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Edge vertex prime labeling of some graphs

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Abstract

Let $G=(V(G),E(G))$ be a graph with p vertices and q edges. A bijective function $f:V(G)\bigcup E(G)\rightarrow\{1,2,3,...,p+q\}$ is said to be an *edge vertex prime labeling*, if for any edge $xy\in E(G)$, it is satisfies that $f(x),f(y)$ and $f(xy)$ are pairwise relatively prime. In this paper, we investigate several families of edge vertex prime labeling for triangular and rectangular book, Butterfly graph, Drums graph D_n, Jahangir graph $J_{n,3}$ and $J_{n,4}$.

Keywords

Prime labeling, edge vertex prime labeling, relatively prime, triangular and rectangular book, butterfly graph.

AMS Subject Classification

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1. Introduction

We begin with finite, connected and undirected graph $G=(V(G),E(G))$ without loops and multiple edges. Here elements of $V(G)$ and $E(G)$ are known as vertex set and edge set, respectively. For all other terminology and notations in graph theory we follow Balakrishnan and Ranganathan [1]. The concept of an edge vertex prime labeling of a graph is a variation of a prime labeling, which was developed by Roger Entriger and first introduced in [7] by Tout, Dabboucy and Howalla. For a simple graph G with n vertices in the vertex set $V(G)$, a *prime labeling* is an assignment of the integers 1 to n as labels of the vertices such that each pair of labels from adjacent vertices is relatively prime. A graph that has such a labeling is called *prime*. Gallian's dynamic graph labeling survey [2] contains a detailed list of graphs that have been proven to be prime. Recently, much focus has been on variations of prime labeling, such as an *edge vertex prime labeling*, which involves for any edge $e=xy$, they are pairwise relatively prime. We will give brief summary of definitions which are useful for the present investigations. Let $G=(V(G),E(G))$ be a graph with p vertices and q edges. A bijective function $f:V(G)\bigcup E(G)\rightarrow\{1,2,3,...,p+q\}$ is called an *edge vertex prime labeling* if for each edge $e=xy$, $gcd(f(x),f(y))=1$, $gcd(f(x),f(xy))=1$, $gcd(f(y),f(xy))=1$. A graph which admits an edge vertex prime labeling is called an *edge vertex prime graph.* Jagadesh and Baskar Babujee [3] was introduced the concept of an edge vertex prime labeling and they proved the existence of the same paths, cycles and star $K_{1,n}$. In [4], if $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ are two connected graphs, then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1\hat{O}G_2$. The resultant graph $G=G_1\hat{O}G_2$ contains p_1+p_2-1 vertices q_1+q_2 edges. In general, there are p_1p_2 possibilities of getting $G_1\hat{O}G_2$ from G_1 and G_2. In [4], they also proved that an edge vertex prime labeling, for some class of graphs such as generalized star, generalized cycle star, $p+q$ is odd for $G\hat{O}P_n$, $G\hat{O}C_n$, $G\hat{O}K_{1,n}$. Parmer [5] proved that wheel W_n, fan f_n, friendship graph F_n are edge vertex prime labeling. In [6], they also proved that $K_{2,n}$, for all n and $K_{3,n}$ for $n=\{2,3,...,29\}$ are edge vertex prime labeling. The *triangular book* with n pages is defined as n copies of cycles C_3 sharing a common edge. The common edge is called the *spine or base of the book.* This graph is denoted by $B_{3,n}$. The *rectangular book* with n pages is defined as n copies of cycles C_4 sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B_{4,n}$. A *shell* S_n is the graph obtained by taking $(n-3)$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the *apex*. The shell is also called fan f_{n-1}. Parmer [5] proved that the fan graph f_n is an edge vertex prime graph. In this case, we conclude that the shell S_n that is, f_{n-1} is an edge vertex prime graph. A *bow* *graph* is a double shell in which each shell has any order. A *butterfly graph* is a bow graph with exactly two pendant edges at the apex. A *multiple shell* is a collection of edge disjoint shells that have their apex in common. Hence a double shell consists of two edge disjoint shells with a common apex.

The *Drums graph* D_n, $n \geq 3$ can be constructed by two cycles $2C_n$, $n \geq 3$ joining two paths $2P_n$, $n \geq 3$ with sharing a common vertex and it is denoted by $D_n = 2C_n + 2P_n$.

In this paper, we investigate triangular and rectangular book, Butterfly graph, multiple shell, Drums graph D_n, Jahangir graph $J_{n,3}$, $J_{n,4}$ and that some classes of graphs are edge vertex prime labeling.

2. Main Results

Theorem 2.1. *The triangular book $B_{3,n}$ admits edge vertex prime labeling for all n.*

Proof. Let $B_{3,n}$ be a triangular book. Then $V(B_{3,n}) = \{x, y, z_i : 1 \leq i \leq n\}$ and $E(B_{3,n}) = \{xy, xz_i, yz_i : 1 \leq i \leq n\}$. Also, $|V(B_{3,n})| = n+2$ and $|E(B_{3,n})| = 2n+1$. Define a bijective function $f : V(B_{3,n}) \bigcup E(B_{3,n}) \to \{1,2,...,3n+3\}$ by $f(x) = 1$, $f(xy) = 2$, $f(y) = 3$, for each $1 \leq i \leq \lceil \frac{n}{2} \rceil$,
$f(z_{2i+1}) = 6i-1$, $f(z_{2i}) = 6i+1$, $f(xz_{2i-1}) = 6i$, $f(xz_{2i}) = 6i+3$, $f(yz_{2i-1}) = 6i-2$, $f(yz_{2i}) = 6i+2$, Clearly, (i) $f(x), f(y)$ and $f(xy)$, (ii) $f(x), f(z_i)$ and $f(xz_i)$, (iii) $f(y), f(z_i)$ and $f(yz_i)$ are pairwise relatively prime. Therefore, for any edge $xy \in E(B_{3,n})$, the numbers $f(x), f(y)$ and $f(xy)$ are pairwise relatively prime. Hence the triangular book $B_{3,n}$ is an edge vertex prime labeling for all n. □

Theorem 2.2. *The rectangular book $B_{4,n}$ admits edge vertex prime labeling for all n.*

Proof. Let $B_{4,n}$ be a rectangular book. Then $V(B_{4,n}) = \{x, x_i, y, y_i : 1 \leq i \leq n\}$ and $E(B_{4,n}) = \{xx_i, yy_i, x_iy_i : 1 \leq i \leq n\} \bigcup \{xy\}$. Also, $|V(B_{4,n})| = 2n+2$ and $|E(B_{4,n})| = 3n+1$. Define a bijective function $f : V(B_{4,n}) \bigcup E(B_{4,n}) \to \{1,2,...,5n+3\}$ by $f(x) = 1$, $f(y) = 5$, $f(xy) = 4$, $f(xx_1) = 2$, $f(x_1) = 3$, $f(yy_1) = 6$, $f(y_1) = 7$, $f(x_1y_1) = 8$. For each $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $f(x_{2i}) = 10i-1$, $f(y_{2i}) = 10i+3$, $f(x_{2i}y_{2i}) = 10i+1$, $f(xx_{2i}) = 10i$, $f(yy_{2i}) = 10i+2$. For each $1 \leq i \leq \lceil \frac{n-2}{2} \rceil$, $f(x_{2i+1}) = 10i+5$, $f(y_{2i+1}) = 10i+7$, $f(x_{2i+1}y_{2i+1}) = 10i+6$, $f(xx_{2i+1}) = 10i+4$, $f(yy_{2i+1}) = 10i+8$. Clearly, (i) $f(x)$, $f(y)$ and $f(xy)$, (ii) $f(x), f(x_i)$ and $f(xx_i)$, (iii) $f(y), f(y_i)$ and $f(yy_i)$, (iv) $f(x_i), f(y_i)$ and $f(x_iy_i)$ are pairwise relatively prime. Therefore, for any edge $xy \in E(B_{4,n})$, the numbers $f(x), f(y)$ and $f(xy)$ are pairwise relatively prime. Hence the rectangular book $B_{4,n}$ is an edge vertex prime labeling. □

Theorem 2.3. *The butterfly graph with shell is an edge vertex prime labeling.*

Proof. Let G be a butterfly graph with shells of order m and n excluding the apex. Without loss of generality, assume that $m \leq n$. Then $V(G) = \{w_0, w_1, w_2, u_i : 1 \leq i \leq m, v_j : m+1 \leq j \leq m+n\}$ and $E(G) = \{w_0w_1, w_0w_2, w_0u_i : 1 \leq i \leq m, u_iu_{i+1} : 1 \leq i \leq m-1, w_0v_j : m+1 \leq j \leq m+n, v_jv_{j+1} : m+1 \leq j \leq m+n-1\}$. Also, $|V(G)| = m+n+3$ and $|E(G)| = 2m+2n$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,3m+3n+3\}$ by $f(w_0) = 1$,

$$f(u_i) = \begin{cases} 3i : & i \text{ is odd} \\ 3i-1 : & i \text{ is even} \end{cases}$$

$$f(u_iu_{i+1}) = 3i+1, i = 1,2,3,...,m-1.$$

Consider the following cases.

Case 1a. m is even.

$$f(wu_i) = \begin{cases} 3i-1 : & i = 1,3,...,m-1 \\ 3i : & i = 2,4,...,m \end{cases}$$

Case 1b. m is odd.

$$f(wu_i) = \begin{cases} 3i-1 : & i = 1,3,5,...,m \\ 3i : & i = 2,4,6,...,m-1 \end{cases}$$

Case 2a. Both m, n is even.

$$f(v_j) = \begin{cases} 3j-2 : & j = m+1, m+3, ..., m+n-1 \\ 3j-1 : & j = m+2, m+4, ..., m+n \end{cases}$$

Case 2b. Both m, n is odd.

$$f(v_j) = \begin{cases} 3j-2 : & j = m+1, m+3, ..., m+n-1 \\ 3j-1 : & j = m+2, m+4, ..., m+n \end{cases}$$

Case 2c. m is even and n is odd.

$$f(v_j) = \begin{cases} 3j+1 : & j = m+1, m+3, ..., m+n-1 \\ 3j+2 : & j = m+2, m+4, ..., m+n-2 \end{cases}$$

Case 3a. Both m, n is even and both m, n is odd.

$$f(wv_j) = \begin{cases} 3j-1 : & j = m+1, m+3, ..., m+n-1 \\ 3j-2 : & j = m+2, m+4, ..., m+n \end{cases}$$

Case 3b. m is odd and n is even or m is even and n is odd.

$$f(wv_j) = \begin{cases} 3j+2 : & j = m+1, m+3, ..., m+n-1 \\ 3j+1 : & j = m+2, m+4, ..., m+n \end{cases}$$

Case 4a. $3m+3n-1$ is even.
$f(w_1) = 3m+3n$, $f(ww_1) = 3m+3n+1$,
$f(w_2) = 3m+3n+2$, $f(ww_2) = 3m+3n+3$.

Case 4b. $3m+3n-1$ is odd.
$f(w_1) = 3m+3n+1$, $f(ww_1) = 3m+3n$,
$f(w_2) = 3m+3n+3$, $f(ww_2) = 3m+3n+2$.
Clearly, (i) $f(w), f(u_i)$ and $f(wu_i)$, (ii) $f(u_i), f(u_{i+1})$ and $f(u_iu_{i+1})$, (iii) $f(w), f(v_i)$ and $f(wv_i)$, (iv) $f(v_i), f(v_{i+1})$ and $f(v_iv_{i+1})$, (v) $f(w), f(w_1)$ and $f(ww_1)$, (vi) $f(w), f(w_2)$ and $f(ww_2)$ are pairwise relatively prime. Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the butterfly graph with shell is an edge vertex prime labeling. □

Corollary 2.4. *A multiple shell is an edge vertex prime graph.*

Theorem 2.5. *The Drums graph D_n, $n \geq 3$ is an edge vertex prime labeling.*

Proof. Let D_n be the Drums graph. Then $V(D_n) = \{u_i : 1 \leq i \leq 4n-3\}$ and $E(D_n) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_1u_n\} \bigcup \{u_1u_{n+1}\} \bigcup \{u_iu_{i+1} : n+1 \leq i \leq 2n-2\} \bigcup \{u_1u_{2n}\} \bigcup \{u_iu_{i+1} : 2n \leq i \leq 3n-3\} \bigcup \{u_1u_{3n}\} \bigcup \{u_iu_{i+1} : 3n-1 \leq i \leq 4n-2\}$. Also, $|V(D_n)| = 4n-3$ and $|E(D_n)| = 4n-2$. Define a bijective function $f : V(D_n) \bigcup E(D_n) \to \{1,2,...,8n-5\}$ by $f(u_i) = 2i-1$ for $1 \leq i \leq 4n-3$, $f(u_iu_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(u_1u_n) = 2n$, $f(u_iu_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-2$, $f(u_1u_{n+1}) = 4n-2$, $f(u_iu_{i+1}) = 2i$ for $2n \leq i \leq 3n-3$, $f(u_iu_{i+1}) = 2i$ for $3n-1 \leq i \leq 4n-4$. Consider the following cases.
Case (i). When $n \equiv 0,2 \pmod 3$.
$f(u_1u_{2n}) = 8n-5$, $f(u_1u_{4n-3}) = 8n-6$.
Case (ii). When $n \equiv 1 \pmod 3$.
$f(u_1u_{2n}) = 8n-6$, $f(u_1u_{4n-3}) = 8n-5$.
For any edge $u_iu_{i+1} \in E(D_n)$, $gcd(f(u_i), f(u_{i+1})) = gcd(2i-1, 2i+1) = 1$, $gcd(f(u_i), f(u_iu_{i+1})) = gcd(2i-1, 2i) = 1$, $gcd(f(u_{i+1}), f(u_iu_{i+1})) = gcd(2i+1, 2i) = 1$, Clearly, for any edge $uv \in E(D_n)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the Drums graph D_n, $n \geq 3$ admits edge vertex prime labeling. □

Theorem 2.6. *The following graphs are edge vertex prime labeling.*
(a) $\overline{K_n} \bigcup K_{1,m}$ for all $m,n \geq 1$, (b) $K_{1,m} + K_1$ for all $m \geq 1$,
(c) $\overline{K_m} \bigcup \overline{K_n}$ for all $m,n \geq 1$.

Proof. (a) Without loss of generality, we assume that $m \leq n$. Now, let the vertex set of $K_{1,m}$ be $L = \{u\}$ and $M = \{u_i : 1 \leq i \leq m\}$ and edge set $N = \{uu_i : 1 \leq i \leq m\}$ and let the vertex set of $\overline{K_n}$ be $O = \{v_j : 1 \leq i \leq n\}$. Then label the vertices and edges of the sets L, M and N as $f(u) = 1$ and $f(u_i) = 2i+1$ for $1 \leq i \leq m$, $f(uu_i) = 2i$ for $1 \leq i \leq m$, and label the vertex set O by remaining labels are $\{2m+2, 2m+3, ..., 2m+n, 2m+n+1\}$. Hence, we can easily verify that the considered graph is an edge vertex prime labeling.
(b) Let $G = K_{1,m} + K_1$ be a graph. Then $V(G) = \{u, v, v_i : 1 \leq i \leq m\}$ and $E(G) = \{uu_i, vu_i : 1 \leq i \leq m\}$. Also, $|V(G)| = m+2$ and $|E(G)| = 2m$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,3m+2\}$ by $f(u) = 1$, $f(u_i) = 2i+1$ for $1 \leq i \leq m$, $f(uu_i) = 2i$ for $1 \leq i \leq m$, $f(v) = p$, where p is choose the greatest prime number in the set $\{2m+2, 2m+3, ..., 3m+2\}$ and label the edge set $\{vv_i : 1 \leq i \leq m\}$ by remaining labels. Hence, we can easily verify that the considered graph is an edge vertex prime labeling.
(c) Let $G = \overline{K_m} \bigcup \overline{K_n}$ for all $m,n \geq 1$. Without loss of generality, assume that $m \leq n$. Now, we label the vertices of $\overline{K_m}$ as $1,2,3,...,m$ and label the vertices of $\overline{K_n}$ as $m+1, m+2, ..., m+n$. Hence, we can easily verify that the considered graph is an edge vertex prime labeling. □

Theorem 2.7. *The graph G obtained by joining the path P_2 of two copies of cycle C_n is an edge vertex prime labeling, where $n \equiv 0,2 \pmod 2$.*

Proof. Let $\{v_1, v_2, ..., v_n\}$, and $\{v_{n+1}, v_{n+2}, , ..., v_{2n}\}$ are the vertices of first and second cycle C_n, respectively. Then $V(G) = \{v_i : 1 \leq i \leq 2n\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_1v_n\} \bigcup \{v_iv_{i+1} : n+1 \leq i \leq 2n-1\} \bigcup \{v_{n+1}v_{2n}\} \bigcup \{v_1v_{n+1}\}$. Also, $|V(G)| = 2n$ and $|E(G)| = 2n+1$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,4n+1\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 2n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(v_1v_n) = 2n$, $f(v_iv_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-1$, $f(v_{n+1}v_{2n}) = 4n$, $f(v_1v_{n+1}) = 4n+1$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence joining the path P_2 of two copies of cycle C_n admits edge vertex prime labeling. □

Theorem 2.8. *The graph G obtained by joining the path P_3 of three copies of cycle C_n is an edge vertex prime labeling, where $n \equiv 0,2 \pmod 3$.*

Proof. Let $\{v_1, v_2, ..., v_n\}$, $\{v_{n+1}, v_{n+2}, , ..., v_{2n}\}$ and $\{v_{2n+1}, v_{2n+2}, ..., v_{3n}\}$ are the vertices of first, second, and third cycle C_n, respectively. Then $V(G) = \{v_i : 1 \leq i \leq 3n\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_1v_n\} \bigcup \{v_iv_{i+1} : n+1 \leq i \leq 2n-1\} \bigcup \{v_{n+1}v_{2n}\} \bigcup \{v_iv_{i+1} : 2n+1 \leq i \leq 3n-1\} \bigcup \{v_{2n+1}v_{3n}\} \bigcup \{v_1v_{n+1}\} \bigcup \{v_{n+1}v_{2n+1}\}$. Also, $|V(G)| = 3n$ and $|E(G)| = 3n+2$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,..., 6n+2\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 3n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(v_1v_n) = 2n$, $f(v_iv_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-1$, $f(v_{n+1}v_{2n}) = 4n$, $f(v_iv_{i+1}) = 2i$ for $2n+1 \leq i \leq 3n-1$, $f(v_{2n+1}v_{n+1}) = 6n+1$, $f(v_{n+1}v_1) = 6n+2$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence joining the path P_3 of three copies of cycle C_n admits edge vertex prime labeling. □

Theorem 2.9. *The graph G obtained by joining the path P_4 of four copies of cycle C_n is an edge vertex prime labeling, where $n \equiv 0,2 \pmod 3$.*

Proof. Let $\{v_1, v_2, ..., v_n\}$, $\{v_{n+1}, v_{n+2}, , ..., v_{2n}\}$, $\{v_{2n+1}, v_{2n+2}, ..., v_{3n}\}$, and $\{v_{3n+1}, v_{3n+2}, ..., v_{4n}\}$ are the vertices of first, second, third and fourth cycle C_n, respectively. Then $V(G) = \{v_i : 1 \leq i \leq 4n\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_1v_n\} \bigcup \{v_iv_{i+1} : n+1 \leq i \leq 2n-1\} \bigcup \{v_{n+1}v_{2n}\} \bigcup \{v_iv_{i+1} : 2n+1 \leq i \leq 3n-1\} \bigcup \{v_{2n+1}v_{3n}\} \bigcup \{v_iv_{i+1} : 3n+1 \leq i \leq 4n-1\} \bigcup \{v_{3n+1}v_{4n}\} \bigcup \{v_1v_{n+1}\} \bigcup \{v_{n+1}v_{2n+1}\} \bigcup \{v_{2n+1}v_{3n+1}\}$. Also, $|V(G)| = 4n$ and $|E(G)| = 4n+3$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,8n+3\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 4n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(v_1v_n) = 2n$, $f(v_iv_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-1$, $f(v_{n+1}v_{2n}) = 4n$, $f(v_iv_{i+1}) = 2i$ for $2n+1 \leq i \leq 3n-1$, $f(v_{2n+1}v_{3n}) = 6n$, $f(v_iv_{i+1}) = 2i$ for $3n+1 \leq i \leq 4n-1$, $f(v_{3n+1}v_{4n}) = 8n$, $f(v_{3n+1}v_{2n+1}) = 8n+1$, $f(v_{2n+1}v_{n+1}) = 8n+3$, $f(v_1v_{n+1}) = 8n+2$, Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence joining the path P_4 of four copies of cycle C_n admits edge vertex prime labeling. □

Theorem 2.10. *The graph $J_{n,3}$, $n \geq 1$ is an edge vertex prime labeling.*

Proof. Let G be a $J_{n,3}$ graph. Then $V(G) = \{u, v_i : 1 \leq i \leq 3n\}$ and $E(G) = \{v_iv_{i+1}, v_1v_{3n} : 1 \leq i \leq 3n-1\} \bigcup \{uv_1, uv_{n+1}, uv_{2n+1}\}$. Also, $|V(G)| = 3n+1$ and $|E(G)| = 3n+3$. Define a bijective function $f : V(G) \bigcup E(G) \rightarrow \{1, 2, ..., 6n+4\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 3n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq 3n-1$, $f(v_1v_{3n}) = 6n, f(u) = 6n+1$.
Consider the following cases.
Case (i). When $n \equiv 0, 1 \pmod 3$.
$f(uv_1) = 6n+4$, $f(uv_{n+1}) = 6n+2$, $f(uv_{2n+1}) = 6n+3$.
Case (ii). When $n \equiv 2 \pmod 3$.
$f(uv_1) = 6n+3$, $f(uv_{n+1}) = 6n+2$, $f(uv_{2n+1}) = 6n+4$.
Next, we prove that the property of an edge vertex prime labeling. For each $1 \leq i \leq 3n-1$,
$gcd(f(v_i), f(v_{i+1})) = gcd(2i-1, 2i+1) = 1$,
$gcd(f(v_i), f(v_iv_{i+1})) = gcd(2i-1, 2i) = 1$,
$gcd(f(v_{i+1}), f(v_iv_{i+1})) = gcd(2i+1, 2i) = 1$,
$gcd(f(v_1), f(v_{3n})) = gcd(1, 6n-1) = 1$,
$gcd(f(v_1), f(v_1v_{3n})) = gcd(1, 6n) = 1$,
$gcd(f(v_{3n}), f(v_1v_{3n})) = gcd(6n-1, 6n) = 1$.
Verification of **Case (i).**
$gcd(f(u), f(v_1)) = gcd(6n+1, 1) = 1$,
$gcd(f(u), f(uv_1)) = gcd(6n+1, 6n+4) = 1$,
$gcd(f(v_1), f(uv_1)) = gcd(1, 6n+4) = 1$,
$gcd(f(u), f(v_{n+1})) = gcd(6n+1, 2n+1) = 1$,
$gcd(f(u), f(uv_{n+1})) = gcd(6n+1, 6n+2) = 1$,
$gcd(f(v_{n+1}), f(uv_{n+1})) = gcd(2n+1, 6n+2) = 1$,
$gcd(f(u), f(v_{2n+1})) = gcd(6n+1, 4n+1) = 1$,
$gcd(f(u), f(uv_{2n+1})) = gcd(6n+1, 6n+3) = 1$,
$gcd(f(v_{2n+1}), f(uv_{2n+1})) = gcd(4n+1, 6n+3) = 1$.
Similarly the other case (ii) are verified. Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $J_{n,3}$, $n \geq 1$ has an edge vertex prime labeling. □

Theorem 2.11. *The graph G is obtained by subdividing the edges which are all incident with the centre vertex of $J_{n,3}$ is an edge vertex prime labeling, where n is congruent to 0 modulo 3.*

Proof. Let G be a graph which is obtained by subdividing the edges which are all adjacent with the centre vertex of $J_{n,3}$, where $n \equiv 0 \pmod 3$. Then $V(G) = \{u, v_i, w_j : 1 \leq i \leq 3n, 1 \leq j \leq 3\}$ and $E(G) = \{v_iv_{i+1}, v_1v_{3n} : 1 \leq i \leq 3n-1\} \bigcup \{uw_j : 1 \leq j \leq 3\} \bigcup \{w_1v_1, w_2v_{n+1}, w_3v_{2n+1}\}$.
Here $|V(G)| = 3n+4$ and $|E(G)| = 3n+6$. Define a bijective function $f : V(G) \bigcup E(G) \rightarrow \{1, 2, ..., 6n+10\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 3n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq 3n-1$, $f(v_1v_{3n}) = 6n$, $f(u) = 6n+1$, $f(uw_1) = 6n+2$, $f(w_1) = 6n+3$, $f(w_1v_1) = 6n+4$, $f(uw_2) = 6n+5$, $f(w_2) = 6n+6$, $f(w_2v_{n+1}) = 6n+7$, $f(uw_3) = 6n+8$, $f(w_3) = 6n+9$, $f(w_3v_{2n+1}) = 6n+10$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the graph G is an edge vertex prime labeling. □

Theorem 2.12. *The graph $J_{n,4}$, $n \geq 1$ is an edge vertex prime labeling.*

Proof. Let G be a $J_{n,4}$ graph. Then $V(G) = \{u, v_i : 1 \leq i \leq 4n\}$ and $E(G) = \{v_iv_{i+1}, v_1v_{4n} : 1 \leq i \leq 4n-1\} \bigcup \{uv_1, uv_{n+1}, uv_{2n+1}, uv_{3n+1}\}$. Here $|V(G)| = 4n+1$ and $|E(G)| = 4n+4$. Define a bijective function $f : V(G) \bigcup E(G) \rightarrow \{1, 2, ..., 8n+5\}$ by $f(v_i) = 2i-1$ for $1 \leq i \leq 4n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq 4n-1$, $f(v_1v_{4n}) = 8n$.
Consider the following cases.
Case (i). When $n \equiv 0, 1 \pmod 3$.
$f(u) = 8n+5$, $f(uv_1) = 8n+1$, $f(uv_{n+1}) = 8n+2$, $f(uv_{2n+1}) = 8n+3$, $f(uv_{3n+1}) = 8n+4$.
Case(ii). When $n \equiv 2 \pmod 3$.
$f(u) = 8n+1$, $f(uv_1) = 8n+2$, $f(uv_{n+1}) = 8n+3$, $f(uv_{2n+1}) = 8n+4$, $f(uv_{3n+1}) = 8n+5$.
Next, we prove the property of an edge vertex prime labeling. For each $1 \leq i \leq 4n-1$,
$gcd(f(v_i), f(v_{i+1})) = gcd(2i-1, 2i+1) = 1$,
$gcd(f(v_i), f(v_iv_{i+1})) = gcd(2i-1, 2i) = 1$,
$gcd(f(v_{i+1}), f(v_iv_{i+1})) = gcd(2i+1, 2i) = 1$,
$gcd(f(v_1), f(v_{4n})) = gcd(1, 8n-1) = 1$,
$gcd(f(v_1), f(v_1v_{4n})) = gcd(1, 8n) = 1$,
$gcd(f(v_{4n}), f(v_1v_{4n})) = gcd(8n-1, 8n) = 1$.
Verification of **Case(i)**.
$gcd(f(u), f(v_1)) = gcd(8n+5, 1) = 1$,
$gcd(f(u), f(uv_1)) = gcd(8n+5, 8n+1) = 1$,
$gcd(f(v_1), f(uv_1)) = gcd(1, 8n+1) = 1$,
$gcd(f(u), f(v_{n+1})) = gcd(8n+5, 2n+1) = 1$,
$gcd(f(u), f(uv_{n+1})) = gcd(8n+5, 8n+2) = 1$,
$gcd(f(v_{n+1}), f(uv_{n+1})) = gcd(2n+1, 8n+2)$,
$gcd(f(u), f(v_{2n+1})) = gcd(8n+5, 4n+1) = 1$,
$gcd(f(u), f(uv_{2n+1})) = gcd(8n+5, 8n+3) = 1$,
$gcd(f(v_{2n+1}), f(uv_{2n+1})) = gcd(4n+1, 8n+3) = 1$,
$gcd(f(u), f(v_{3n+1})) = gcd(8n+5, 6n+1) = 1$,
$gcd(f(u), f(uv_{3n+1})) = gcd(8n+5, 8n+4)$,
$gcd(f(v_{3n+1}), f(uv_{3n+1})) = gcd(6n+1, 8n+4) = 1$.
Similarly the case (ii) are verified. Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $J_{n,4}$, $n \geq 1$ has an edge vertex prime labeling.

□

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