each  $x \in W$ , there exists an open neighbourhood  $U$  of  $x$  such that  $\gamma(U) \subset W$ . The empty set  $\phi$  is  $\gamma - open$  for any operation  $\gamma: \tau \rightarrow \wp(X)$ . Let  $\tau_\gamma$  be the collections of all  $\gamma - open$  sets of  $(X, \tau)$  with  $\tau_\gamma$ . For any topological space  $(X, \tau)$ ,  $\tau_\gamma \subset \tau$  [7]. Complements of  $\gamma - open$  sets are defined as  $\gamma - closed$ . The  $\gamma - closure$  of  $W \subset X$  with an operation  $\gamma$  is denoted by  $Cl_\gamma(W)$ , is defined as

$$Cl_\gamma(W) = \cap \{B : B \text{ is } \gamma - closed \text{ and } W \subset B\}.$$

The  $\gamma - interior$  of  $W \subset X$  with an operation  $\gamma$  on  $\tau$  is denoted by  $Int_\gamma(W)$ , is defined as

$$Int_\gamma(W) = \cup \{B : B \text{ is a } \gamma - open \text{ set and } B \subset W\}.$$

A topological space  $X$  with an operation  $\gamma$  on  $\tau$  is said to be  $\gamma - regular$  if for each  $x \in X$  and each neighbourhood  $V$  of  $x$ , there exists an open neighbourhood  $U$  of  $x$  with  $\gamma(U) \subset V$ . According to this notion,  $\tau = \tau_\gamma \Leftrightarrow X$  is a  $\gamma - regular$  space [7].

In this paper,  $(X, \tau)$  and  $(Y, \sigma)$  denotes topological space. Furthermore, there is no separation axioms on them unless

otherwise mentioned.  $Cl(W)$  and  $Int(W)$  denote the closure of  $W$  and the interior of  $W$ , respectively, in topological space  $(X, \tau)$ . Let us recall some of basic definitions.

**Definition 2.1.** Let  $(X, \tau)$  be a topological space and  $W \subset X$ . Then

1.  $W$  is called an  $\alpha$ -open set [12] if  $W \subset Int(Cl(Int(W)))$ ,
2.  $W$  is called a pre-open set [2] if  $W \subset Int(Cl(W))$ ,
3.  $W$  is called a semi-open set [10] if  $W \subset Cl(Int(W))$ ,
4.  $W$  is called a  $\beta$ -open set [9] if  $W \subset Cl(Int(Cl(W)))$ ,
5.  $W$  is called an  $\alpha^*$ -set [4] if  $Int(Cl(Int(W))) = Int(W)$ ,
6.  $W$  is called a  $C$ -set [4] if  $W = U \cap V$ , where  $U \in \tau$  and  $V$  is an  $\alpha^*$ -set,
7.  $W$  is called a  $t$ -set [8] if  $Int(Cl(W)) = Int(W)$ ,
8.  $W$  is called a  $B$ -set [8] if  $W = U \cap V$ , where  $U \in \tau$  and  $V$  is a  $t$ -set,

**Definition 2.2.** Let  $(X, \tau)$  be a topological space and  $W \subset X$  with an operation  $\gamma$  on  $\tau$ . Then

1.  $W$  is called an  $\alpha$ - $\gamma$ -open set [11] if  $W \subset Int_\gamma(Cl_\gamma(Int_\gamma(W)))$ ,
2.  $W$  is called a pre- $\gamma$ -open set [6] if  $W \subset Int_\gamma(Cl_\gamma(W))$ ,
3.  $W$  is called a semi- $\gamma$ -open set [5] if  $W \subset Cl_\gamma(Int_\gamma(W))$ ,
4.  $W$  is called a  $\beta$ - $\gamma$ -open set [6] if  $W \subset Cl_\gamma(Int_\gamma(Cl_\gamma(W)))$ ,
5.  $W$  is called a  $\gamma$ -regular open set [1] if  $Int_\gamma(Cl_\gamma(W)) = W$ .

Let  $(X, \tau)$  be a topological space and  $\gamma$  be an operation on  $\tau$ . The  $\beta$ - $\gamma$ -interior of  $W \subset X$  with an operation  $\gamma$  is denoted by  $\beta Int_\gamma(W)$  [3], is defined as

$\beta Int_\gamma(W) = \cap \{B : B \text{ is } \beta\text{-}\gamma\text{-open and } B \subset W\}$ . Complements of  $\beta$ - $\gamma$ -open sets are defined as  $\beta$ - $\gamma$ -closed. Therefore, we have  $\beta Int_\gamma(W) = W \cap Cl_\gamma(Int_\gamma(Cl_\gamma(W)))$ .

**Definition 2.3.** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces and let  $\gamma : \tau \rightarrow \wp(X)$  be the operation on  $\tau$ . A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\gamma$ -continuous [3] (resp.  $\alpha$ - $\gamma$ -continuous [11], pre- $\gamma$ -continuous [6], semi- $\gamma$ -continuous [5],  $\beta$ - $\gamma$ -continuous [6]) if for each  $x \in X$  and each open set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ -open set  $U$  containing  $x$  (resp.  $\alpha$ - $\gamma$ -open set, pre- $\gamma$ -open set, semi- $\gamma$ -open set,  $\beta$ - $\gamma$ -open set) such that  $f(U) \subset V$ .

### 3. $C_\gamma$ -sets, $B_\gamma$ -sets, $S_\gamma$ -sets and $\beta_\gamma$ -sets

**Definition 3.1.** Let  $(X, \tau)$  be a topological space and  $W \subset X$  with an operation  $\gamma$  on  $\tau$ . Then

1.  $W$  is called an  $\alpha^*$ - $\gamma$ -set if  $Int_\gamma(Cl_\gamma(Int_\gamma(W))) = Int_\gamma(W)$ ,
2.  $W$  is called a  $t$ - $\gamma$ -set if  $Int_\gamma(Cl_\gamma(W)) = Int_\gamma(W)$ ,
3.  $W$  is called a  $s$ - $\gamma$ -set if  $Cl_\gamma(Int_\gamma(W)) = Int_\gamma(W)$ ,
4.  $W$  is called a  $\beta^*$ - $\gamma$ -set if  $Cl_\gamma(Int_\gamma(Cl_\gamma(W))) = Int_\gamma(W)$ .

**Proposition 3.2.** The following are equivalent for a subset  $W$  of a space  $(X, \tau)$  with an operator  $\gamma$ ,

1.  $W$  is  $\alpha^*$ - $\gamma$ -set,
2.  $W$  is  $\beta$ - $\gamma$ -closed set,
3.  $Int_\gamma(W)$  is  $\gamma$ -regular-open set.

*Proof.* Straightforward. □

**Proposition 3.3.** Let  $W$  be a subset of a space  $(X, \tau)$  with an operator  $\gamma$ ,

1. A semi- $\gamma$ -open set  $W$  is a  $t$ - $\gamma$ -set if and only if  $W$  is an  $\alpha^*$ - $\gamma$ -set.
2.  $W$  is an  $\alpha$ - $\gamma$ -open set and  $W$  is  $\alpha^*$ - $\gamma$ -set if and only if  $W$  is  $\gamma$ -regular-open set.

*Proof.* 1. Let  $W$  be a semi- $\gamma$ -open and  $W$  be an  $\alpha^*$ - $\gamma$ -set. Since  $W$  is a semi- $\gamma$ -open,  $Cl_\gamma(Int_\gamma(W)) = Cl_\gamma(W)$  and  $Int_\gamma(Cl_\gamma(W)) = Int_\gamma(Cl_\gamma(Int_\gamma(W))) = Int_\gamma(W)$ . Therefore,  $W$  is a  $t$ - $\gamma$ -set.

2. Let  $W$  be an  $\alpha$ - $\gamma$ -open set and  $W$  be an  $\alpha^*$ - $\gamma$ -set. By Proposition 1 and the definition of  $\alpha$ - $\gamma$ -open set, we have  $Int_\gamma(Cl_\gamma(W)) = W$  and hence  $Int_\gamma(Cl_\gamma(W)) = Int_\gamma(Cl_\gamma(Int_\gamma(W))) = W$ .

The converse is obvious. □

**Definition 3.4.** Let  $(X, \tau)$  be a topological space and  $W \subset X$  with an operation  $\gamma$  on  $\tau$ . Then

1.  $W$  is called a  $C_\gamma$ -set if  $W = U \cap V$ , where  $U \in \tau_\gamma$  and  $V$  is an  $\alpha^*$ - $\gamma$ -set,
2.  $W$  is called a  $B_\gamma$ -set if  $W = U \cap V$ , where  $U \in \tau_\gamma$  and  $V$  is a  $t$ - $\gamma$ -set,
3.  $W$  is called a  $S_\gamma$ -set if  $W = U \cap V$ , where  $U \in \tau_\gamma$  and  $V$  is a  $s$ - $\gamma$ -set,
4.  $W$  is called a  $\beta_\gamma$ -set if  $W = U \cap V$ , where  $U \in \tau_\gamma$  and  $V$  is a  $\beta^*$ - $\gamma$ -set,
5.  $W$  is called a  $\gamma$ - $\gamma$ - $\beta$ -open set [3] if  $\beta Int_\gamma(W) = Int_\gamma(W)$ .

**Proposition 3.5.** Let  $(X, \tau)$  be a topological space with an operation  $\gamma$  and  $W \subset X$ . Then the following hold:

1. If  $W$  is a  $t$ - $\gamma$ -set, then  $W$  is an  $\alpha^*$ - $\gamma$ -set,
2. If  $W$  is a  $s$ - $\gamma$ -set, then  $W$  is an  $\alpha^*$ - $\gamma$ -set,
3. If  $W$  is a  $\beta^*$ - $\gamma$ -set, then  $W$  is both  $t$ - $\gamma$ -set and  $s$ - $\gamma$ -set.
4.  $t$ - $\gamma$ -set and  $s$ - $\gamma$ -set are independent.

*Proof.* Straightforward from the definitions of  $\gamma$ -interior and  $\gamma$ -closure. □

**Remark 3.6.** The converses are false. See the following examples.

**Example 3.7.** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ . We define an operator  $\gamma : \tau \rightarrow \wp(X)$  by  $\gamma(W) = W \cup \{a, c\}$  if  $W \neq \{a\}$  and  $\gamma(W) = W$  if  $W = \{a\}$ . Then  $\tau_\gamma = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ . If we take  $W = \{a\}$ , then  $W$  is an  $\alpha^*$ - $\gamma$ -set and a  $t$ - $\gamma$ -set, but it is not a  $s$ - $\gamma$ -set and not a  $\beta^*$ - $\gamma$ -set.

**Example 3.8.** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ . We define an operator  $\gamma : \tau \rightarrow \wp(X)$  by  $\gamma(W) = W$  if  $W = \{a, c\}$  or  $W = \phi$  and  $\gamma(W) = X$  if otherwise. Then  $\tau_\gamma = \{\phi, X\}$ . If we take  $W = \{b\}$ , then  $W$  is an  $\alpha^*$ - $\gamma$ -set and a  $s$ - $\gamma$ -set, but it is not a  $t$ - $\gamma$ -set and not a  $\beta^*$ - $\gamma$ -set.



**Proposition 3.9.** Let  $(X, \tau)$  be a topological space with an operation  $\gamma$  and  $W \subset X$ . Then the following hold:

1. If  $W$  is an  $\alpha^* - \gamma$ -set, then  $W$  is  $C_\gamma$ -set,
2. If  $W$  is a  $t - \gamma$ -set, then  $W$  is  $B_\gamma$ -set,
3. If  $W$  is a  $s - \gamma$ -set, then  $W$  is  $S_\gamma$ -set,
4. If  $W$  is a  $\beta^* - \gamma$ -set, then  $W$  is  $\beta_\gamma$ -set.

*Proof.* 1. Let  $W$  be an  $\alpha^* - \gamma$ -set. If we take  $U = X \in \tau_\gamma$ , then  $W = U \cap W$  and hence  $W$  is a  $C_\gamma$ -set.

The proof of (2), (3) and (4) are same. □

**Remark 3.10.** The converses are false. See the following examples.

**Example 3.11.** In Example 1, if we take  $W = \{a, c\}$ , then  $W$  is a  $C_\gamma$ -set (resp.  $B_\gamma$ -set,  $S_\gamma$ -set,  $\beta_\gamma$ -set), but it is not an  $\alpha^* - \gamma$ -set (resp.  $t - \gamma$ -set,  $s - \gamma$ -set,  $\beta^* - \gamma$ -set).

- Proposition 3.12.** 1. A  $B_\gamma$ -set is a  $C_\gamma$ -set,  
 2. A  $S_\gamma$ -set is a  $C_\gamma$ -set,  
 3. A  $\beta_\gamma$ -set is both a  $B_\gamma$ -set and a  $S_\gamma$ -set.

**Remark 3.13.** The converses are false.  $B_\gamma$ -set and  $S_\gamma$ -set are independent notions. See the following examples.

**Example 3.14.** In Example 1, if we take  $W = \{a, b\}$ , then  $W$  is a  $B_\gamma$ -set, but it is not a  $S_\gamma$ -set and not a  $\beta_\gamma$ -set.

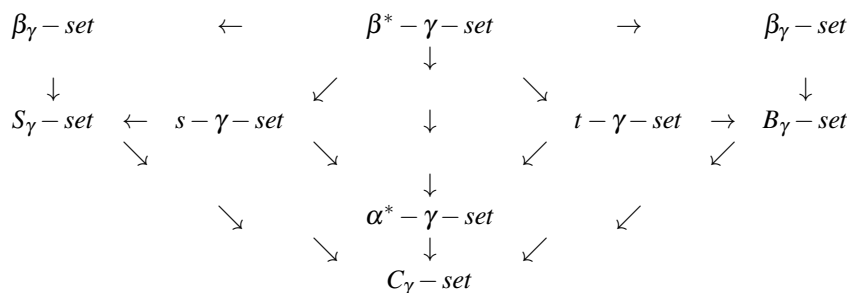
In Example 2, if we take  $W = \{b\}$ , then  $W$  is a  $C_\gamma$ -set and a  $S_\gamma$ -set, but it is not a  $B_\gamma$ -set and not a  $\beta_\gamma$ -set.

**Proposition 3.15.** Let  $(X, \tau)$  be a topological space with an operation  $\gamma$  and  $W \subset X$ . Then  $\gamma - \gamma - \beta$ -open set [3] and  $\beta_\gamma$ -set are equivalent.

*Proof.* Let  $W$  be a  $\beta^* - \gamma$ -set. Then  $Cl_\gamma(Int_\gamma(Cl_\gamma(W))) = Int_\gamma(W)$ . Hence by Proposition 4(4),  $W$  is  $\beta_\gamma$ -set. Therefore,  $\beta Int_\gamma(W) = W \cap Cl_\gamma(Int_\gamma(Cl_\gamma(W))) = W \cap Int_\gamma(W) = Int_\gamma(W)$ . Thus  $W$  is  $\gamma - \gamma - \beta$ -open set.

Conversely, let  $W$  be a  $\gamma - \gamma - \beta$ -open set. Then  $\beta Int_\gamma(W) = Int_\gamma(W)$ . Hence  $\beta Int_\gamma(W)$  is a  $\gamma$ -open set. Since  $W = W \cap X$ ,  $W$  is  $\beta_\gamma$ -set. □

**Remark 3.16.** We have the following diagram according to sets defined above. It is shown in Examples 1-2 that the notion of  $S_\gamma$ -sets is different from that of  $B_\gamma$ -sets.



**Theorem 3.17.** For a subset  $W$  of a space  $(X, \tau)$  with an operation  $\gamma$ , the following properties are equivalent:

1.  $W$  is  $\gamma$ -open,
2.  $W$  is an  $\alpha - \gamma$ -open set and a  $C_\gamma$ -set,
3.  $W$  is a pre- $\gamma$ -open set and a  $B_\gamma$ -set,
4.  $W$  is a semi- $\gamma$ -open set and a  $S_\gamma$ -set,
5.  $W$  is a  $\beta - \gamma$ -open set and a  $\beta_\gamma$ -set.

*Proof.* The proof of (1) $\Rightarrow$ (2), (1) $\Rightarrow$ (3), (1) $\Rightarrow$ (4), (1) $\Rightarrow$ (5) are obvious.

(5) $\Rightarrow$ (1) Let  $W$  be a  $\beta - \gamma$ -open set and a  $\beta_\gamma$ -set. Since  $W$  is a  $\beta_\gamma$ -set, we have  $W = U \cap V$ , where  $U$  is a  $\gamma$ -open set and  $V$  is a  $\beta^* - \gamma$ -set. By the hypothesis,  $W$  is also  $\beta - \gamma$ -open and we have

$$\begin{aligned}
 W &\subset Cl_\gamma(Int_\gamma(Cl_\gamma(W))) = Cl_\gamma(Int_\gamma(Cl_\gamma(U \cap V))) \\
 &\subset Cl_\gamma(Int_\gamma(Cl_\gamma(U) \cap Cl_\gamma(V))) \\
 &= Cl_\gamma(Int_\gamma(Cl_\gamma(U)) \cap Int_\gamma(Cl_\gamma(V))) \\
 &\subset Cl_\gamma(Int_\gamma(Cl_\gamma(U))) \cap Cl_\gamma(Int_\gamma(Cl_\gamma(V))) \\
 &\subset Cl_\gamma(Int_\gamma(Cl_\gamma(U))) \cap Int_\gamma(V).
 \end{aligned}$$

Hence

$$\begin{aligned}
 W &= U \cap V = (U \cap V) \cap U \\
 &\subset (Cl_\gamma(Int_\gamma(Cl_\gamma(U))) \cap Int_\gamma(V)) \cap U \\
 &= (Cl_\gamma(Int_\gamma(Cl_\gamma(U))) \cap U) \cap Int_\gamma(V).
 \end{aligned}$$

Notice  $W = U \cap V \supset U \cap Int_\gamma(V)$ . Therefore, we obtain  $W = U \cap Int_\gamma(V)$ .

(2) $\Rightarrow$ (1), (3) $\Rightarrow$ (1), (4) $\Rightarrow$ (1) are shown similarly. □

**Remark 3.18.** If  $(X, \tau)$  is a  $\gamma$ -regular space, then the concept of  $\alpha - \gamma$ -open and  $\alpha$ -open (resp. pre- $\gamma$ -open and pre-open, semi- $\gamma$ -open and semi-open,  $\beta - \gamma$ -open and  $\beta$ -open,  $B_\gamma$ -set and  $B$ -set,  $C_\gamma$ -set and  $C$ -set) coincide.

#### 4. Decompositions of $\gamma$ -continuity

**Definition 4.1.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a function and let  $\gamma : \tau \rightarrow \wp(X)$  be the operation on  $\tau$ . If for each  $V \in \sigma$ ,  $f^{-1}(V)$  is a  $C_\gamma$ -set (resp.  $B_\gamma$ -set,  $S_\gamma$ -set,  $\beta_\gamma$ -set), then  $f$  is said to be  $C_\gamma$ -continuous (resp.  $B_\gamma$ -continuous,  $S_\gamma$ -continuous,  $\beta_\gamma$ -continuous).

By Proposition 5, we get the following proposition.

- Proposition 4.2.** 1. A  $B_\gamma$ -continuous function is  $C_\gamma$ -continuous,  
 2. A  $S_\gamma$ -continuous function is  $C_\gamma$ -continuous,  
 3. A  $\beta_\gamma$ -continuous is both  $B_\gamma$ -continuous and  $S_\gamma$ -continuous.



**Theorem 4.3.** For a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$  with the operation  $\gamma$  on  $\tau$ , the following properties are equivalent:

1.  $f$  is  $\gamma$ -continuous
2.  $f$  is  $\alpha$ - $\gamma$ -continuous and  $C_\gamma$ -continuous,
3.  $f$  is pre- $\gamma$ -continuous and  $B_\gamma$ -continuous, .
4.  $f$  is semi- $\gamma$ -continuous and  $S_\gamma$ -continuous,
5.  $f$  is  $\beta$ - $\gamma$ -continuous and  $\beta_\gamma$ -continuous.

*Proof.* This is an immediate consequence of Theorem 1.  $\square$

**Remark 4.4.**  $\alpha$ - $\gamma$ -continuity and  $C_\gamma$ -continuity, pre- $\gamma$ -continuity and  $B_\gamma$ -continuity, semi- $\gamma$ -continuity and  $S_\gamma$ -continuity,  $\beta$ - $\gamma$ -continuity and  $\beta_\gamma$ -continuity are independent of each other. See the following examples.

**Example 4.5.** Let  $X = Y = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$  and  $\sigma = \{\phi, Y, \{a\}, \{c\}, \{a, c\}\}$ . We define an operator  $\gamma : \tau \longrightarrow \wp(X)$  by  $\gamma(W) = W \cup \{a, c\}$  if  $W \neq \{a\}$  and  $\gamma(W) = W$  if  $W = \{a\}$ . Then  $\tau_\gamma = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ . Define a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$  as  $f(a) = f(b) = a, f(c) = c$ . Then  $f$  is  $C_\gamma$ -continuous (resp.  $B_\gamma$ -continuous, semi- $\gamma$ -continuous and  $\beta$ - $\gamma$ -continuous), but it is not  $\alpha$ - $\gamma$ -continuous (resp. pre- $\gamma$ -continuous,  $S_\gamma$ -continuous and  $\beta_\gamma$ -continuous).

**Example 4.6.** Let  $X = Y = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{a, b\}\}$  and  $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$ . We define an operator  $\gamma : \tau \longrightarrow \wp(X)$  by  $\gamma(W) = W$  if  $W = \{a, c\}$  or  $W = \phi$  and  $\gamma(W) = X$  if otherwise. Then  $\tau_\gamma = \{\phi, X\}$ . Define a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$  as  $f(a) = f(c) = a, f(b) = b$ . Then  $f$  is both  $S_\gamma$ -continuous and pre- $\gamma$ -continuous, but it is neither semi- $\gamma$ -continuous nor  $B_\gamma$ -continuous.

**Example 4.7.** Let  $X = Y = \{a, b, c, d\}$  and  $\tau = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$  and  $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$ . We define an operator  $\gamma : \tau \longrightarrow \wp(X)$  by  $\gamma(W) = Cl(W)$  if  $W \neq \{a\}$  and  $\gamma(W) = Int(Cl(W))$  if  $W = \{a\}$ . Then  $\tau_\gamma = \{\phi, \{a\}, \{c\}, \{a, c\}, \{a, b, d\}, X\}$ . Define a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$  as  $f(a) = f(c) = a, f(b) = f(d) = b$ . Then  $f$  is  $\beta_\gamma$ -continuous, but it is not  $\beta$ - $\gamma$ -continuous.

**Example 4.8.** Let  $X = Y = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$  and  $\sigma = \{\phi, Y, \{a\}\}$ . We define an operator  $\gamma : \tau \longrightarrow \wp(X)$  by  $\gamma(W) = Int(Cl(W))$  if  $W = \{a\}$  and  $\gamma(W) = X$  if  $W \neq \{a\}$ . Then  $\tau_\gamma = \{\phi, \{a\}, X\}$ . Define a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$  as  $f(a) = f(c) = a, f(b) = b$ . Then  $f$  is  $\alpha$ - $\gamma$ -continuous, but it is not  $C_\gamma$ -continuous.

**Corollary 4.9.** Let  $(X, \tau)$  be a  $\gamma$ -regular space. For a function  $f : (X, \tau) \longrightarrow (Y, \sigma)$ , the following properties are equivalent:

1.  $f$  is continuous,
2.  $f$  is pre-continuous and  $B$ -continuous [8],
3.  $f$  is  $\alpha$ -continuous and  $C$ -continuous [4].

*Proof.* In  $\gamma$ -regular space, we have  $\tau = \tau_\gamma$ .  $\square$

## 5. Conclusion

A decomposition of  $\gamma$ -continuity is a pair of properties of functions between topological spaces with an operation  $\gamma$  each of which is weaker than  $\gamma$ -continuity, and which are together equivalent to  $\gamma$ -continuity. One member of the pair is a  $\gamma$ -continuity dual of the other. In this paper, we have obtain decompositions of  $\gamma$ -continuity.

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