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Trevigintic and quattuorvigintic functional equations in matrix normed spaces

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Abstract

In this work, we determine the general solution of trevigintic functional equation and we investigate the stability of trevigintic and quattuorvigintic functional equations in matrix normed space with the help fixed point method.

Keywords

Hyers-Ulam-Rassias stability, Generalized Hyers-Ulam-Rassias stability, Ulam-Gavruta-Rassias stability, J. M. Rassias stability, fixed point, trevigintic and quattuorvigintic functional equations, matrix normed space.

AMS Subject Classification

39B52, 39B82, 46L07, 47H10, 47L25

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Contents

1. Introduction

In 1940, S. M. Ulam [21] triggered the study of stability problems for various functional equations. He raised a question relating to the stability of homomorphism. In the following year, D. H. Hyers [5] was able to give a partial solution to Ulam's question. The result of Hyers was generalized by Aoki [1] for additive mappings. In 1978, Th. M. Rassias [12] succeeded in extending the result of Hyers theorem by weakening the condition for the Cauchy difference.

In 1982, J. M. Rassias [14] solved the Ulam problem for different mappings and for many Euler-Lagrange type quadratic mappings, by involving a product of different powers of norms. In 1994, a generalization of the Rassias theorem was obtained by Gavruta [4] by replacing the unbounded Cauchy difference by a general control function. A further generalization of the stability for a large class of mapping was obtained by Isac and Th. M. Rassias [6]. They also presented some applications in non-linear analysis, especially in fixed point theory. This terminology may also be applied to the cases of other functional equations [2, 3, 10, 13, 16, 19, 20, 23]. Also, the generalized Hyers-Ulam stability of functional equations and inequalities in matrix normed space has been studied by number of authors [7, 8, 15, 18, 22].

Very recently, R. Murali et. al., [11] discussed the general solution of viginti duo functional equation and its stability in multi-Banach spaces.

In this paper, we introduce the following new functional equation

$$\begin{aligned}
&\zeta(u+12v)-23\zeta(u+11v)+253\zeta(u+10v)\\
&-1771\zeta(u+9v)+8855\zeta(u+8v)\\
&-33649\zeta(u+7v)+100947\zeta(u+6v)\\
&-245157\zeta(u+5v)+490314\zeta(u+4v)\\
&-817190\zeta(u+3v)+1144066\zeta(u+2v)\\
&-1352078\zeta(u+v)+1352078\zeta(u)\\
&-1144066\zeta(u-v)+817190\zeta(u-2v)\\
&-490314\zeta(u-3v)+245157\zeta(u-4v)\\
&-100947\zeta(u-5v)+33649\zeta(u-6v)\\
&-8855\zeta(u-7v)+1771\zeta(u-8v)-\zeta(u-11v)\\
&-253\zeta(u-9v)+23\zeta(u-10v)=23!\zeta(v), \qquad (1.1)
\end{aligned}$$

where $23! = 25852016740000000000000$, is said to be trevigintic functional equation if the function $\zeta(u)=au^{23}$ is its solution and Murali et al.,[17] found the general solution of the following equation

$$\begin{aligned}
&\zeta(u+12v)-24\zeta(u+11v)+276\zeta(u+10v)\\
&-2024\zeta(u+9v)+10626\zeta(u+8v)\\
&-42504\zeta(u+7v)+134596\zeta(u+6v)\\
&-346104\zeta(u+5v)+735471\zeta(u+4v)\\
&-1307504\zeta(u+3v)+1961256\zeta(u+2v)\\
&-2496144\zeta(u+v)+2704156\zeta(u)\\
&-2496144\zeta(u-v)+1961256\zeta(u-2v)\\
&-1307504\zeta(u-3v)+735471\zeta(u-4v)\\
&+134596\zeta(u-6v)-42504\zeta(u-7v)\\
&+10626\zeta(u-8v)-2024\zeta(u-9v)\\
&+276\zeta(u-10v)-346104\zeta(u-5v)-24\zeta(u-11v)\\
&+\zeta(u-12v)=1.124000728\times 10^{21}\zeta(v), \qquad (1.2)
\end{aligned}$$

in matrix paranormed spaces. The above equation is called quattuordecic functional equation if the function $f(u)=au^{24}$ is its solution.In this paper, we determine the general solution of the functional equation (1.1) and we also prove the stability of the functional equations (1.1) and (1.2) in matrix normed space with the help of fixed point method.

2. Trevigintic functional equation (1.1) and its general solution

Throughout this segment, let us consider $(X,\|.\|_n)$ be a matrix normed space, $(Y,\|.\|_n)$ be a matrix Banach space and let n be a fixed non-negative integer.

In this part, we derive the general solution of trevigintic functional equation (1.1). For this, let us consider $\mathscr{D}$ and $\mathscr{E}$ be real vector spaces.

Theorem 2.1. *If $\zeta:\mathscr{D}\to\mathscr{E}$ be a mapping satisfying (1.1) for all $x,y\in\mathscr{D}$, then $\zeta(2u)=2^{23}\zeta(u)$ for all $u,v\in\mathscr{D}$.*

Proof. Letting $u=v=0$ in (1.1), one gets $\zeta(0)=0$. Refilling $u=0$, $v=u$ and $u=u$, $v=-u$ in (1.1) and adding the two out coming equations, we get $\zeta(-u)=-\zeta(u)$. Hence, ζ is an odd mapping. Refilling $u=0$, $v=2u$ and $u=12u$, $v=u$ in (1.1) and subtracting the two out coming equations, one gets

$$\begin{aligned}
&23\zeta(23u)-275\zeta(22u)+1771\zeta(21u)\\
&-8625\zeta(20u)+33649\zeta(19u)-102465\zeta(18u)\\
&+245157\zeta(17u)-483230\zeta(16u)+817190\zeta(15u)\\
&-1168860\zeta(14u)+1352078\zeta(13u)\\
&+1144066\zeta(11u)-961400\zeta(10u)+490314\zeta(9u)\\
&+100947\zeta(7u)-360525\zeta(6u)+8855\zeta(5u)\\
&+325105\zeta(4u)+253\zeta(3u)-23!\zeta(u)\\
&-(208035+23!)\zeta(2u)-1284780\zeta(12u)=0 \qquad (2.1)
\end{aligned}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(11u,u)$ in (1.1), and increasing the out coming equation by 23, and then subtracting the out coming equation from (2.1), we get

$$\begin{aligned}
&254\zeta(22u)-4048\zeta(21u)-29953728\zeta(11u)\\
&-170016\zeta(19u)+671462\zeta(18u)-2076624\zeta(17u)\\
&+5155381\zeta(16u)-10460032\zeta(15u)\\
&-24961440\zeta(13u)+29813014\zeta(12u)\\
&+25352118\zeta(10u)-18305056\zeta(9u)\\
&+32108\zeta(20u)-11277222\zeta(8u)\\
&-5537664\zeta(7u)+1961256\zeta(6u)-765072\zeta(5u)\\
&+528770\zeta(4u)-40480\zeta(3u)+17626510\zeta(14u)\\
&-(202216+23!)\zeta(2u)+23!(24)\zeta(u)=0 \qquad (2.2)
\end{aligned}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(10u,u)$ in (1.1), and increasing the out coming equation by 254, and then subtracting the out coming equation from (2.2) , we have

$$\begin{aligned}
&1794\zeta(21u)-32154\zeta(20u)+279818\zeta(19u)\\
&-1577708\zeta(18u)+6470222\zeta(17u)\\
&+51809846\zeta(15u)-106913246\zeta(14u)\\
&+182604820\zeta(13u)-260779750\zeta(12u)\\
&+313474084\zeta(11u)-318075694\zeta(10u)\\
&+272287708\zeta(9u)-196289038\zeta(8u)\\
&+119002092\zeta(7u)-60308622\zeta(6u)\\
&+24875466\zeta(5u)-8018076\zeta(4u)\\
&+2208690\zeta(3u)-(652050+23!)\zeta(2u)\\
&+23!(278)\zeta(u)-20485157\zeta(16u)=0 \qquad (2.3)
\end{aligned}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(9u,u)$ in (1.1), and increasing the out coming equation by 1794, and then subtracting the out coming equation from (2.3), we have

$$\begin{aligned}
&9108\zeta(20u)-174064\zeta(19u)+1599466\zeta(18u)\\
&-9415648\zeta(17u)+39881149\zeta(16u)\\
&-129289072\zeta(15u)+332898412\zeta(14u)\\
&-697018496\zeta(13u)+1205259110\zeta(12u)\\
&-1738980320\zeta(11u)+2107552238\zeta(10u)\\
&-2153340224\zeta(9u)+1856165366\zeta(8u)\\
&-1347036768\zeta(7u)+819314694\zeta(6u)\\
&-414936192\zeta(5u)+173080842\zeta(4u)\\
&-58157616\zeta(3u)+(15232026-23!)\zeta(2u)\\
&+23!(2072)\zeta(u)=0 \qquad (2.4)
\end{aligned}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(8u,u)$ in (1.1), and increasing the out coming equation by 9108, and then subtracting the out coming equation from (2.4), we arrive at

$$\begin{aligned}
&35420\zeta(19u)-704858\zeta(18u)+6714620\zeta(17u)\\
&-40770191\zeta(16u)+177186020\zeta(15u)\\
&-586526864\zeta(14u)+1535871460\zeta(13u)\\
&-3260520802\zeta(12u)+5703986200\zeta(11u)\\
&-8312600890\zeta(10u)+10161386200\zeta(9u)\\
&-10458561060\zeta(8u)+9073116360\zeta(7u)\\
&-6623651826\zeta(6u)+4050843720\zeta(5u)\\
&-2059809114\zeta(4u)+861258552\zeta(3u)\\
&-(291033582+23!)\zeta(2u)+23!(11180)\zeta(u)=0
\end{aligned} \tag{2.5}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(7u,u)$ in (1.1), and increasing the out coming equation by 35420, and then subtracting the out coming equation from (2.5), we arrive at

$$\begin{aligned}
&109802\zeta(18u)-2246640\zeta(17u)+21958629\zeta(16u)\\
&-136458080\zeta(15u)+605320716\zeta(14u)\\
&-2039671280\zeta(13u)+5422940138\zeta(12u)\\
&-11662935680\zeta(11u)+20632268910\zeta(10u)\\
&-30361431520\zeta(9u)+37432041700\zeta(8u)\\
&-38817486400\zeta(7u)+33899165890\zeta(6u)\\
&-24894026080\zeta(5u)+15307077350\zeta(4u)\\
&-7821387728\zeta(3u)+(3275547898-23!)\zeta(2u)\\
&+23!(46600)\zeta(u)=0
\end{aligned} \tag{2.6}$$

$\forall\ u\in\mathscr{D}$. Refilling (u,v) by $(6u,u)$ in (1.1), and increasing the out coming equation by 109802, and then subtracting the out coming equation from (2.6), we have

$$\begin{aligned}
&278806\zeta(17u)-5821277\zeta(16u)+58001262\zeta(15u)\\
&-366975994\zeta(14u)+1655056218\zeta(13u)\\
&-5661242356\zeta(12u)+15255793230\zeta(11u)\\
&-33205188920\zeta(10u)+59367664860\zeta(9u)\\
&-88188693230\zeta(8u)+109643382200\zeta(7u)\\
&-114561702700\zeta(6u)+100726599100\zeta(5u)\\
&-74419493590\zeta(4u)+45988290190\zeta(3u)\\
&-(23448721670+23!)\zeta(2u)\\
&+23!(156402)\zeta(u)=0
\end{aligned} \tag{2.7}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(5u,u)$ in (1.1), and increasing the out coming equation by 278806, and then subtracting the out coming equation from (2.7), we have

$$\begin{aligned}
&591261\zeta(16u)-12536656\zeta(15u)\\
&-813770912\zeta(13u)+23!(435208)\zeta(u)\\
&+3720300738\zeta(12u)-12888836050\zeta(11u)\\
&+35146053620\zeta(10u)-77334820220\zeta(9u)\\
&+139648781900\zeta(8u)-209329083000\zeta(7u)\\
&+262405477400\zeta(6u)-276234447300\zeta(5u)\\
&+244482433700\zeta(4u)-181355419500\zeta(3u)\\
&+126789432\zeta(14u)\\
&-(110784936300-23!)\zeta(2u)=0
\end{aligned} \tag{2.8}$$

for all $u\in\mathscr{D}$. Refilling (u,v) by $(4u,u)$ in (1.1), and increasing the out coming equation by 591261, and then subtracting the out coming equation from (2.8), we have

$$\begin{aligned}
&1062347\zeta(15u)+233352319\zeta(13u)\\
&-1515315417\zeta(12u)+7006505341\zeta(11u)\\
&-24539970540\zeta(10u)+67616952750\zeta(9u)\\
&-150254764000\zeta(8u)+273842902300\zeta(7u)\\
&-414022530800\zeta(6u)+523046954000\zeta(5u)\\
&-553901433400\zeta(4u)+489850571600\zeta(3u)\\
&-(23!+352492298900)\zeta(2u)-22799601\zeta(14u)\\
&+23!(1026469)\zeta(u)=0
\end{aligned} \tag{2.9}$$

$\forall\ u\in\mathscr{D}$. Refilling (u,v) by $(3u,u)$ in (1.1), and increasing the out coming equation by 1062347, and then subtracting the out coming equation from (2.9), we have

$$\begin{aligned}
&1634380\zeta(14u)+(755662041400-23!)\zeta(2u)\\
&-2400577344\zeta(11u)+11206943660\zeta(10u)\\
&-39623789860\zeta(9u)+110185977100\zeta(8u)\\
&-247016270700\zeta(7u)+453848040300\zeta(6u)\\
&-690466712300\zeta(5u)+873067490900\zeta(4u)\\
&-910778521300\zeta(3u)-35421472\zeta(13u)\\
&+366101120\zeta(12u)+23!(2088816)\zeta(u)=0
\end{aligned} \tag{2.10}$$

$\forall\ u\in\mathscr{D}$. Refilling (u,v) by $(2u,u)$ in (1.1), and increasing the out coming equation by 1634380, and then subtracting the out coming equation from (2.10), we have

$$\begin{aligned}
&2169268\zeta(13u)-47397020\zeta(12u)\\
&+493909636\zeta(11u)-3265491240\zeta(10u)\\
&+15369828380\zeta(9u)-54762190030\zeta(8u)\\
&+153249928800\zeta(7u)-344616868000\zeta(6u)\\
&+630659845000\zeta(5u)-941775845500\zeta(4u)\\
&+1134044963000\zeta(3u)+23!(3723196)\zeta(u)\\
&-(23!+1053467503000)\zeta(2u)=0
\end{aligned} \tag{2.11}$$

$\forall\ u\in\mathscr{D}$. Refilling (u,v) by (u,u) in (1.1), and increasing the out coming equation by 2169268, and then subtracting the out coming equation from (2.11), we have

$$\begin{aligned}&2496144\zeta(12u)+(815929566100-23!)\zeta(2u)\\&+574113120\zeta(10u)-3789146592\zeta(9u)\\&+1768268410\zeta(8u)-61889394340\zeta(7u)\\&+167985498900\zeta(6u)-359968926200\zeta(5u)\\&+611947174600\zeta(4u)-815929566100\zeta(3u)\\&-54915168\zeta(11u)+23!(5892464)\zeta(u)=0 \quad (2.12)\end{aligned}$$

$\forall\, u\in\mathscr{D}$. Refilling (u,v) by $(0,u)$ in (1.1), and increasing the out coming equation by 2496144, and then subtracting the out coming equation from (2.12), we have $\zeta(2u)=2^{23}\zeta(u)$ for all $u\in\mathscr{D}$. Thus $\zeta:\mathscr{D}\to\mathscr{E}$ is a trevigintic mapping. □

3. Stability of functional equations (1.1) and (1.2)

We will prove the Generalized Hyers-Ulam-Rassias stability for the functional equations (1.1) and (1.2) in matrix normed space with the help of fixed point method.

For a mapping $\zeta: X\to Y$, define $\mathscr{H}\zeta: X^2\to Y$ and $\mathscr{H}\zeta_n: M_n(X)^2\to M_n(Y)$ by,

$$\begin{aligned}\mathscr{H}\zeta(c,d)=&+8855\zeta(c+8d)-23\zeta(c+11d)\\&+253\zeta(c+10d)-1771\zeta(c+9d)\\&-33649\zeta(c+7d)+100947\zeta(c+6d)\\&-245157\zeta(c+5d)+490314\zeta(c+4d)\\&-817190\zeta(c+3d)+1144066\zeta(c+2d)\\&-1352078\zeta(c+d)+1352078\zeta(c)\\&-1144066\zeta(c-d)+817190\zeta(c-2d)\\&-490314\zeta(c-3d)+245157\zeta(c-4d)\\&-100947\zeta(c-5d)+33649\zeta(c-6d)\\&-8855\zeta(c-7d)+1771\zeta(c-8d)\\&-253\zeta(c-9d)+23\zeta(c-10d)\\&-\zeta(c-11d)-23!\zeta(d)\zeta(c+12d)\end{aligned}$$

for all $c,d\in X$.

$$\begin{aligned}\mathscr{H}\zeta(x_{rs},y_{rs})=&\ \zeta([x_{rs}+12y_{rs}])-23\zeta([x_{rs}+11y_{rs}])\\&+253\zeta([x_{rs}+10y_{rs}])-1771\zeta([x_{rs}+9y_{rs}])\\&+8855\zeta([x_{rs}+8y_{rs}])-33649\zeta([x_{rs}+7y_{rs}])\\&+100947\zeta([x_{rs}+6y_{rs}])-245157\zeta([x_{rs}+5y_{rs}])\\&+490314\zeta([x_{rs}+4y_{rs}])-817190\zeta([x_{rs}+3y_{rs}])\\&+1144066\zeta([x_{rs}+2y_{rs}])-1352078\zeta([x_{rs}+y_{rs}])\\&+1352078\zeta([x_{rs}])-1144066\zeta([x_{rs}-y_{rs}])\\&+817190\zeta([x_{rs}-2y_{rs}])-490314\zeta([x_{rs}-3y_{rs}])\\&+245157\zeta([x_{rs}-4y_{rs}])-100947\zeta([x_{rs}-5y_{rs}])\\&+33649\zeta([x_{rs}-6y_{rs}])-8855\zeta([x_{rs}-7y_{rs}])\\&+1771\zeta([x_{rs}-8y_{rs}])-253\zeta([x_{rs}-9y_{rs}])\\&+23\zeta([x_{rs}-10y_{rs}])-\zeta([x_{rs}-11y_{rs}])-23!\zeta([y_{rs}])\end{aligned}$$

where $23!=25852016740000000000000$ and all $x=[x_{rs}],y=[y_{rs}]\in M_n(X)$. Similarly, we can define the another functional equation (1.2) in the above form.

Theorem 3.1. *Let $t=\pm1$ be fixed and $\sigma: X^2\to[0,\infty)$ be a function such that there exists a $\lambda<1$ with*

$$\sigma(c,d)\le 2^{23t}\lambda\sigma\left(\frac{c}{2^t},\frac{d}{2^t}\right)\qquad\forall\, c,d\in X. \tag{3.1}$$

Let $\zeta: X\to Y$ be a mapping satisfying

$$\|\mathscr{H}\zeta_n([x_{rs}],[y_{rs}])\|\le\sum_{r,s=1}^{n}\sigma(x_{rs},y_{rs}) \tag{3.2}$$

$\forall\ x=[x_{rs}],y=[y_{rs}]\in M_n(X)$. *Then there exists a unique trevigintic mapping $\mathbb{V}: X\to Y$ such that*

$$\|\zeta_n([x_{rs}])-\mathbb{V}_n([x_{rs}])\|_n\le\sum_{r,s=1}^{n}\frac{\lambda^{\frac{1-t}{2}}}{2^{23}(1-\lambda)}\sigma^*(x_{rs}) \tag{3.3}$$

$\forall\ x=[x_{rs}]\in M_n(X)$, *where*

$$\begin{aligned}\sigma^*(x_{rs})=&\frac{1}{23!}[\sigma(0,2x_{rs})+2496144\sigma(0,x_{rs})\\&+\sigma(12x_{rs},x_{rs})+23\sigma(11x_{rs},x_{rs})\\&+254\sigma(10x_{rs},x_{rs})+1794\sigma(9x_{rs},x_{rs})\\&+9108\sigma(8x_{rs},x_{rs})+35420\sigma(7x_{rs},x_{rs})\\&+109802\sigma(6x_{rs},x_{rs})+1062347\sigma(3x_{rs},x_{rs})\\&+278806\sigma(5x_{rs},x_{rs})+591261\sigma(4x_{rs},x_{rs})\\&+1634380\sigma(2x_{rs},x_{rs})+2169268\sigma(x_{rs},x_{rs})]\end{aligned}$$

Proof. Switching $n=1$ in (3.2), we get

$$\|\mathscr{H}\zeta(c,d)\|\le\sigma(c,d) \tag{3.4}$$

By utilizing Theorem 2.1, we can get

$$\begin{aligned}\left\|-\zeta(2c)+2^{23}\zeta(c)\right\|\le&\frac{1}{23!}[\sigma(0,2c)+\sigma(12c,c)\\&+23\sigma(11c,c)+254\sigma(10c,c)+591261\sigma(4c,c)\\&+1794\sigma(9c,c)+9108\sigma(8c,c)+35420\sigma(7c,c)\\&+109802\sigma(6c,c)+278806\sigma(5c,c)\\&+1062347\sigma(3c,c)+1634380\sigma(2c,c)\\&+2169268\sigma(c,c)+2496144\sigma(0,c)]\end{aligned}$$

Therefore,

$$\left\|\zeta(2c)-2^{23}\zeta(c)\right\|\le\sigma^*(c) \tag{3.5}$$

$\forall\ c\in X$. Hence

$$\left\|\zeta(c)-\frac{1}{2^{23t}}\zeta(2^tc)\right\|\le\frac{\lambda^{\left(\frac{1-t}{2}\right)}}{2^{23}}\sigma^*(c) \tag{3.6}$$

$\forall\ c\in X$. Taking $\mathscr{T}=\{\zeta: X\to Y\}$ and the generalized metric ρ on $\mathscr{T}$ as follows:

$$\rho(\zeta,\zeta_1)=\inf\left\{\tau\in\mathbb{R}_+:\|\zeta(c)-\zeta_1(c)\|\le\tau\sigma^*(c),\forall c\in X\right\},$$

It is easy to check that $(\mathscr{T}, \rho)$ is a complete generalized metric (see also [9]). Define the mapping $\mathscr{S}: \mathscr{T} \to \mathscr{T}$ by

$$\mathscr{S}\zeta(c) = \frac{1}{2^{23t}}\zeta(2^t c) \qquad \forall\, \zeta \in \mathscr{T} \text{ and } c \in X.$$

Letting $\zeta, \zeta_1 \in \mathscr{T}$ and ν be an arbitrary constant with $\rho(\zeta, \zeta_1) = \nu$. Then $\|\zeta(c) - \zeta_1(c)\| \le \nu\sigma^*(c)$ for all $c \in X$. Utilizing (3.1), we find that

$$\|\mathscr{S}\zeta(c) - \mathscr{S}\zeta_1(c)\| = \left\| \frac{1}{2^{23t}}\zeta(2^t c) - \frac{1}{2^{23t}}(2^t c)\right\| \le \lambda\nu\sigma^*(c)$$

for all $c \in X$.
Hence it holds that $\rho(\mathscr{S}\zeta, \mathscr{S}\zeta_1) \le \lambda\nu$, that is, $\rho(\mathscr{S}\zeta, \mathscr{S}\zeta_1) \le \lambda\rho(\zeta, \zeta_1)$ for all $\zeta, \zeta_1 \in \mathscr{T}$.

By (3.6), we have $\rho(\zeta, \mathscr{S}\zeta_1) \le \frac{\lambda^{\left(\frac{1-t}{2}\right)}}{2^{23}}$.
By Theorem 2.2 in [3], there exists a mapping $\mathbb{V}: X \to Y$ which satisfying:

1. $\mathbb{V}$ is a unique fixed point of $\mathscr{S}$, which is satisfied $\mathbb{V}(2^t c) = 2^{23t}\mathbb{V}(c)\ \forall\, c \in X$.
2. $\rho(\mathscr{S}^k\zeta, \mathbb{V}) \to 0$ as $k \to \infty$. This implies that $\lim_{k\to\infty} \frac{1}{2^{23kt}}\zeta(2^{kt}c) = \mathbb{V}(c)\ \forall\, c \in X$.
3. $\rho(\zeta, \mathbb{V}) \le \frac{1}{1-\lambda}\rho(\zeta, \mathscr{S}\zeta) \Rightarrow$

$$\|\zeta(c) - \mathbb{V}(c)\| \le \frac{\lambda^{\frac{1-t}{2}}}{2^{23}(1-\lambda)}\sigma^*(c) \quad \forall\, c \in X. \tag{3.7}$$

It follows from (3.1) and (3.2) that

$$\begin{aligned}\|\mathscr{H}\mathbb{V}(c,d)\| &= \lim_{k\to\infty}\frac{1}{2^{23kt}}\left\|\mathscr{H}\zeta(2^{kt}c, 2^{kt}d)\right\| \\ &\le \lim_{k\to\infty}\frac{1}{2^{23kt}}\sigma(2^{kt}c, 2^{kt}d) \\ &\le \lim_{k\to\infty}\frac{2^{kt}\lambda^t}{2^{23kt}}\sigma(c,d) = 0\end{aligned}$$

for all $c, d \in X$. Therefore, the mapping $\mathbb{V}: X \to Y$ is treviginticmapping. By Lemma 2.1 in [7] and (3.7), we can get (3.3) Hence $\mathbb{V}: X \to Y$ is a unique trevigintic mapping satisfying (3.3). □

Theorem 3.2. *Let $t = \pm 1$ be fixed and $\sigma: X^2 \to [0, \infty)$ be a function such that there exists a $\lambda < 1$ with*

$$\sigma(c,d) \le 2^{24t}\lambda\sigma\left(\frac{c}{2^t}, \frac{d}{2^t}\right) \tag{3.8}$$

$\forall\, c, d \in X$. Let $\zeta: X \to Y$ be a mapping satisfying

$$\|\mathscr{H}\zeta_n([x_{rs}], [y_{rs}])\| \le \sum_{r,s=1}^{n}\sigma(x_{rs}, y_{rs}) \tag{3.9}$$

$\forall\, x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there exists a unique quattuorvigintic mapping $\mathbb{Q}: X \to Y$ such that

$$\|\zeta_n([x_{rs}]) - \mathbb{Q}_n([x_{rs}])\|_n \le \sum_{r,s=1}^{n}\frac{\lambda^{\frac{1-t}{2}}}{2^{24}(1-\lambda)}\sigma^*(x_{rs}) \tag{3.10}$$

*$\forall\, x = [x_{rs}] \in M_n(X)$,
where*

$$\begin{aligned}\sigma^*(x_{rs}) = \frac{2}{24!}[&\frac{1}{2}\sigma(0, 2x_{rs}) + \sigma(12x_{rs}, x_{rs}) \\ &+ 24\sigma(11x_{rs}, x_{rs}) + 276\sigma(10x_{rs}, x_{rs}) \\ &+ 2024\sigma(9x_{rs}, x_{rs}) + 10626\sigma(8x_{rs}, x_{rs}) \\ &+ 42504\sigma(7x_{rs}, x_{rs}) + 134596\sigma(6x_{rs}, x_{rs}) \\ &+ 346104\sigma(5x_{rs}, x_{rs}) + 735471\sigma(4x_{rs}, x_{rs}) \\ &+ 1307504\sigma(3x_{rs}, x_{rs}) + 1961256\sigma(2x_{rs}, x_{rs}) \\ &+ 2496144\sigma(x_{rs}, x_{rs}) + 1352078\sigma(0, x_{rs})]\end{aligned}$$

Proof. The proof is similar to the proof of Theorem 3.1. □

The following corollary gives the Hyers-Ulam-Rassias stability for the trevigintic and quattuorvigintic functional equations (1.1), (1.2) respectively. This stability involving the sum of powers of norms.

Corollary 3.3. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l \neq 23$. Let $\zeta: X \to Y$ be a mapping such that*

$$\|\mathscr{H}\zeta_n([x_{rs}], [y_{rs}])\|_n \le \sum_{r,s=1}^{n}\omega(\|x_{rs}\|^l + \|y_{rs}\|^l) \tag{3.11}$$

$\forall\, x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there exists a unique trevigintic mapping $\mathbb{V}: X \to Y$ such that

$$\|\zeta_n([x_{rs}]) - \mathbb{V}_n([x_{rs}])\|_n \le \sum_{r,s=1}^{n}\frac{\omega_0}{|2^{23} - 2^l|}\|x_{rs}\|^l$$

$\forall\, x = [x_{rs}] \in M_n(X)$, where

$$\begin{aligned}\omega_0 = \frac{\omega}{23!}\Big[&10557876 + 1634381(2^l) \\ &+ 1062347(3^l) + 591261(4^l) + 278806(5^l) \\ &+ 35420(7^l) + 9108(8^l) + 1794(9^l) \\ &+ 254(10^l) + 23(11^l) + (12^l) + 109802(6^l)\Big]\end{aligned}$$

Proof. The proof is identical to the proof of Theorem 3.1 by taking $\sigma(c,d) = \omega(\|c\|^l + \|d\|^l)$ for all $c, d \in X$. Then we can choose $\lambda = 2^{t(l-23)}$, and we can obtain the required result. □

Corollary 3.4. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l \neq 24$. Let $\zeta: X \to Y$ be a mapping satisfying (3.11). Then there exists a unique quattuorvigintic mapping $\mathbb{Q}: X \to Y$ such that*

$$\|\zeta_n([x_{rs}]) - \mathbb{Q}_n([x_{rs}])\|_n \le \sum_{r,s=1}^{n}\frac{\omega_0}{|2^{24} - 2^l|}\|x_{rs}\|^l$$

$\forall\, x = [x_{rs}] \in M_n(X)$, where

$$\begin{aligned}\omega_0 = \frac{2\omega}{24!}[&10884752 + 196125.5(2^l) + 1307504(3^l) \\ &+ 735471(4^l) + 346104(5^l) + 134596(6^l) \\ &+ 42504(7^l) + 10626(8^l) + 2024(9^l) \\ &+ 276(10^l) + 24(11^l) + (12^l)]\end{aligned}$$

Proof. The proof is identical to the proof of Theorem 3.2. □

The following corollary gives the Ulam-Gavruta-Rassias stability for the trevigintic and quattuorvigintic functional equations (1.1), (1.2) respectively. This stability involving the product of powers of norms.

Corollary 3.5. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l = a+b \neq 23$. Let $\zeta : X \to Y$ be a mapping such that*

$$\|\mathscr{H}\zeta_n([x_{rs}],[y_{rs}])\|_n \leq \sum_{r,s=1}^{n} \omega(\|x_{rs}\|^a \cdot \|y_{rs}\|^b) \quad (3.12)$$

$\forall\ x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. *Then there exists a unique trevigintic mapping $\mathbb{V} : X \to Y$ such that*

$$\|\zeta_n([x_{rs}]) - \mathbb{V}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{23}-2^l|} \|x_{rs}\|^l$$

$\forall\ x = [x_{rs}] \in M_n(X)$, *where*

$$\omega_0 = \frac{\omega}{23!}[2169268 + 1634380(2^a) + 1062347(3^a) + 591261(4^a) + 278806(5^a) + 109802(6^a) + 35420(7^a) + 9108(8^a) + 1794(9^a) + 254(10^a) + 23(11^a) + (12^a)]$$

Proof. The proof is resembling to the proof of Theorem 3.1. □

Corollary 3.6. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l = a+b \neq 24$. Let $\zeta : X \to Y$ be a mapping satisfies (3.12). Then there exists a unique quattuorvigintic mapping $\mathbb{Q} : X \to Y$ such that*

$$\|\zeta_n([x_{rs}]) - \mathbb{Q}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{24}-2^l|} \|x_{rs}\|^l \quad \forall\ x = [x_{rs}] \in M_n(X),$$

$\forall\ x = [x_{rs}] \in M_n(X)$, *where*

$$\omega_0 = \frac{2\omega}{24!}[2496144 + 1961256(2^a) + 1307504(3^a) + 735471(4^a) + 346104(5^a) + 134596(6^a) + 42504(7^a) + 10626(8^a) + 2024(9^a) + 276(10^a) + 24(11^a) + (12^a)]$$

Proof. The proof is resembling to the proof of Theorem 3.2. □

The following corollary gives the Ulam J Rassias stability for the trevigintic and quattuorvigintic functional equations (1.1), (1.2) respectively. This stability involving the mixed product of sum of powers of norms.

Corollary 3.7. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l = a+b \neq 23$. Let $\zeta : X \to Y$ be a mapping such that*
$\|\mathscr{H}\zeta_n([x_{rs}],[y_{rs}])\|_n \leq$

$$\sum_{r,s=1}^{n} \omega(\|x_{rs}\|^a \cdot \|y_{ij}\|^b + \|x_{rs}\|^{a+b} + \|y_{rs}\|^{a+b}) \quad (3.13)$$

$\forall\ x = [x_{rs}], y = [y_{rs}] \in M_n(X)$.*Then there exists a unique trevigintic mapping $\mathbb{V} : X \to Y$ such that*

$$\|\zeta_n([x_{rs}]) - \mathbb{V}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{23}-2^l|} \|x_{rs}\|^l$$

$\forall\ x = [x_{rs}] \in M_n(X)$, *where*

$$\omega_0 = \frac{\omega}{23!}\Big[12727144 + 1634380(2^a) + 1634381(2^l) + 1062347(3^l + 3^a) + 591261(4^l + 4^a) + 278806(5^l + 5^a) + 109802(6^l + 6^a) + 35420(7^l + 7^a) + 9108(8^l + 8^a) + 1794(9^l + 9^a) + (12^l + 12^a) + 254(10^l + 10^a) + 23(11^l + 11^a)\Big]$$

Proof. The proof is identical to the proof of Theorem 3.1. □

Corollary 3.8. *Let $t = \pm 1$ be fixed and let l, ω be non-negative real numbers with $l = a+b \neq 24$. Let $\zeta : X \to Y$ be a mapping satisfying (3.13). Then there exists a unique quattuorvgintic mapping $\mathbb{Q} : X \to Y$ such that*

$$\|\zeta_n([x_{rs}]) - \mathbb{Q}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{24}-2^l|} \|x_{rs}\|^l$$

$\forall\ x = [x_{rs}] \in M_n(X)$, *where*

$$\omega_0 = \frac{2\omega}{24!}[13380896 + 1961256(2^a) + 1961256.5(2^l) + 1307504(3^l + 3^a) + 346104(5^l + 5^a) + 134596(6^l + 6^a) + 42504(7^l + 7^a) + 10626(8^l + 8^a) + 2024(9^l + 9^a) + 276(10^l + 10^a) + (12^l + 12^a) + 735471(4^l + 4^a) + 24(11^l + 11^a)]$$

Proof. The proof is identical to the proof of Theorem 3.2. □

Now we will provide an example to illustrate that the functional equations (1.1) is not stable for $t = 23$ in corollary 3.3.

Example 3.9. *Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a function defined by*

$$\sigma(x) = \begin{cases} \omega_0 x^{23}, & if\ |x| < 1 \\ \omega_0, & otherwise \end{cases}$$

where $\omega_0 > 0$ *is a constant, and define a function* $\zeta : \mathbb{R} \to \mathbb{R}$ *by* $\zeta(x) = \sum_{n=0}^{\infty} \frac{\sigma(2^n x)}{2^{23n}}$*for all* $x \in \mathbb{R}$. *Then* ζ *satisfies the inequality*

$$|\mathscr{M}\zeta(x,y)| \leq \frac{258520167388849766 4000}{8388607}(8388608)^2\varepsilon(|x|^{23}+|x|^{23}) \tag{3.14}$$

for all $x,y \in \mathbb{R}$. *Then there does not exist a trevigintic mapping* $\mathbb{V} : \mathbb{R} \to \mathbb{R}$ *and a constant* $\lambda > 0$ *such that*

$$|\zeta(x) - \mathbb{V}(x)| \leq \lambda\, |x|^{23} \qquad \forall\, x \in \mathbb{R}. \tag{3.15}$$

Proof. It is easy to see that h is bounded by $\dfrac{8388608\varepsilon}{8388607}$ on $\mathbb{R}$. If $|x|^{23}+|c|^{23} = 0$, then (3.14) is trivial. If $|x|^{23}+|c|^{23} \geq \frac{1}{2^{23}}$, then L.H.S of (3.14) is $< \dfrac{2585201673888497664000}{8388607}(8388608)^2$.
Suppose that $0 < |x|^{23}+|x|^{23} < \frac{1}{2^{23}}$, then there exists a non-negative integer k such that

$$\frac{1}{2^{23(k+1)}} \leq |b|^{23}+|c|^{23} < \frac{1}{2^{23k}}, \tag{3.16}$$

so that $2^{23(k-1)}|b|^{23} < \frac{1}{2^{23}}, 2^{23(k-1)}|c|^{23} < \frac{1}{2^{23}}$, and $2^n(y), 2^n(x\pm 7y), 2^n(x\pm 6y), 2^n(x\pm 5y), 2^n(x\pm 4y)$, $2^n(x\pm 3y), 2^n(x\pm 8y), 2^n(x\pm 2y), 2^n(x\pm 9y)$, $2^n(x\pm 10y), 2^n(x\pm 11y), 2^n(x\pm 12y), 2^n(x) \in (-1,1)$ for all $n = 0,1,2,...,k-1$. Hence $\mathscr{M}\zeta(2^n x, 2^n y) = 0$ for $n = 0,1,2,...,k-1$. From the definition of ζ and (3.16), we obtain that

$$|\mathscr{M}\zeta(x,y)| \leq \sum_{n=0}^{\infty} \frac{1}{2^{23n}}\left[\mathscr{M}\zeta(2^n x, 2^n y)\right]$$
$$\leq \frac{2585201673888497664000}{8388607}(8388608)^2\varepsilon(|x|^{23}+|x|^{23}).$$

Therefore, ζ satisfies (3.14) for all $x,y \in \mathbb{R}$. Suppose on the contrary that there exists a trevigintic mapping $\mathscr{V} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ satisfying (3.15). Then there exists a constant $c \in \mathbb{R}$ such that $\mathscr{V}(x) = cx^{23}$ for any $x \in \mathbb{R}$. Thus we obtain the following inequality $|h(b)| \leq (\lambda + |c|)\,|b|^{23}$. Let $m \in \mathbb{N}$ with $m\omega_0 > \lambda + |c|$. If $b \in (0, \frac{1}{2^{m-1}})$, then $2^n x \in (0,1)$ for all $n = 0,1,2,...,m-1$, and for this case we get $\zeta(x) = \sum_{n=0}^{\infty} \dfrac{\psi(2^n x)}{2^{23n}} \geq \sum_{n=0}^{m-1} \dfrac{\omega_0 (2^n x)^{23}}{2^{23n}} > (\lambda + |c|)\,|x|^{23}$ which is a contradiction to above inequality $|h(b)| \leq (\lambda + |c|)\,|b|^{23}$. Therefore the trevigintic functional equation (1.1) is not stable for $t = 23$. □

4. Conclusion

In this investigation, we identified a general solution of trevigintic functional equation (1.1) and we establised the generalized Hyers-Ulam-Rassias stability, Hyers-Ulam-Rassias stability, Ulam-Gavruta-Rassias stability and J. M. Rassias stability of these functional equations (1.1) and (1.2)in matrix normed spaces with the help of fixed point method and also provided the example for nonstability.

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