



A queueing-inventory system with perishable items and retrial of customers

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Abstract

In this paper, we consider a continuous review perishable (s, Q) inventory system in which the customers arrive according to a Poisson process. Service time and lead time are assumed to be independent exponential distributions. A customer who arrives during server busy or stock out period either enters into an orbit of infinite capacity or leaves the system. The time between any two successive retrials of the orbiting customer is distributed as an exponential with parameter depending on the number of customers in the orbit. Decay time of items is also assumed to be exponentially distributed with linear rate. Some relevant system performance measures are derived. A suitable cost function is constructed and analyzed. Some numerical and graphical illustrations are also included to highlight the results.

Keywords

Cost Analysis, Matrix Analytic Method, Perishable Inventory, Retrials.

AMS Subject Classification

60K25, 90B05, 91B70.

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1. Introduction

The stochastic modeling of perishable inventory received attention of researchers in the past few decades. An unrealistic assumption in most of the inventory models was indefinite life time of items to meet future demands. But, the practical situation is different; there are valuable inventories which are deteriorating. This leads to the necessity of the assumption of randomness in decay time. In the case of inventory system with service facility, the customer demand is satisfied only

after a random time of service. Furthermore, the procurement of items actually takes place with a minimum delay of transportation or production of goods. These facts force to the assumption of positive service time and lead time. Besides, these assumptions make the modeling of inventory systems more realistic.

Kalpakam and shanthi [1] analyzed a lost sale $(S-1, S)$ perishable system in which reorders are placed at every demand epoch with arbitrary resupply time distribution. Jayaraman et al.[2] modeled a continuous review perishable inventory system in which the customer who arrives during the stock-out periods are offered to join a pool of infinite capacity or leaves the system. The demands in the pool are selected one by one by the server only when the inventory level is above s . Jeganathan et al. [3] considered an inventory model with finite customer waiting area and instantaneous replenishment of items having exponential life times. Kalpakam and Arivarignan [4] discussed an inventory model with Poisson demand and negative exponential life times. The items are removed from stock one at a time either due to random demand or random failure of item is considered. Jayaraman et al. [5] studied a continuous review perishable inventory system with infinite pool and the waiting customer independently renege the system after an exponentially distributed amount of

3. System stability

Define, Lyapunov test function (see Falin and Templeton [16]) as

$$\phi(r) = i, \text{ if } r \text{ is a state in the level } i$$

The mean drift y_r , for any r belonging to the level $i \geq 1$ is given by,

$$\begin{aligned} y_r &= \sum_{p \neq r} q_{rp}(\phi(p) - \phi(r)) \\ &= \sum_u q_{ru}(\phi(u) - \phi(r)) + \sum_v q_{rv}(\phi(v) - \phi(r)) \\ &\quad + \sum_w q_{rw}(\phi(w) - \phi(r)) \end{aligned}$$

where u, v and w vary over the states belonging to the levels $(i - 1), i$ and $(i + 1)$ respectively.

$$y_r = \begin{cases} -i\theta, & \text{if server idle with positive inventory level} \\ -i\theta(1 - \delta) + \lambda\gamma, & \text{otherwise} \end{cases}$$

Since $(1 - \delta) > 0$, for any $\epsilon > 0$, we can find N' large enough so that $y_r < -\epsilon$, for any r belonging to the level $i \geq N'$. According to Tweedie[17], the system under consideration is stable.

4. System Performance Measures

Let the steady state probability vector be $\mathbf{x} = (x_0, x_1, x_2, \dots)$;

$$x_i = (y_{i,0,0}, y_{i,0,1}, \dots, y_{i,0,S}, y_{i,1,1}, y_{i,1,2}, \dots, y_{i,1,S}) (i \geq 0)$$

Expected inventory level,

$$E_{inv} = \sum_{i=0}^{\infty} \sum_{j=0}^S j y_{i,0,j} + \sum_{i=0}^{\infty} \sum_{j=1}^S j y_{i,1,j}$$

Expected number of customers in the orbit,

$$E_{orbit} = \left(\sum_{i=1}^{\infty} i x_i \right) \mathbf{e}$$

Expected reorder rate,

$$E_{ro} = \mu \sum_{i=0}^{\infty} y_{i,1,s+1} + (s+1)\omega \left(\sum_{i=0}^{\infty} \sum_{k=0}^1 y_{i,k,s+1} \right)$$

Expected perishable rate,

$$E_p = \omega \sum_{i=0}^{\infty} \sum_{j=0}^S j y_{i,0,j} + \omega \sum_{i=0}^{\infty} \sum_{j=1}^S j y_{i,1,j}$$

Expected number of departures,

$$E_{ds} = \mu \sum_{i=0}^{\infty} \sum_{j=1}^S y_{i,1,j}$$

Expected number of customers lost before entering the orbit,

$$E_{lb} = (1 - \gamma)\lambda \sum_{i=0}^{\infty} \left(y_{i,0,0} + \sum_{j=1}^S y_{i,1,j} \right)$$

Expected number of customers lost due to retrials,

$$E_{lr} = \theta(1 - \delta) \sum_{i=1}^{\infty} i \left(y_{i,0,0} + \sum_{j=1}^S y_{i,1,j} \right)$$

Overall retrial rate,

$$\theta_1^* = \theta \left(\sum_{i=1}^{\infty} i x_i \right) \mathbf{e}$$

Successful retrial rate,

$$\theta_2^* = \theta \sum_{i=0}^{\infty} i \left(\sum_{j=1}^S y_{i,0,j} \right)$$

5. Cost Analysis

we define the expected total cost per unit time as

$$\begin{aligned} C(s, S) &= (C_F + (S - s)c_1)E_{ro} + c_2E_{inv} + c_3E_{orbit} \\ &\quad + c_4(E_{lb} + E_{lr}) + c_5E_{ds} + c_6E_p \end{aligned}$$

where, C_F =fixed cost, c_1 =procurement cost per unit per unit time, c_2 =holding cost of inventory per unit per unit time, c_3 =holding cost of customers per unit per unit time, c_4 =cost due to loss of customers per unit per unit time, c_5 =cost due to service per unit per unit time, c_6 =cost due to decay of items per unit per unit time.

5.1 Numerical Results and Interpretations

The following tables represent the effect of variation different parameters on the overall and successful rate of retrials

Table 1. Variation in λ

λ	θ_1^*	θ_2^*
2.0	1.4098	0.080379
2.1	1.5070	0.086583
2.2	1.6075	0.093005
2.3	1.7111	0.099638
2.4	1.8179	0.106470
2.5	1.9276	0.113490
2.6	2.0403	0.120700
2.7	2.1557	0.128070
2.8	2.2738	0.135600
2.9	2.3945	0.143280

Fix $(S, s, \mu, \omega, \beta, \theta, \gamma, \delta) = (20, 5, 3, 0.3, 1.5, 1.2, 0.7, 0.6)$



Table 2. Variation in μ

μ	θ_1^*	θ_2^*
3.0	1.4098	0.080379
3.1	1.3881	0.080238
3.2	1.3676	0.080114
3.3	1.3483	0.080005
3.4	1.3302	0.079909
3.5	1.3131	0.079826
3.6	1.2969	0.079754
3.7	1.2817	0.079692
3.8	1.2672	0.079639
3.9	1.2535	0.079595

Fix $(S, s, \lambda, \omega, \beta, \theta, \gamma, \delta) = (20, 5, 2, 0.3, 1.5, 1.2, 0.7, 0.6)$

Table 3. Variation in ω

ω	θ_1^*	θ_2^*
0.3	1.4098	0.080379
0.4	1.4104	0.081409
0.5	1.4136	0.082939
0.6	1.4202	0.084894
0.7	1.4302	0.087203
0.8	1.4438	0.089797
0.9	1.4609	0.092604
1.0	1.4813	0.095555
1.2	1.5049	0.098588

Fix $(S, s, \lambda, \mu, \beta, \theta, \gamma, \delta) = (20, 5, 2, 3, 1.5, 1.2, 0.7, 0.6)$

Table 4. Variation in β

β	θ_1^*	θ_2^*
1.1	1.4275	0.095610
1.2	1.4202	0.090481
1.3	1.4153	0.086416
1.4	1.4120	0.083116
1.5	1.4098	0.080379
1.6	1.4084	0.078063
1.7	1.4075	0.076072
1.8	1.4069	0.074335
1.9	1.4066	0.072799

Fix $(S, s, \lambda, \mu, \omega, \theta, \gamma, \delta) = (20, 5, 2, 3, 0.3, 1.2, 0.7, 0.6)$

Table 5. Variation in θ

θ	θ_1^*	θ_2^*
0.4	2.1769	0.13384
0.5	2.2081	0.13619
0.6	2.2393	0.13824
0.7	2.2697	0.13993
0.8	2.2986	0.14126
0.9	2.3257	0.14223
1.0	2.3507	0.14287
1.1	2.3736	0.14321
1.2	2.3945	0.14328

Fix $(S, s, \lambda, \mu, \omega, \beta, \gamma, \delta) = (20, 5, 2.9, 3, 0.3, 1.5, 0.7, 0.6)$

Table 6. Variation in γ

γ	θ_1^*	θ_2^*
0.1	0.17556	0.007849
0.2	0.35950	0.016904
0.3	0.55190	0.027191
0.4	0.75293	0.038710
0.5	0.96276	0.051439
0.6	1.18160	0.065344
0.7	1.40980	0.080379
0.8	1.64760	0.096491
0.9	1.89510	0.113630

Fix $(S, s, \lambda, \mu, \omega, \beta, \theta, \delta) = (20, 5, 2, 3, 0.3, 1.5, 1.2, 0.6)$

Table 7. Variation in δ

δ	θ_1^*	θ_2^*
0.1	0.9167	0.043151
0.2	0.9833	0.047769
0.3	1.0613	0.053363
0.4	1.1542	0.060268
0.5	1.2675	0.068996
0.6	1.4098	0.080379
0.7	1.5967	0.095891
0.8	1.8592	0.118510
0.9	2.2750	0.155740

Fix $(S, s, \lambda, \mu, \omega, \beta, \theta, \gamma) = (20, 5, 2, 3, 0.3, 1.5, 1.2, 0.7)$

The number of customers in the orbit increases when the arrival rate λ increases. From table 1, it is clear that the overall and successful rate of retrials increase with the increase in λ . From table 3, if ω increases then the inventory level reduces due to decay, the overall and successful rates of retrials increase because the number of customers in the orbit gets increased. The increase in either the service rate μ or the replenishment rate β , the number of orbiting customers get decreased so that the overall and successful rate of retrials decrease (see tables 2, 4). When the probabilities γ and δ increase, the number of customers in the orbit also increases so that the overall and successful rate of retrials from the orbit increase. (see tables 6 and 7). It is obvious that as θ increases, the overall and successful rate of retrials also increase. (see table 5).

5.2 Graphical Illustrations and Interpretation

The optimum value of the expected total cost per unit time by varying the parameter one at a time and keeping others fixed. Here, we fixed maximum inventory level as 20 unit and reorder level as 5 unit. By fixing all the parameter except the arrival rate λ . It is clear from fig.1 that the cost function attains its minimum value 131.8 at $\lambda = 2.5$. As perishable rate increases and keeping other parameters fixed, one can observe that the cost function attains the minimum value 145.05 at $\omega = 0.4$ (see fig.3). One can also observe the minimum value of the objective function by changing other parameters $\mu, \beta, \theta, \gamma$ and δ (see fig.2, fig.4, fig.5, fig.6 and fig.7).



$\mu = 3; \omega = 0.3; \beta = 1.5; \theta = 1.2; \gamma = 0.7; \delta = 0.6;$
 $C_F = 20; c_1 = 1; c_2 = 12; c_3 = c_4 = c_5 = 1; c_6 = 0.01;$

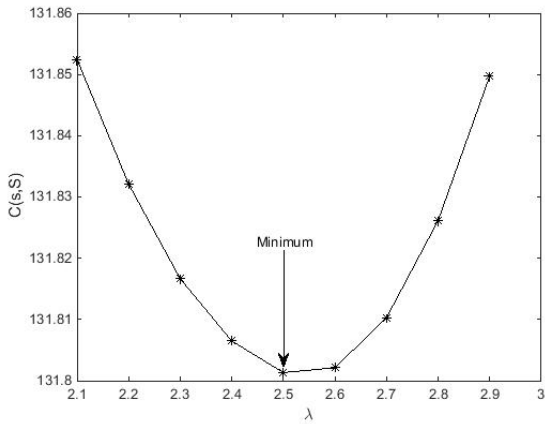


Figure 1. λ vs $C(s,S)$

$\lambda = 2; \omega = 0.3; \beta = 1.5; \theta = 1.2; \gamma = 0.7; \delta = 0.6;$
 $C_F = 20; c_1 = c_2 = 1; c_3 = 6.3; c_4 = 1; c_5 = 0.1; c_6 = 1;$

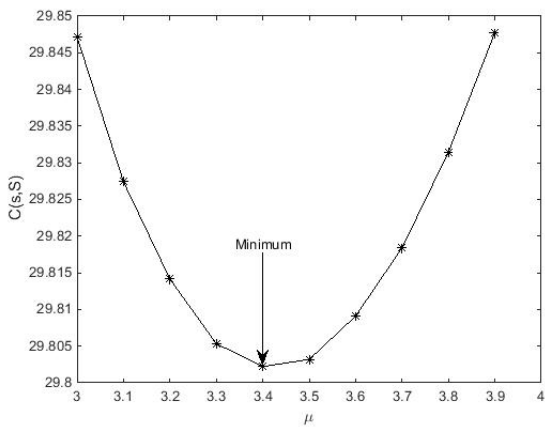


Figure 2. μ vs $C(s,S)$

$\lambda = 2; \mu = 3; \beta = 1.5; \theta = 1.2; \gamma = 0.7; \delta = 0.6;$
 $C_F = 20; c_1 = 1.1; c_2 = 13; c_3 = c_4 = c_5 = c_6 = 1;$

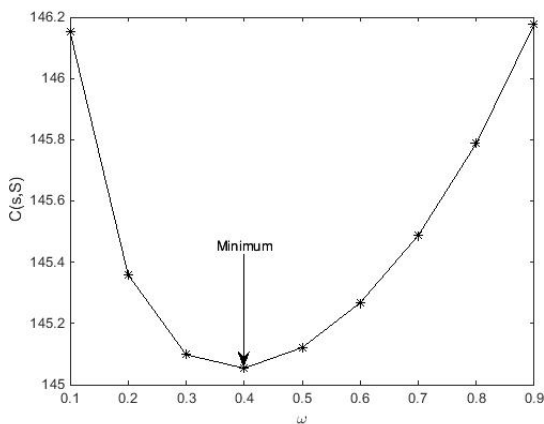


Figure 3. ω vs $C(s,S)$

$\lambda = 2; \mu = 3; \omega = 0.3; \theta = 1.2; \gamma = 0.7; \delta = 0.6;$
 $C_F = 20; c_1 = c_2 = 1; c_3 = 46; c_4 = 1; c_5 = 3; c_6 = 1;$

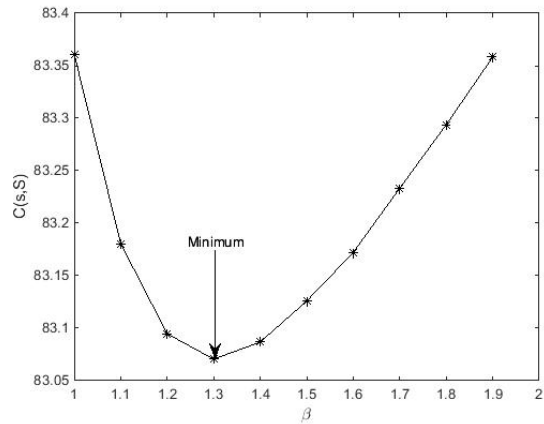


Figure 4. β vs $C(s,S)$

$\lambda = 2; \mu = 3; \omega = 0.3; \beta = 1.5; \gamma = 0.7; \delta = 0.6;$
 $C_F = 20; c_1 = c_2 = c_3 = c_4 = 1; c_5 = 0.01; c_6 = 1;$

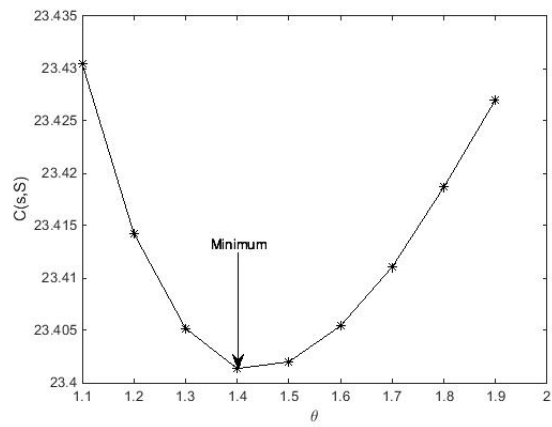


Figure 5. θ vs $C(s,S)$

$\lambda = 2; \mu = 3; \omega = 0.3; \beta = 1.5; \theta = 1.2; \delta = 0.6;$
 $C_F = 20; c_1 = 4; c_2 = c_6 = 1; c_3 = 2.3; c_4 = 1; c_5 = 0.01;$

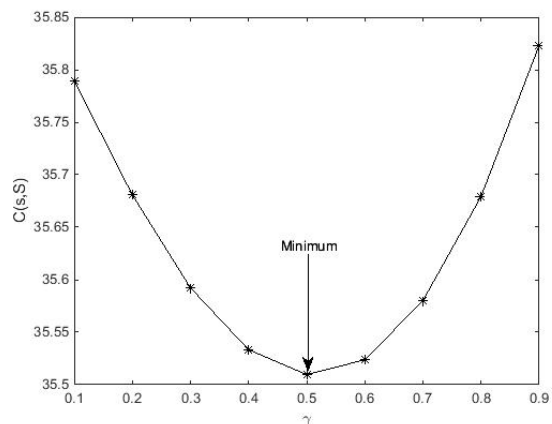


Figure 6. γ vs $C(s,S)$



$\lambda = 2; \mu = 3; \beta = 1.5; \theta = 1.2; \omega = 0.3; \gamma = 0.7;$
 $C_F = 20; c_1 = 1; c_2 = 0.6; c_3 = c_4 = c_5 = c_6 = 1;$

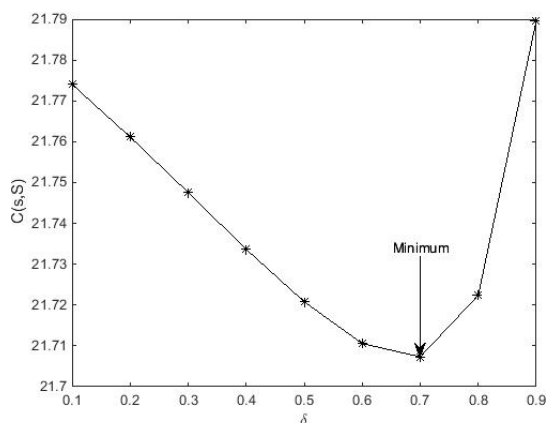


Figure 7. δ vs $C(s, S)$

6. Concluding remarks

We studied a continuous review perishable inventory system with exponentially distributed service time and lead time. We assumed that the inter arrival time of customers follows exponential distribution. The successful rates of retrial out of all retrials were tabulated. The minimum values of the expected total cost for the variations of different parameters were illustrated graphically. One can extend this model by considering the changes in the arrival process or service time distribution.

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