



Uniform eventual practical stability of impulsive differential system in terms of two measures

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Abstract

In the present paper, an impulsive differential system is investigated for uniform eventual practical stability. Sufficient criteria have been obtained for the uniform eventual practical stability of the impulsive differential system in terms of two measures by using Lyapunov-like function. The results that are obtained to investigate the stability are significantly dependent on the impulse moments. The results have been verified with the help of an example.

Keywords

Impulsive differential system, Eventual Practical Stability, Two Measures, Lyapunov function.

AMS Subject Classification

34D20.

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Article History: Received 29 January 2019; Accepted 2 April 2019

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1. Introduction

The theory of impulsive differential equations has been developing as an important area of investigation due to its wide applications in the area of control theory, population dynamics, disease control etc. and a significant progress has been made in this theory in the past [4, 9, 13]. As stability is one of the major hurdle in the theory of differential systems, there are several concepts of stabilities (e.g. exponential stability, asymptotic stability, practical stability etc.) studied in literature [1, 7, 14, 15]. To combine all various concepts of stability and to have a common framework to explore the stability theory, the potential of stability concepts in terms of two measures has been successfully demonstrated [5, 8].

In many real world applications, it is necessarily required that the state of a system may not be stable, yet, the system

may fluctuate closely to this desired state and its outcome is reasonably good. Consider an example of a projectile, which may fluctuate about an unstable trajectory, yet its final path may be acceptable. Similarly, the problem in a chemical reaction where temperature has to be kept within some specific bounds without compromising with the final outcome. Likewise, the effectiveness of vaccines in certain infectious diseases may not give complete results but it is practically acceptable etc. In these cases, it is appropriate to use the practical stability which stabilizes a system into certain subsets of the phase space. However, sometimes we only need to study the ultimate state of the stability of the solutions, this kind of stability is called eventual stability. The concept of eventual stability was introduced by La Salle and Rath [6] for ordinary differential systems to study the stabilities which are not equilibrium states but nevertheless act more and more like equilibrium states as time passes. For example, the stability of a damaged ship, in short, is its ability to survive after flooding. Both the theories of practical stability as well as eventual stability have been developing intensively and are investigated by many researchers [3, 10, 12]. The eventual stability for a nonlinear differential systems without impulses is investigated by Weijie Feng and Xilin Fu [1]. The concept of eventual stability for impulsive differential system has been discussed by various authors [11, 17]. Yu Zhang and Jitao Sun investigated the practical stability for impulsive differential

systems in terms of two measures[15]. A new kind of stability- 'eventual practical stability' is investigated by Yu Zhang and Jitao Sun for impulsive differential system with delay in terms of two measures [16]. Johnny Henderson and Snezhana Hristova investigated the same stability for differential equations with maxima [2].

In this paper, the main motive is to study the uniform eventual practical stability for a more general impulsive differential system in terms of two measures. We deduced the results by using Lyapunov-like function. The paper is arranged into three sections: In preliminaries, we have introduced some basic definitions and notations where as in main results, we have obtained some criteria to bring the uniform eventual practical stability of an impulsive differential system in terms of two measures. Finally the derived results are illustrated with the help of an example.

2. Preliminaries

Let R^n denotes n dimensional Euclidean space and let $R^+ = [0, \infty)$,

Considering the impulse effect on the differential system:

$$\begin{cases} x' = f(t,x) + g(t,y), & t \neq t_i; & \Delta x = P_t(x) + Q_t(y), & t = t_i \\ y' = p(t,x,y), & t \neq t_i; & \Delta y = S_t(x,y), & t = t_i \end{cases} \quad (1)$$

where the inter-relationship on their differentials are governed by different functions $f, g : R^+ \times R^n \rightarrow R^n$ and $p : R^+ \times R^n \times R^n \rightarrow R^n$ such that $t \in R^+, x, y \in R^n, P_t, Q_t : R^n \rightarrow R^n$ and $S_t : R^n \times R^n \rightarrow R^n$. Here $i = 1, 2, 3, \dots$

Let us define,

$$\Delta x = x(t_i) - x(t_i^-), \quad t = t_i ;$$

$$\Delta y = y(t_i) - x(t_i^-), \quad t = t_i$$

Let $t_0 \in R^+$ and $x_0, y_0 \in R^n$. Let us consider the solutions of the above mentioned differential system (1) as $x(t; t_0, x_0, y_0)$ and $y(t; t_0, x_0, y_0)$ which satisfy the following conditions:

$$x(t_0^+; t_0, x_0, y_0) = x_0 \quad \text{and} \quad y(t_0^+; t_0, x_0, y_0) = y_0$$

Throughout this paper, we will use the following assumptions:

- (a) The functions f, g , and p are continuous functions such that at $x = 0$ both $f(t, x) = g(t, y) = 0$ and at $x = 0 = y$, $p(t, x, y) = 0$ for $t_0 \in R^+$;
- (b) The functions P_t, Q_t and S_t are also continuous such that at $x = 0 = y$ both $P_t(x) = 0, Q_t(y) = 0$ and $S_t(x, y) = 0$;
- (c) $\|x + P_t(x) + Q_t(y)\| \leq \|x\|$ and $\|y + S_t(x, y)\| \leq \|y\|$ holds for $x, y \in R^n$;
- (d) $0 < t_1 < t_2 < t_3 < \dots < t_i < t_{i+1} < \dots$ and $t_i \rightarrow +\infty$ as $i \rightarrow +\infty$;

- (e) Both the solutions $x(t_0^+; t_0, x_0, y_0)$ and $y(t_0^+; t_0, x_0, y_0)$ for $(t_0, x_0, y_0) \in R^+ \times R^n \times R^n$, of the system (1) are unique.

In order to investigate the stability behaviour of the impulsive differential system (1), firstly we define the following:

Definition 2.1. The set K, K_1 and Γ be defined as the class of continuous functions:

$$K = \{ \phi : \phi \in C(R^+, R^+) \text{ is strictly increasing and } \phi(0) = 0 \}$$

$$K_1 = \{ \psi : \psi \in C(R^+, R^+) \text{ is increasing and } \psi(s) < s \text{ for } s > 0 \}$$

$$\Gamma = \{ h : h \in C(R^+ \times R^n \times R^n, R^+) \text{ where } \inf h(t, x, y) = 0. \}$$

Definition 2.2. [17]. Consider the function $V : R^+ \times R^n \times R^n \rightarrow R^n$ which belongs to the class v_0 , if it satisfies the following conditions:

- (i) $V(t, x, y)$ is piecewise continuous for $(t, x, y) \in [t_{i-1}, t_i) \times R^n \times R^n$ for $i \in N$, and $\lim_{(t,x,y) \rightarrow (t_i^+, x_0, y_0)} V(t, x, y) = V(t_i^+, x_0, y_0)$ exists;

- (ii) V is Lipschitz in the local neighborhood of x and y ;

- (iii) $V(t, 0, 0) = 0 \quad \forall t \in R^+$.

Definition 2.3. [5]. Let $h_0, h \in \Gamma$, then

- (i) h_0 is finer than h , iff $h(t, x, y) \leq \phi(h_0(t, x, y))$ whenever $h_0(t, x, y) < \delta$ holds, for a function $\phi \in K$ where $\delta > 0$;

- (ii) h -positive definite, iff $\beta(h(t, x, y)) \leq V(t, x, y)$ whenever $h(t, x, y) < \rho$ holds, for a function $\beta \in K$ where $\rho > 0$;

- (iii) h_0 -decreasing, iff $V(t, x, y) \leq \alpha(h_0(t, x, y))$ whenever $h_0(t, x, y) < \delta$ holds, for a function $\alpha \in K$ where $\delta > 0$;

- (iv) $S(h, \rho) = \{(t, x, y) \in R^+ \times R^n \times R^n, h(t, x, y) < \rho\}$.

Definition 2.4. [17]. The right hand derivative of $V \in v_0$ is defined as:

$$V'(t, x, y) = \lim_{s \rightarrow 0^+} \sup \frac{1}{s} \left[V(t+s, x+sf(t,x) + sg(t,y), y + p(t,x,y)) - V(t, x, y) \right]$$

Definition 2.5. [16]. The stability of system (1) in terms of two measures - (h_0, h) about equilibrium point is:

- (i) eventual practical if for a given (A, B) with $0 < A < B$, and for $t_0 \in R^+$, there exists a $\tau(A, B) > 0$ such that $h(t, x, y) < B$ whenever $h_0(t_0, x_0, y_0) < A$ for some $t \geq t_0 \geq \tau(A, B)$;

- (ii) uniform eventual practical if the above definition holds for all $t_0 \in R^+$.



3. Main Results

In this section, we study the uniform eventual practical stability of the system (1) in terms of two measures by using Lyapunov method with the following theorems.

Theorem 3.1. *Assume the following conditions:*

- (I.1) given $0 < A < B$;
- (I.2) Let $h, h_0 \in \Gamma$ such that h_0 is finer than h for $\varphi \in K$ whenever $h_0(t, x, y) < A$;
- (I.3) Let function $V \in v_0$ exists such that $\beta(h(t, x, y)) \leq V(t, x, y) \leq \alpha(h_0(t, x, y))$ for $\alpha, \beta \in K$;
- (I.4) $V'(t, x, y) \leq 0$, where $(t, x, y) \in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n$;
- (I.5) For all $i \in \mathbb{N}$,
 $V(t_i, x(t_i), y(t_i)) \leq (1 + c_i)(V(t_i^-, x(t_i^-), y(t_i^-)))$
 where $c_i \geq 0$ and $\sum_{i=1}^{\infty} c_i < \infty$;
- (I.6) $\varphi(A) < B$ and $M\alpha(A) < \beta(B)$,
 where $M = \prod_{i=1}^{\infty} (1 + c_i)$.

Then, the stability of the system (1) in terms of two measures - (h_0, h) is uniform eventual practical.

Proof. Let us consider the two solutions of the impulsive differential system (1) as $x(t; t_0, x_0, y_0)$ and $y(t; t_0, x_0, y_0)$.

Let us assume, for a given (A, B) with $0 < A < B$, there exists a $\tau(A, B) > 0$.

Let $h_0(t_0, x_0, y_0) < A$ for $(t_0, x_0, y_0) \in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n$ and $t \geq t_0 \geq \tau(A, B)$.

As $\sum_{i=1}^{\infty} c_i < \infty$, it follows that $1 \leq M < \infty$.

In order to prove uniform eventual practical stability, we need to verify that

$$V(t, x, y) \leq M\alpha(A), \quad t \geq t_0 > \tau(A, B) \quad (2)$$

To obtain inequality (2), firstly we will prove that

$$V(t, x, y) \leq \alpha(A), \quad t \in [t_0, t_1) \quad (3)$$

From condition (I.3), V is h_0 -decreasing, therefore we have $V(t_0, x_0, y_0) \leq \alpha(h_0(t_0, x_0, y_0)) < \alpha(A)$.

Hence, inequality (3) holds for $t = t_0$.

Let, if possible (3) doesn't hold for $t \in (t_0, t_1)$.

Then, there exists an $\hat{s} \in (t_0, t_1)$ such that

$$V(\hat{s}, x(\hat{s}), y(\hat{s})) > \alpha(A).$$

Consider $\bar{s} = \inf \{t : V(t, x(t), y(t)) > \alpha(A), t \in (t_0, t_1)\}$

This means, $V(\bar{s}, x(\bar{s}), y(\bar{s})) = \alpha(A)$ which implies

$$V'(\bar{s}, x(\bar{s}), y(\bar{s})) \geq 0$$

But, from condition (I.4), we get $V'(t, x, y) \leq 0$.

Hence, we get a contradictory results, so eq.(3) holds.

Consider

$$\begin{aligned} V(t_1, x(t_1), y(t_1)) &= V(t_1, x(t_1^-) + P_t(x) + Q_t(y), y(t_1^-) \\ &\quad + S_t(x, y)) \\ &\leq (1 + c_1)\alpha(A) \end{aligned}$$

Next, we will verify that

$$V(t, x, y) \leq (1 + c_1)\alpha(A), \quad t_1 < t < t_2 \quad (4)$$

Let if possible, inequality (4) doesn't hold. Then, there exists a $\hat{u} \in (t_1, t_2)$ such that

$$V(\hat{u}, x(\hat{u}), y(\hat{u})) > (1 + c_1)\alpha(A).$$

Let $\bar{u} = \inf \{t : V(t, x(t), y(t)) > (1 + c_1)\alpha(A), t \in (t_1, t_2)\}$

This means, $V(\bar{u}, x(\bar{u}), y(\bar{u})) = (1 + c_1)\alpha(A)$ which implies

$$V'(\bar{u}, x(\bar{u}), y(\bar{u})) \geq 0.$$

Hence, again we get a contradiction as $V'(t, x, y) \leq 0$, so inequality (4) holds.

Now Consider,

$$\begin{aligned} V(t_2, x(t_2), y(t_2)) &= V(t_2, x(t_2^-) + P_t(x) + Q_t(y), y(t_2^-) \\ &\quad + S_t(x, y)) \\ &\leq (1 + c_1)(1 + c_2)\alpha(A) \end{aligned}$$

Continuing this induction process, we have

$$V(t_i, x(t_i), y(t_i)) \leq (1 + c_1)(1 + c_2)\dots(1 + c_i)\alpha(A) \quad (5)$$

where $t_i \leq t < t_{i+1}, i \in \mathbb{N}$

As $M = \prod_{i=1}^{\infty} (1 + c_i)$, the above inequality becomes

$$V(t, x, y) \leq M\alpha(A), \quad t \geq t_0 \geq \tau(A, B)$$

Also by using condition (I.6), it follows that

$$V(t, x, y) \leq M\alpha(A) < \beta(B), \quad t \geq t_0 \geq \tau(A, B)$$

From condition (I.3), as $V(t, x, y)$ is h -positive definite, therefore we have

$$h(t, x(t), y(t)) \leq \beta^{-1}(V(t, x(t), y(t))) < \beta^{-1}(\beta(B)) < B$$

where $t \geq t_0 \geq \tau(A, B)$

Hence, the system (1) is uniformly eventually practically stable. \square

Now, we prove the same kind of stability as proved in Theorem 3.1 for the impulsive differential system (1) with more stronger criteria in the following theorem.

Theorem 3.2. *Let the following conditions hold:*

(II.1) same as condition (I.1);

(II.2) same as condition (I.2);



(II.3) same as condition (I.3);

(II.4) $V'(t, x, y) \leq \gamma(t)\lambda(V(t, x, y))$, where $(t, x, y) \in [t_{i-1}, t_i) \times R^n \times R^n$ for $i \in N$ such that $\gamma, \lambda : R^+ \rightarrow R^+$ are locally integrable;

(II.5) For all $i \in N$, and $(t, x, y) \in R^+ \times R^n \times R^n$, there exists a $\psi \in K_1$ such that $V(t_i, x(t_i), y(t_i)) \leq \psi(V(t_i^-, x(t_i^-), y(t_i^-)))$;

(II.6) $\int_{t_{i-1}}^{t_i} \gamma(s)ds < L$ and $\int_{\mu}^{\psi^{-1}(\mu)} \frac{ds}{\lambda(s)} \geq L$, holds for a constant $L > 0$ such that $\mu > 0$ where $i \in N$;

(II.7) $\varphi(A) < B$ and $\alpha(A) < \psi(\beta(B))$.

Then the stability of the system (1) in terms of two measures (h_0, h) is uniform eventual practical.

Proof. Let for a given (A, B) with $0 < A < B$, there exists a $\tau(A, B) > 0$. Also, let the solutions of impulsive differential system (1) as $x(t; t_0, x_0, y_0)$ and $y(t; t_0, x_0, y_0)$. Then for $(t_0, x_0, y_0) \in R^+ \times R^n \times R^n$, let $h_0(t_0, x_0, y_0) < A$ for $t \geq t_0 \geq \tau(A, B)$.

As h is finer than h_0 , we have $h(t_0, x_0, y_0) < \varphi(h_0(t_0, x_0, y_0)) < \varphi(A) < B$. In order to prove uniform eventual practical stability, firstly we will prove that

$$V(t, x, y) \leq \psi^{-1}(\alpha(A)), \quad t \geq t_0 > \tau(A, B) \quad (6)$$

In order to prove inequality (6), first of all we will claim that

$$V(t, x, y) \leq \psi^{-1}(\alpha(A)), \quad t \in [t_0, t_m) \quad (7)$$

As, V is h_0 -decescent.

We have, $V(t_0, x_0, y_0) \leq \alpha(h_0(t_0, x_0, y_0)) < \alpha(A) < \psi^{-1}(\alpha(A))$. Let, if possible, (7) doesn't hold for $t \in (t_0, t_m)$, then there must be a $\hat{t} \in (t_0, t_m)$ for which

$$V(\hat{t}, x(\hat{t}), y(\hat{t})) > \psi^{-1}(\alpha(A)) > \alpha(A) > V(t_0, x_0, y_0)$$

Since $V(t, x, y)$ is continuous in $[t_{m-1}, t_m)$, there must exists a $l_1 \in (t_0, \hat{t})$ for which

$$\begin{aligned} V(l_1, x(l_1), y(l_1)) &= \psi^{-1}(\alpha(A)), \\ V(t, x(t), y(t)) &> \psi^{-1}(\alpha(A)), \quad l_1 < t < \hat{t}, \\ V(t, x(t), y(t)) &\leq \psi^{-1}(\alpha(A)), \quad t_0 < t < l_1 \end{aligned} \quad (8)$$

Also, there exists a $l_2 \in (t_0, l_1)$ such that

$$\begin{aligned} V(l_2, x(l_2), y(l_2)) &= \alpha(A), \\ V(t, x(t), y(t)) &\geq \alpha(A), \quad l_2 < t < l_1 \end{aligned} \quad (9)$$

From condition (II.4), we have

$$V'(t, x, y) \leq \gamma(t)\lambda(V(t, x, y))$$

Integrate the above inequality in $[l_2, l_1]$, then by using condition (II.6), we have

$$\int_{V(l_2, x(l_2), y(l_2))}^{V(l_1, x(l_1), y(l_1))} \frac{du}{\lambda(u)} \leq \int_{l_2}^{l_1} \gamma(t)dt \leq \int_{t_{m-1}}^{t_m} \gamma(t)dt < L \quad (10)$$

On the other hand, by using (8), (9) and condition (II.6), we have

$$\int_{V(l_2, x(l_2), y(l_2))}^{V(l_1, x(l_1), y(l_1))} \frac{du}{\lambda(u)} = \int_{\alpha(A)}^{\psi^{-1}(\alpha(A))} \frac{du}{\lambda(u)} \geq L \quad (11)$$

On comparing (10) and (11), there is a contradiction.

Hence inequality (7) holds.

By using (II.5), we have

$$\begin{aligned} V(t_m, x(t_m), y(t_m)) &= V(t_m, x(t_m^-), y(t_m^-)) + P_t(x) \\ &\quad + Q_t(y), y(t_m^-) + S_t(x, y) \\ &\leq \psi(V(t_m^-, x(t_m^-), y(t_m^-))) \\ &\leq \psi(\psi^{-1}(\alpha(A))) \\ &= \alpha(A) \end{aligned} \quad (12)$$

Also, as $\psi \in K_1$, hence $V(t_m, x(t_m), y(t_m)) \leq \psi^{-1}(\alpha(A))$.

Now, we will claim that

$$V(t, x, y) \leq \psi^{-1}(\alpha(A)), \quad t \in (t_m, t_{m+1}) \quad (13)$$

Let, if possible, inequality (13) doesn't exist, then there must exists a $\hat{q} \in (t_m, t_{m+1})$ for which

$$V(\hat{q}, x(\hat{q}), y(\hat{q})) > \psi^{-1}(\alpha(A)) > \alpha(A) > V(t_m, x(t_m), y(t_m))$$

As $V(t, x, y)$ is continuous in $[t_m, t_{m+1})$, there must holds a $q_1 \in (t_m, \hat{q})$ such that

$$\begin{aligned} V(q_1, x(q_1), y(q_1)) &= \psi^{-1}(\alpha(A)), \\ V(t, x(t), y(t)) &> \psi^{-1}(\alpha(A)), \quad q_1 < t < \hat{q}, \\ V(t, x(t), y(t)) &\leq \psi^{-1}(\alpha(A)), \quad t_0 < t < q_1 \end{aligned} \quad (14)$$

Also, there must holds a $q_2 \in (t_m, q_1)$ for which

$$\begin{aligned} V(q_2, x(q_2), y(q_2)) &= \alpha(A), \\ V(t, x(t), y(t)) &\geq \alpha(A), \quad q_2 < t < q_1 \end{aligned} \quad (15)$$

On integrating the inequality $V'(t, x, y) \leq \gamma(t)\lambda(V(t, x, y))$ in $[q_2, q_1]$ and by using (14), (15) and condition (II.6), again we will get a contradiction as obtained in first case. Hence inequality (13) holds.

Now consider

$$\begin{aligned} V(t_{m+1}, x(t_{m+1}), y(t_{m+1})) &= V(t_{m+1}, x(t_{m+1}^-), y(t_{m+1}^-)) + P_t(x) \\ &\quad + Q_t(y), y(t_{m+1}^-) + S_t(x, y) \\ &\leq \psi(V(t_{m+1}^-, x(t_{m+1}^-), y(t_{m+1}^-))) \\ &\leq \psi(\psi^{-1}(\alpha(A))) \\ &= \alpha(A) \end{aligned}$$



(16) (2). Also, $\beta(h(t, x, y)) \leq V(t, x, y) \leq \alpha(h_0(t, x, y))$ is satisfied.

Therefore, by following the simple induction process, we have

$$V(t, x(t), y(t)) \leq \psi^{-1}(\alpha(A)), \quad t_{m+i} \leq t < t_{m+i+1}$$

and

$$V(t_{m+i+1}, x(t_{m+i+1}), y(t_{m+i+1})) \leq \alpha(A)$$

As $\alpha(A) < \psi^{-1}(\alpha(A))$, by condition (II.7), we have

$$V(t, x(t), y(t)) \leq \psi^{-1}(\alpha(A)) < \beta(B) \tag{17}$$

Hence, by using condition (II.3) and (17) we have

$$h(t, x, y) \leq \beta^{-1}(V(t, x, y)) < \beta^{-1}(\beta(B)) < B,$$

where $t \geq t_0 \geq \tau(A, B)$.

Thus, the system (1) is uniform eventually practically stable. \square

4. Example

Following is an illustrative example to verify the above obtained results for uniform eventual practical stability. Consider the following system:

Example 4.1.

$$\begin{cases} x' = lx(t) + my(t), t \neq t_i; x(t_i) = ux(t_i^-) + vy(t_i^-) \\ y' = qy(t), t \neq t_i; y(t_i) = wy(t_i^-), i = 1, 2, 3, \dots \end{cases} \tag{18}$$

where $0 < t_1 < t_2 < t_3 < \dots < t_i < t_{i+1} < \dots$ and $t_i \rightarrow \infty$ as $i \rightarrow +\infty$, $l > 0, m > 0, q > 0, u > 0, v > 0, w > 0$. Let the following conditions be satisfied:

(1) $l > q, v^2 + w^2 < u^2, 3u^2 + v^2 < 2;$

(2) $t_i - t_{i-1} < \frac{-\ln(3u^2+v^2)+\ln 2}{2l+m}$.

Then, the differential system (18) under consideration is (h_0, h) - uniform eventually practically stable.

Proof. Let us define the functions as follows:

$$V(t, x, y) = \frac{1}{2}(x^2 + y^2), \quad h_0(t, x, y) = h(t, x, y) = x^2 + y^2,$$

$$\alpha(s) = ns, \quad \beta(s) = \frac{1}{n}s, \quad n > 1, \quad \varphi(s) = s$$

$$\psi(s) = \frac{3u^2 + v^2}{2}s, \quad \lambda(s) = s, \quad \gamma(s) = 2l + m$$

Now, we will investigate the conditions of Theorem II as follows:

(1). For a given $0 < A < \frac{3u^2+v^2}{2} \frac{1}{n^2} B$, if $h_0(t, x, y) < A$, then clearly $h(t, x, y) \leq \varphi(h_0(t, x, y))$

$$\begin{aligned} (3). V'(t, x, y) &= xx' + yy' \\ &\leq l(x^2(t) + y^2(t)) + m \left(\frac{x^2(t) + y^2(t)}{2} \right) \\ &= (2l + m) \left(\frac{x^2(t) + y^2(t)}{2} \right) \\ &= \gamma(t)\lambda(V(t, x, y)). \end{aligned}$$

$$\begin{aligned} (4). V(t_i, x(t_i), y(t_i)) &= V(t_i, x(t_i^-) + P_t(x) + Q_t(y), y(t_i^-) + S_t(x, y)) \\ &= \frac{1}{2} [(ux(t_i^-) + vy(t_i^-))^2 + (wy(t_i^-))^2] \\ &= \frac{1}{2} [u^2x^2(t_i^-) + v^2y^2(t_i^-) + 2uvx(t_i^-)y(t_i^-) + w^2y^2(t_i^-)] \\ &\leq \frac{1}{2} [u^2(x^2(t_i^-) + y^2(t_i^-)) + uv(x^2(t_i^-) + y^2(t_i^-))] \\ &= \frac{1}{2} (u^2 + uv) [x^2(t_i^-) + y^2(t_i^-)] \\ &\leq \frac{1}{2} [x^2(t_i^-) + y^2(t_i^-)] \frac{3u^2 + v^2}{2} \\ &= \psi(V(t_i^-, x(t_i^-), y(t_i^-))) \end{aligned}$$

(5). Let $L = \ln \frac{2}{3u^2+v^2}$, clearly $L > 0$
Now

$$\int_{t_{i-1}}^{t_i} \gamma(s) ds < (2l + m) \frac{-\ln(3u^2 + v^2) + \ln 2}{2l + m} = \ln \frac{2}{3u^2 + v^2} = L.$$

Also, for any value of $v > 0$, we have

$$\int_v^{\psi^{-1}(v)} \frac{ds}{\lambda(s)} = \int_v^{\frac{2}{3u^2+v^2}v} \frac{ds}{s} = \ln \frac{2}{3u^2 + v^2} = L.$$

(6). As $0 < A < \frac{3u^2+v^2}{2} \frac{1}{n^2} B$.

It is clear that $\varphi(A) = A < B$ and

$$\alpha(A) = nA < n \frac{3u^2 + v^2}{2} \frac{1}{n^2} B = \frac{3u^2 + v^2}{2} \frac{1}{n} B = \psi(\beta(B))$$

Here all the conditions as mentioned in the Theorem 3.2 are well satisfied. Therefore, the impulsive differential system (18) is (h_0, h) - uniform eventually practically stable. \square

5. Conclusion

In this paper, the uniform eventual practical stability of an impulsive differential system has been studied. Earlier, the authors [2, 16] investigated the stability for impulsive differential systems with comparison principle in terms of two measures. Here in the present investigation same kind



of stability is derived for an impulsive differential system by using Lyapunov-like functions in terms of two measures. The results obtained above indicate that the impulses also significantly contribute towards the system's eventual practical stability. An example is also given to illustrate the results proved in Theorem II.

Acknowledgment

One of us (PM) would like to thank IKGPTU for providing online library facility.

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ISSN(P):2319 – 3786

Malaya Journal of Matematik

ISSN(O):2321 – 5666

