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Impact of profession and surroundings on spread of swine flu: A mathematical study

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Abstract

Swine flu is a respiratory disease caused by H1N1 virus which was introduced in 2009. People who work in poultry farms and are in contact with live swine have high risk of swine flu infection. Use of mask by workers can be taken as prevention against disease. Here, we have formulated a SEIR mathematical model in two compartments to see the effect of profession and surroundings on spread of swine flu. Moreover, disease free equilibrium point, endemic equilibrium point and basic reproduction number have been calculated. It is seen that disease free equilibrium point always exists and is stable when $R_0 < 1$. Similarly, endemic equilibrium point exist and is stable when $R_0 > 1$. Sensitivity analysis of equilibrium point and basic reproduction number indicates the impact of parameter on spread of disease. Optimal value for efficiency of mask (a) is derived. Further, using MATLAB, numerical simulation has been done with respect to suitable parameter values and appropriate graphs have been obtained for all populations to understand the transmission behavior of swine flu.

Keywords

Swine flu infection and its prevention strategy, SEIR Mathematical model, Basic reproduction number, Stability analysis, Optimal control, Sensitivity analysis.

AMS Subject Classification

34D20, 34D23, 37C75, 49Q12, 90C31, 93C15, 93D05.

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Contents

1. Introduction

Swine flu is an infectious disease which rapidly spreads to susceptible people through respiratory droplets produced by coughing and sneezing of an infectious one. Symptoms of swine flu are fever, coughing, weakness, headache, muscle joint pain, and discomfort. Children, adults, old aged, pregnant women and heart disease patients have high risk of infection. It has an initial incubation period of 1-4 days followed by a latency period of 2-10 days [1].

The transmission of swine flu from swine to humans mainly occurs in swine farms, where farmers are in close contact with live pigs. People who are associated with swine have high chance to get swine flu infection, so farmers and veterinarians are suggested to use face mask while dealing with infected animals [9]. In 2004, University of Lowa performed a small surveillance study in which humans who work with swine were recorded. This study was on the basis of recommendation that people whose jobs involve handling poultry farms and swine be the focus of increased public health surveillance. Other professions at particular risk of infection are veterinarians and meat processing workers, although the risk of infection for both of these groups is lower than that of farm workers [2].

Many models have been proposed and analyzed for swine flu disease. Das et al. [3] studied mathematical model on stability analysis of swine flu transmission. They have done local and global stability of the model. They concluded that the disease is endemic in nature still the disease is under control if we are enough cautious to control the contact rate. Shrivastav et al. [4] have formulated mathematical model on analysis of symptomatic and asymptomatic of swine flu with optimal control by assuming simple mass-action type incidence. They evaluated basic reproduction number of the model and studied the local and the global stabilities of different equilibria of the model. Further, this model was extended to optimal control model. Finally, numerical simulation was performed to see the effect of optimal control on the infected population and they observed that optimal control model gives better result compared to the model without optimal control as it reduces the number of infective persons. Jin et al. [5] discussed an epidemic model of influenza A (H1N1) based on networks. They calculated the basic reproduction number and studied the effects of various immunization schemes. The final size relation was derived for the network epidemic model. The model parameters were estimated via least-squares fitting of the model solution to the observed data in China. Aldila et al. [6] proposed a compartmental SIR dynamical model for swine flu disease. Prevention against transmission was done with the use of medical mask with given rate in each compartment. The equilibrium points and basic reproductive ratio as the epidemic indicator before medical mask intervention have been illustrated analytically. Numerical optimal control results are shown to see the effectiveness of the treatment for various parameter values. It was concluded that as long as the worsening effect of medical mask not in a high number, then medical mask intervention will reduce swine flu epidemic. Cador et al. [7] suggested a stochastic metapopulation model on control of endemic swine flu persistence in farrow−to−finish pig farms. They developed a stochastic metapopulation model representing the co-circulation of two distinct swIAVs within a typical farrow-to-finish pig herd to evaluate the risk of reassortant viruses generation due to co-infection events. Control strategies related to herd management and/or vaccination schemes (batch-to-batch or mass vaccination of the sow herd and vaccination of growing pigs) were implemented to assess their relative efficacy regarding viral persistence. Although some vaccination schemes (batch-to-batch vaccination) had a beneficial effect in breeding sows by reducing the persistence of swIAVs within this subpopulation, none of vaccination strategies achieved swIAVs fade-out within the entire farrow-to-finish pig herd. Further, Chitnis et al. [12], Marsudi et al. [13] and Rani et al. [18] have done sensitivity analysis for basic reproduction numbers in their proposed outbreak models. Move over, Athithan et al. [14], Srivastav et al. [15], Sisodiya et al. [16] and Goswami et al. [17] have measured optimal controls in several epidemic models for underlying diseases.

Do profession and surrounding also helps in prevalence of infectious disease? In this paper, we argue they can do. In this series of analysis, we have wished to outline a mathematical model to see the impact of profession and surroundings on spread of swine flu in two compartments. The structure of the paper is as follows- In section 2, we have formulated SEIR model in two compartments. In section 3, We have found bounded region for the solutions. In section 4, we have calculated disease free, endemic equilibrium points and basic reproduction. In section 5, Conditions for stability of disease free equilibrium points and endemic equilibrium points are carried out. In section 6, value of optimal control is measured to minimize infected person from swine flu infection. In section 7, Numerical simulation has been done. Moreover sensitivity analysis of equilibrium points and basic reproduction number is executed and also appropriate graphs have been obtained. In section 8, conclusion has been discussed with the help of outcomes.

2. Model Formulation

In the model, we have assumed that total population is $N = N_1 + N_2$. where, N_1 is total population, who is not associated with poultry farms i.e. live far from animals and N_2 is total population, who is associated with poultry farms i.e. deal with animals. Suppose S_1, E_1, I_1 and R_1 are the susceptible, exposed, infectious and recovered populations, who are not associated with poultry farms, respectively. Similarly, S_2, E_2, I_2 and R_2 are the susceptible, exposed, infectious and recovered population, who are associated with poultry farms, respectively.

Π_1 is recruitment rate of susceptible population who is not associated with poultry farms, so $(1-\Pi_1)$ will be recruitment rate of susceptible population who is associated with poultry farms. Use of mask is taken as prevention parameter against swine flu by population while dealing with animals. Susceptible of both compartments are decreased through infected individuals who deal with animals by transmission rate of swine flu infection (λ). Initially, population of both compartments gets exposed by infected individuals who deal with animals then this exposed population joins infectious class by conversion rate from exposed to infected one (ξ). Infected population is increased by conversion rate from exposed to infected one (ξ) and is decreased by disease induced death rate (σ) and recovery rate (υ). Recovered population of both the compartments is increased by recovery of infected individuals. All the population classes are decreased by natural death (d).

The spread of swine flu among the populations of different surroundings has been described by the following system of

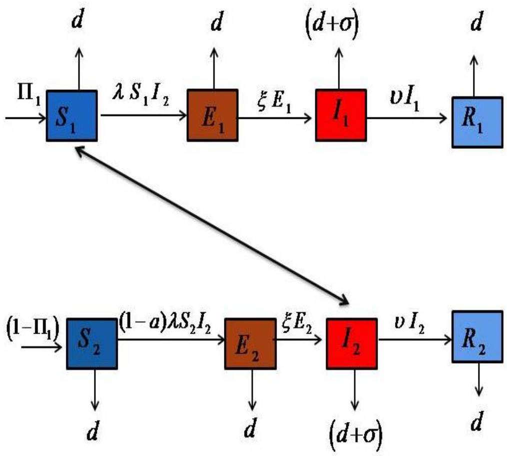

Figure 1. Spread of swine flu in two compartments

ordinary differential equations;

$$\frac{dS_1}{dt} = \Pi_1 - \lambda S_1 I_2 - dS_1 \tag{2.1}$$

$$\frac{dE_1}{dt} = \lambda S_1 I_2 - (d+\xi)E_1 \tag{2.2}$$

$$\frac{dI_1}{dt} = \xi E_1 - (d+\sigma+\upsilon)I_1 \tag{2.3}$$

$$\frac{dR_1}{dt} = \upsilon I_1 - dR_1 \tag{2.4}$$

$$\frac{dS_2}{dt} = (1-\Pi_1) - (1-a)\lambda S_2 I_2 - dS_2 \tag{2.5}$$

$$\frac{dE_2}{dt} = (1-a)\lambda S_2 I_2 - (d+\xi)E_2 \tag{2.6}$$

$$\frac{dI_2}{dt} = \xi E_2 - (d+\sigma+\upsilon)I_2 \tag{2.7}$$

$$\frac{dR_2}{dt} = \upsilon I_2 - dR_2 \tag{2.8}$$

With initial conditions,
$S_1(0) = S_{10} > 0, E_1(0) = E_{10} > 0, I_1(0) = I_{10} > 0,$
$R_1(0) = R_{10} > 0, S_2(0) = S_{20} > 0, E_2(0) = E_{20} > 0,$
$I_2(0) = I_{20} > 0, R_2(0) = R_{20} > 0.$
Here, all the parameters of model (2.1) to (2.8) are non negative constants.

Table 1. Description of Variables and parameters

Symbols	Parameters
S_1	Susceptible population who is not associated with poultry farms.
E_1	Exposed population who is not associated with poultry farms.
I_1	Infected population who is not associated with poultry farms.
R_1	Recovered population who is not associated with poultry farms.
S_2	Susceptible population who is associated with poultry farms.
E_2	Exposed population who is associated with poultry farms.
I_2	Infected population who is associated with poultry farms.
R_2	Recovered population who is associated with poultry farms.
Π_1	Recruitment rate of susceptible population who is not associated with poultry farms i.e. deal with animals.
$1-\Pi_1$	Recruitment rate of susceptible population who is associated with poultry farms i.e. deal with animals.
λ	Transmission rate of swine flu.
d	Natural death rate.
ξ	Conversion rate from exposed to infected one.
σ	Disease induced death rate.
υ	Recovery rate.
a	Efficiency of mask.

3. Bounded Region

To determine the bounded region for model system (2.1) to (2.8) let us assume total population is $N = N_1 + N_2$. where $N_1 = S_1 + E_1 + I_1 + R_1$ is total population, not associated with poultry farms and $N_2 = S_2 + E_2 + I_2 + R_2$ is total population, associated with poultry farms. Now,

$$N_1(t) = S_1(t) + E_1(t) + I_1(t) + R_1(t)$$

Differentiating above equation with respect to time, we obtain;

$$\frac{dN_1}{dt} = \frac{dS_1}{dt} + \frac{dE_1}{dt} + \frac{dI_1}{dt} + \frac{dR_1}{dt}$$

Now adding equations (2.1) to (2.4), we get;

$$\frac{dN_1}{dt} = \Pi_1 - d(S_1 + E_1 + I_1 + R_1) - \sigma I_1$$

$$\frac{dN_1}{dt} = \Pi_1 - dN_1 - \sigma I_1$$

This implies;

$$\frac{dN_1}{dt} + dN_1 \leq \Pi_1$$

On solving above differential equation, we get;

$N_1(t) \le \frac{\Pi_1(1-e^{-dt})}{d} + N_1(S_1(0), E_1(0), I_1(0), R_1(0))e^{-dt}$
Further as $t \to \infty$ then $N_1(t) \le \frac{\Pi_1}{d}$
Again, $N_2(t) = S_2(t) + E_2(t) + I_2(t) + R_2(t)$
Differentiating above equation with respect to time, we get;

$$\frac{dN_2}{dt} = \frac{dS_2}{dt} + \frac{dE_2}{dt} + \frac{dI_2}{dt} + \frac{dR_2}{dt}$$

Now adding equations (2.5) to (2.8), we have;

$$\frac{dN_2}{dt} = (1-\Pi_1) - d(S_2+E_2+I_2+R_2) - \sigma I_2$$

$$\frac{dN_2}{dt} = (1-\Pi_1) - dN_2 - \sigma I_2$$

This implies;

$$\frac{dN_2}{dt} + dN_2 \le (1-\Pi_1)$$

On solving above differential equation, we get;

$N_2(t) \le \frac{(1-\Pi_1)(1-e^{-dt})}{d} + N_2(S_2(0), E_2(0), I_2(0), R_2(0))e^{-dt}$
Now, as $t \to \infty$ then $N_2(t) \le \frac{(1-\Pi_1)}{d}$.
Thus all the solutions of model system (2.1) to (2.8) will lie in the region.
$\Upsilon = \{(S_1, E_1, I_1, R_1, S_2, E_2, I_2, R_2):$
. $S_1, E_1, I_1, R_1, S_2, E_2, I_2, R_2 \ge 0.$
. $: (S_1+E_1+I_1+R_1) \le \frac{\Pi_1}{d}, (S_2+E_2+I_2+R_2) \le \frac{(1-\Pi_1)}{d}\Big\}$
And clearly Υ is a compact positively invariant region in R_8^+.

4. Equilibrium Points and Basic Reproduction Number

4.1 Disease Free Equilibrium Point

The model system (2.1) to (2.8) has disease free equilibrium point $(S_1, 0, 0, 0, S_2, 0, 0, 0)$ in region Υ, where $S_1 = \frac{\Pi_1}{d}$ and $S_2 = \frac{1-\Pi_1}{d}$. therefore, disease free equilibrium point $A_1\left(\frac{\Pi_1}{d}, 0, 0, 0, \frac{(1-\Pi_1)}{d}, 0, 0, 0\right)$ always exist.

4.2 Basic Reproduction Number (R_0)

The basic reproduction number indicates the average number of secondary infections produced by an infected individual during the entire infection period. The basic reproduction number is an important tool for the assessment of the local stability of the equilibrium points. By applying the Next generation matrix method [8] on model system (2.1) to (2.8), we can calculate the reproductive number. Suppose f is the rate of growth of new infection in infected class and V is the shifting of individuals out of infected class by any mean. Let us we consider the equations (2.2), (2.3), (2.6) and (2.7), we have;.
$f_1(E_1, I_1, E_2, I_2) = \lambda S_1 I_2,\ f_2(E_1, I_1, E_2, I_2) = 0,$
$f_3(E_1, I_1, E_2, I_2) = (1-a)\lambda S_2 I_2,\ f_4(E_1, I_1, E_2, I_2) = 0,$
$V_1(E_1, I_1, E_2, I_2) = (d+\xi)E_1,$
$V_2(E_1, I_1, E_2, I_2) = -\xi E_1 + (d+\sigma+\upsilon)I_1,$
$V_3(E_1, I_1, E_2, I_2) = (d+\xi)E_2,$
$V_4(E_1, I_1, E_2, I_2) = -\xi E_2 + (d+\sigma+\upsilon)I_2.$
Therefore

$$f = \begin{bmatrix} 0 & 0 & 0 & \lambda S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-a)\lambda S_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} (d+\xi) & 0 & 0 & 0 \\ -\xi & (d+\sigma+\upsilon) & 0 & 0 \\ 0 & 0 & (d+\xi) & 0 \\ 0 & 0 & -\xi & (d+\sigma+\xi) \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{(d+\xi)} & 0 & 0 & 0 \\ \frac{\xi}{(d+\xi)(d+\sigma+\xi)} & \frac{1}{(d+\sigma+\xi)} & 0 & 0 \\ 0 & 0 & \frac{1}{(d+\xi)} & 0 \\ 0 & 0 & \frac{\xi}{(d+\xi)(d+\sigma+\xi)} & \frac{1}{(d+\sigma+\xi} \end{bmatrix}$$

$$fV^{-1} = \begin{bmatrix} 0 & 0 & \frac{\lambda S_1 \xi}{(d+\xi)(d+\sigma+\xi)} & \frac{\lambda S_1}{(d+\sigma+\xi)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(1-a)\lambda S_2 \xi}{(d+\xi)(d+\sigma+\xi)} & \frac{(1-a)\lambda S_2}{(d+\sigma+\xi)} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The eigen equation of fV^{-1} is $\psi^3\left(\psi - \frac{(1-a)\xi\lambda S_2}{(d+\xi)(d+\sigma+\upsilon)}\right) = 0.$ Dominating Eigen value of matrix fV^{-1} is known as basic reproduction number, consequently $R_0 = \frac{(1-a)\lambda\xi(1-\Pi_1)}{d(d+\xi)(d+\sigma+\upsilon)}$. The basic reproduction number R_0 indicates the status of the disease i.e. when there will be disease free state or endemic state in the context.

4.3 Endemic Equilibrium Point

Endemic equilibrium point is the steady state solution when disease persists in the population .To evaluate endemic equilibrium point $A_2\left(\overline{S_1}, \overline{E_1}, \overline{I_1}, \overline{R_1}, \overline{S_2}, \overline{E_2}, \overline{I_2}, \overline{R_2}\right)$ in the region Υ. We put $\frac{d\overline{S_1}}{dt} = \frac{d\overline{E_1}}{dt} = \frac{d\overline{I_1}}{dt} = \frac{d\overline{R_1}}{dt} = \frac{d\overline{S_2}}{dt} = \frac{d\overline{E_2}}{dt} = \frac{d\overline{I_2}}{dt} = \frac{d\overline{R_2}}{dt} = 0$ Where,

$$\overline{S_1} = \frac{\Pi_1(1-a)}{d(R_0-a)},$$

$$\overline{E_1} = \frac{\Pi_1(R_0-1)}{(d+\xi)(R_0-a)},$$

$$\overline{I_1} = \frac{\Pi_1\xi(R_0-1)}{(d+\xi)(d+\sigma+\upsilon)(R_0-a)},$$

$$\overline{R_1} = \frac{\Pi_1\xi\upsilon(R_0-1)}{d(d+\xi)(d+\sigma+\upsilon)(R_0-a)},$$

$$\overline{S_2} = \frac{(1-\Pi_1)}{dR_0},$$

$$\overline{E_2} = \frac{d(d+\sigma+\upsilon)}{(1-a)\lambda\xi}(R_0-1),$$

$$\overline{I_2} = \frac{d}{(1-a)\lambda}(R_0 - 1),$$

$$\overline{R_2} = \frac{\upsilon}{(1-a)\lambda}(R_0 - 1)$$

Thus, $\overline{S_1}, \overline{E_1}, \overline{I_1}, \overline{R_1}, \overline{S_2}, \overline{E_2}, \overline{I_2}$ and $\overline{R_2}$ all are positive when $R_0 > 1$ and $0 < a < 1$. Hence $A_2\left(\overline{S_1}, \overline{E_1}, \overline{I_1}, \overline{R_1}, \overline{S_2}, \overline{E_2}, \overline{I_2}, \overline{R_2}\right)$ exists when $R_0 > 1$.

5. Stability Analysis of Equilibrium Points

5.1 Local Stability of the Disease Free Equilibrium Point (A_1)

The variational matrix of the model system (2.1) to (2.8) about (A_1) is given by;

$$J_1 = \begin{bmatrix} -d & 0 & 0 & 0 & 0 & 0 & -\lambda S_1 & 0 \\ 0 & -(d+\xi) & 0 & 0 & 0 & 0 & \lambda S_1 & 0 \\ 0 & \xi & -(d+\sigma+\upsilon) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \upsilon & -d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d & 0 & -(1-a)\lambda S_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(d+\xi) & (1-a)\lambda S_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & -(d+\sigma+\upsilon) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \upsilon & -d \end{bmatrix}$$

The characteristic equation of J_1 is obtained by substituting $|J_1 - \psi I| = 0$. then we have;
$\left(\psi^2 + (2d+\xi+\sigma+\upsilon)\psi + (d+\xi)(d+\sigma+\upsilon)(1-R_0)\right)$
$(\psi+d)^4(\psi+d+\xi)(\psi+d+\sigma+\upsilon)$=0
This implies,
$\psi = -d, \psi = -d, \psi = -d, \psi = -d, \psi = -(d+\xi)$,
$\psi = -(d+\sigma+\upsilon)$ and also
$\psi^2 + (2d+\xi+\sigma+\upsilon)\psi + (d+\xi)(d+\sigma+\upsilon)(1-R_0) = 0$
has negative roots when $R_0 < 1$. Clearly all eight roots of characteristic equation are negative if $R_0 < 1$. Thus we observe that the disease free equilibrium point (A_1) is linearly asymptotically stable if $R_0 < 1$. However, the disease free equilibrium point (A_1) is unstable if $R_0 > 1$.

5.2 Local Stability of the Endemic Equilibrium Point (A_2)

The variational matrix of the model system (2.1) to (2.8) about (A_2) is given by;

$$J_2 = \begin{bmatrix} -(\lambda I_2 + d) & 0 & 0 & 0 & 0 & 0 & -(\lambda S_1) & 0 \\ (\lambda I_2) & -(d+\xi) & 0 & 0 & 0 & 0 & (\lambda S_1) & 0 \\ 0 & \xi & -(d+\sigma+\upsilon) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \upsilon & -d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -((1-a)\lambda I_2 + d) & 0 & -(1-a)\lambda S_2 & 0 \\ 0 & 0 & 0 & 0 & (1-a)\lambda I_2 & -(d+\xi) & (1-a)\lambda S_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & -(d+\sigma+\upsilon) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \upsilon & -d \end{bmatrix}$$

The characteristic equation of J_2 is obtained by substituting $|J_2 - \psi I| = 0$. then we have;
$(\psi+d)^2(\psi+d+\xi)(\psi+d+\sigma+\upsilon)(\psi+d+\lambda I_2)$
$(a_0\psi^3 + a_1\psi^2 + a_2\psi + a_3) = 0$.
$(\psi+d)^2(\psi+d+\xi)(\psi+d+\sigma+\upsilon)\left(\psi + d\left(\frac{R_0-a}{1-a}\right)\right)$
$(a_0\psi^3 + a_1\psi^2 + a_2\psi + a_3) = 0$,
where,
$a_0 = 1, a_1 = dR_0 + 2d + \xi + \sigma + \upsilon$,
$a_2 = dR_0(2d + \xi + \sigma + \upsilon)$,
$a_3 = (d+\xi)(d+\sigma+\upsilon)d(R_0 - 1)$.
Clearly, four roots of characteristic equations are negative and remaining four characteristic roots are obtained by solving the following equations;
$\psi = -d\left(\frac{R_0-a}{1-a}\right)$
Which is negative if $R_0 > 1$ and $0 < a < 1$.
Now, $a_0\psi^3 + a_1\psi^2 + a_2\psi + a_3 = 0$
$a_0 = 1 > 0, a_1 = (dR_0 + 2d + \xi + \sigma + \upsilon) > 0$,
$a_2 = (dR_0(2d + \xi + \sigma + \upsilon)) > 0$,
$a_3 = (d+\xi)(d+\sigma+\upsilon)d(R_0 - 1) > 0$ if $R_0 > 1$.
We can easily see from Descartes' rule of sign that the characteristic equation will have negative real roots if $R_0 > 1$.Clearly all the roots of equation are negative if $R_0 > 1$.
Thus we can conclude that the endemic equilibrium point (A_2) is linearly asymptotically stable if $R_0 > 1$. However, the endemic equilibrium point (A_2) is unstable if $R_0 < 1$.

5.3 Global Stability of the Endemic Equilibrium Point (A_2)

Applying the transformations on model system (2.1) to (2.8) and taking linear terms, we get ;
$S_1 = \overline{S_1} + y_1, E_1 = \overline{E_1} + y_2, I_1 = \overline{I_1} + y_3, R_1 = \overline{R_1} + y_4$,
$S_2 = \overline{S_2} + y_5, E_2 = \overline{E_2} + y_6, I_2 = \overline{I_2} + y_7, R_2 = \overline{R_2} + y_8$.

$$\frac{dy_1}{dt} = -\lambda\left(y_1\overline{I_2} + y_7\overline{S_1}\right) - dy_1$$

$$\frac{dy_2}{dt} = \lambda\left(y_1\overline{I_2} + y_7\overline{S_1}\right) - (d+\xi)\,y_2$$

$$\frac{dy_3}{dt} = \xi y_2 - (d+\sigma+\upsilon)y_3$$

$$\frac{dy_4}{dt} = \upsilon y_3 - dy_4$$

$$\frac{dy_5}{dt} = -(1-a)\lambda\left(y_7\overline{S_2} + y_5\overline{I_2}\right) - dy_5$$

$$\frac{dy_6}{dt} = (1-a)\lambda\left(y_7\overline{S_2} + y_5\overline{I_2}\right) - (d+\xi)y_6$$

$$\frac{dy_7}{dt} = \xi y_6 - (d+\sigma+\upsilon)y_7$$

$$\frac{dy_8}{dt} = \upsilon y_7 - dy_8$$

Let us, consider a positive definite function U given by;
$U = \frac{1}{2}(U_1y_1^2 + U_2y_2^2 + U_3y_3^2 + U_4y_4^2 + U_5y_5^2 + U_6y_6^2 + U_7y_7^2 + U_8y_8^2)$
Differentiating U with respect to time then put the above system of equations in $\frac{dU}{dt}$, we get;
$\frac{dU}{dt} = -\left(\lambda U_1\overline{I_2} + dU_1\right)y_1^2 - (d+\xi)\,U_2y_2^2 - (d+\sigma+\upsilon)U_3y_3^2 - dU_4y_4^2 - \left((1-a)\lambda U_5\overline{I_2} + dU_5\right)y_5^2 - (d+\xi)U_6y_6^2 - (d+\sigma+\upsilon)U_7y_7^2 - dU_8y_8^2 - \lambda\overline{S_1}U_1y_1y_7 + U_2\lambda\overline{I_2}y_1y_2 + U_2\lambda\overline{S_1}y_2y_7 + \xi U_3y_2y_3 + \upsilon U_4y_3y_4 + (1-a)\lambda U_6\overline{S_2}y_6y_7 + (1-a)\lambda U_6\overline{I_2}y_5y_6 - (1-a)\lambda U_5\overline{S_2}y_5y_7 + \xi U_7y_6y_7 + \upsilon U_8y_7y_8$
$\frac{dU}{dt} = -U_{11}y_1^2 - U_{22}y_2^2 - U_{33}y_3^2 - U_{44}y_4^2 - U_{55}y_5^2 - U_{66}y_6^2 - U_{77}y_7^2 - U_{88}y_8^2 + U_{17}y_1y_7 + U_{12}y_1y_2 + U_{27}y_2y_7 + U_{23}y_2y_3 + U_{34}y_3y_4 + U_{56}y_5y_6 + U_{57}y_5y_7 + U_{67}y_6y_7 + U_{78}y_7y_8$
On rearranging the above equation, we have;
$\frac{dU}{dt} = -\left[\left(\frac{U_{11}}{2}y_1^2 - U_{12}y_1y_2 + \frac{U_{22}}{3}y_2^2\right) + \left(\frac{U_{11}}{2}y_1^2 - U_{17}y_1y_7 + \frac{U_{77}}{5}y_7^2\right) + \left(\frac{U_{22}}{3}y_2^2 - U_{27}y_2y_7 + \frac{U_{77}}{5}y_7^2\right) + \left(\frac{U_{22}}{3}y_2^2 - U_{23}y_2y_3 + \frac{U_{33}}{2}y_3^2\right) + \left(\frac{U_{33}}{2}y_3^2 - U_{34}y_3y_4 + U_{44}y_4^2\right) + \left(\frac{U_{55}}{2}y_5^2 - U_{56}y_5y_6 + \frac{U_{66}}{2}y_6^2\right) + \left(\frac{U_{55}}{2}y_5^2 - U_{57}y_5y_7 + \frac{U_{77}}{5}y_7^2\right) + \left(\frac{U_{66}}{2}y_6^2 - U_{67}y_6y_7 + \frac{U_{77}}{5}y_7^2\right) + \left(\frac{U_{77}}{5}y_7^2 - U_{78}y_7y_8 + U_{88}y_8^2\right)\right]$
Where,
$U_{11} = \left(\lambda\overline{I_2} + d\right)U_1, U_{22} = (d+\xi)\,U_2, U_{33} = (d+\sigma+\upsilon)U_3,$
$U_{44} = dU_4, U_{55} = \left((1-a)\lambda\overline{I_2} + d\right)U_5, U_{66} = (d+\xi)\,U_6,$
$U_{77} = (d+\sigma+\upsilon)U_7, U_{88} = dU_8, U_{17} = -\lambda\overline{S_1}U_1,$
$U_{27} = \lambda U_2\overline{S_1}, U_{23} = \xi U_3, U_{34} = \upsilon U_4, U_{56} = (1-a)\lambda U_6\overline{I_2},$
$U_{57} = -(1-a)\lambda U_5\overline{S_2}, U_{67} = \left((1-a)\lambda U_6\overline{S_2} + \xi U_7\right),$
$U_{12} = \lambda U_2\overline{I_2}, U_{78} = \upsilon U_8.$
Using lypenove's theorem of stability, we have;

$$\frac{\left(\lambda\overline{I_2} + d\right)U_1\,(d+\xi)}{6} > \frac{U_2\left(\lambda\overline{I_2}\right)^2}{4},$$

$$\frac{\left(\lambda\overline{I_2} + d\right)(d+\sigma+\upsilon)\,U_7}{10} > \frac{U_1\left(\lambda\overline{S_1}\right)^2}{4},$$

$$\frac{(d+\xi)\,(d+\sigma+\upsilon)\,U_7}{15} > \frac{U_2\left(\lambda\overline{S_1}\right)^2}{4},$$

$$\frac{(d+\xi)\,(d+\sigma+\upsilon)U_2}{6} > \frac{U_3\,(\xi)^2}{4},$$

$$\frac{(d+\sigma+\upsilon)dU_3}{2} > \frac{U_4\,(\upsilon)^2}{4},$$

$$\frac{\left((1-a)\lambda\overline{I_2} + d\right)(d+\xi)\,U_5}{4} > \frac{\left((1-a)\lambda\overline{I_2}\right)^2U_6}{4},$$

$$\frac{\left((1-a)\lambda\overline{I_2} + d\right)(d+\sigma+\upsilon)\,U_7}{10} > \frac{\left((1-a)\lambda\overline{S_2}\right)^2U_5}{4},$$

$$\frac{(d+\xi)\,(d+\sigma+\upsilon)\,U_6U_7}{10} > \frac{\left((1-a)\lambda U_6\overline{S_2} + \xi U_7\right)^2}{4},$$

$$\frac{(d+\sigma+\upsilon)\,dU_7}{5} > \frac{U_8\,(\upsilon)^2}{4}.$$

Hence, we can conclude that under above mentioned conditions the Endemic Equilibrium Point (A_2) is globally stable otherwise unstable.

6. Optimal Control Problem

The main purpose of this section is to reach at an optimal control problem, which influences to swine flu spread. Here, we have to find the optimal control $U(t) = (a(t))^T \in R$. In this paper, efficiency of mask a is one of the parameter, that can minimize the number of infected population. Using Pontryagin's Minimum Principle, we calculate the optimal value of parameter required to control the swine flu transmission. Let us assume the objective functional J, which minimizes the number of infected population and the cost related to control a. here, we tend to values of a which is utilize to optimize the objective function J.Thus, the objective function to be minimized is given below;

$$J(a(t)) = \int_o^{t_f}\left[C_1I_1(t) + C_2I_2(t) + \frac{C_3}{2}(a)^2\right]dt \quad (6.1)$$

The parameters $C_1 > 0$, $C_2 > 0$ and $C_3 > 0$ are unit less weight constants. We have to obtain an optimal control parameter a^* such that,

$$J(a^*) = \min_a [J(a) \mid a \in U]$$

Where $U = \{a|0 \le a \le 1\, and\, t \in [0, t_f]\}$ Before using Pontryagin's Minimum Principle on the model system (2.1) to (2.8) and (6.1), we introduce Lagrangian function for the problem is defined by;

$$L(I_1, I_2, a) = C_1I_1(t) + C_2I_2(t) + \frac{C_3}{2}(a)^2$$

Further, our objective is changed now we have to minimize Hamiltonian function H with respect to a. For this, we choose the Hamiltonian function as follows;
$H = C_1I_1(t) + C_2I_2(t) + \frac{C_3}{2}(a)^2 + \lambda_1(\Pi_1 - \lambda S_1I_2 - dS_1) + \lambda_2(\lambda S_1I_2 - (d+\xi)E_1) + \lambda_3(\xi E_1 - (d+\sigma+\upsilon)I_1) + \lambda_4(\upsilon I_1 - dR_1) + \lambda_5((1-\Pi_1) - (1-a)\lambda S_2I_2 - dS_2)$

$+\lambda_6((1-a)\lambda S_2 I_2-(d+\xi)E_2)+\lambda_7(\xi E_2-(d+\sigma+\upsilon)I_2)$
$+\lambda_8(\upsilon I_2-dR_2)$

Where λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, λ_7 and λ_8 are the adjoint variables or co-state variables. Now transversality conditions are given by,

$\lambda_1(T)=\lambda_2(T)=\lambda_3(T)=\lambda_4(T)=\lambda_5(T)=\lambda_6(T)$
$=\lambda_7(T)=\lambda_8(T)=0$

For above mentioned transversality condition the following holds;

$$\frac{d\lambda_1}{dt}=-\frac{\partial H}{\partial S_1},\ \frac{d\lambda_2}{dt}=-\frac{\partial H}{\partial E_1},\ \frac{d\lambda_3}{dt}=-\frac{\partial H}{\partial I_1},\ \frac{d\lambda_4}{dt}=-\frac{\partial H}{\partial R_1},$$

$$\frac{d\lambda_5}{dt}=-\frac{\partial H}{\partial S_2},\ \frac{d\lambda_6}{dt}=-\frac{\partial H}{\partial E_2},\ \frac{d\lambda_7}{dt}=-\frac{\partial H}{\partial I_2},\ \frac{d\lambda_8}{dt}=-\frac{\partial H}{\partial R_2}$$

Using above mentioned condition, we attain the system of differential equations with respect to the associated adjoint variables as follows;

$$\frac{d\lambda_1}{dt}=\lambda_1(\lambda I_2+d)-\lambda_2\lambda I_2$$
$$\frac{d\lambda_2}{dt}=\lambda_2(d+\xi)-\lambda_3\xi$$
$$\frac{d\lambda_3}{dt}=-C_1+\lambda_3(d+\sigma+\upsilon)-\lambda_4\upsilon$$
$$\frac{d\lambda_4}{dt}=\lambda_4 d$$
$$\frac{d\lambda_5}{dt}=\lambda_5((1-a)\lambda I_2+d)-\lambda_6(1-a)\lambda I_2$$
$$\frac{d\lambda_6}{dt}=\lambda_6(d+\xi)-\lambda_7\xi$$
$$\frac{d\lambda_7}{dt}=-C_2+\lambda_1\lambda S_1-\lambda_2\lambda S_1+\lambda_5(1-a)\lambda S_2$$
$$-\lambda_6(1-a)\lambda S_2+\lambda_7(d+\sigma+\upsilon)-\lambda_8\upsilon$$
$$\frac{d\lambda_8}{dt}=\lambda_8 d$$

The optimal control can be classified by the expressions, given below;

$$a^*(t)=\max\left\{0,\min\left(\hat{a}(t),1\right)\right\}$$

Further, 100% effective controls is not possible avail that's why upper and lower bounds of the control are 0 and 1 respectively, thus we can conclude the following;

$$a^*=\begin{cases}0 & if\ \hat{a}\le 0,\\ \hat{a} & if\ 0<\hat{a}<1,\\ 1 & if\ \hat{a}\ge 1,\end{cases}$$

The control parameter a^* =1 indicates that swine flu infected population should wear mask to reduce the transmission of infection, so that there may be the highest decline in number of infected population. Differentiating H with respect to the effective control a, this implies;

$$\frac{\partial H}{\partial a}=C_3 a+\lambda_5\lambda S_2 I_2-\lambda_6\lambda S_2 I_2$$

The value $\hat{a}$ of the optimal control a^* is obtained by substituting $\frac{\partial H}{\partial a}=0$ this gives,

$$\hat{a}=\frac{(\lambda_6-\lambda_5)\lambda S_2 I_2}{C_3}$$

Table 2. For $\Pi_1=0.6$, $\lambda=0.0055$, $d=0.02$, $\sigma=0.01$, $\upsilon=0.01$, $\xi=0.3$

(a)	(R_0)	Equilibrium points
0.2	2.06	$A_2(12.88,1.06,8.02,4.01,9.69,0.64,4.82,2.41)$
0.4	1.54	$A_2(15.69,0.89,6.70,3.35,12.92,0.44,3.31,1.65)$
0.5	1.28	$A_2(19.00,0.68,5.15,2.57,15.51,0.28,2.10,1.05)$
0.6	1.03	$A_2(27.82,0.13,1.01,0.50,19.39,0.03,0.28,0.14)$
0.7	0.77	$A_1(30,0,0,0,20,0,0,0)$
0.8	0.51	$A_1(30,0,0,0,20,0,0,0)$

Table 3. For $\Pi_1=0.6, a=0.5, d=0.02, \sigma=0.01$, $\upsilon=0.01$, $\xi=0.3$

(λ)	(R_0)	Equilibrium points
0.0035	0.82	$A_1(30,0,0,0,20,0,0,0)$
0.0040	0.93	$A_1(30,0,0,0,20,0,0,0)$
0.0045	1.05	$A_2(72.04,0.2,1.38,0.7,18.96,0.06,0.48,0.24)$
0.0055	1.28	$A_2(19.0,0.7,5.15,2.57,15.51,0.28,2.10,1.05)$
0.0065	1.52	$A_2(14.65,0.95,7.2,3.6,13.12,0.42,3.22,1.61)$

7. Numerical Simulation

In this section, we have done numerical simulations of the model system (2.1) to (2.8) with the help of MATLAB R 2014a (32-bit) software. Here, efficiency of mask (a) and transmission rate of disease (λ) are two important parameters, which affect reproduction number (R_0). The objective of this section is to calculate equilibrium points and reproduction number for different values of parameters, which are stated in Table 2 and Table 3.

From Table 2 , it can be seen that as efficiency of mask increases then basic reproduction number R_0 decreases and disease free state attains. From Table 3, it can be concluded that as transmission rate of swine flu increases then basic reproduction number R_0 increases and endemic state attains.

Further, we have done the sensitivity analysis of the basic reproduction number and endemic equilibrium points, which indicates us the significance of parameters in transmission and prevalence of disease. Sensitivity indices measure the relative change in state variable (X) with respect to state parameter (x_i). Normalized forward sensitivity index method is used to evaluate sensitivity indices of a variable with respect to a parameter, which is given by; $i^X_{x_i}=\frac{\partial X}{\partial x_i}\times\frac{x_i}{X}$

Table 4. Sensitivity indices of state variables in first compartment and basic reproduction number w.r.to parameters

Parameters (x_i)	$i_{x_i}^{R_0}$	$i_{x_i}^{S_1}$	$i_{x_i}^{E_1}$	$i_{x_i}^{I_1}$	$i_{x_i}^{R_1}$
Π_1	-1.50	+3.45	-3.24	-3.24	-3.24
a	-1.00	+1.27	-2.19	-2.19	-2.19
λ	+1.00	-1.63	+2.83	+2.83	+2.83
ξ	+0.06	-0.10	-0.76	+0.24	+0.24
d	-1.56	+1.55	-4.48	-4.98	-5.98
σ	-0.25	+0.41	-0.71	-0.96	-0.96
υ	-0.25	+0.41	-0.71	-0.96	+0.04

Table 5. Sensitivity indices of state variables in second compartment and basic reproduction number w.r.to parameters

Parameters (x_i)	$i_{x_i}^{R_0}$	$i_{x_i}^{S_2}$	$i_{x_i}^{E_2}$	$i_{x_i}^{I_2}$	$i_{x_i}^{R_2}$
Π_1	-1.50	0	-6.69	-6.69	-6.69
a	-1.00	+1.00	-3.46	-3.46	-3.46
λ	+1.00	-1.00	+3.46	+3.46	+3.46
ξ	+0.06	-0.06	-0.72	+0.28	+0.28
d	-1.56	+0.56	-5.47	-5.97	-6.97
σ	-0.25	+0.25	-0.86	-1.11	-1.11
υ	-0.25	+0.25	-0.86	-1.11	-0.11

The values of sensitivity indices of state variables of model system (2.1) to (2.8) and basic reproduction number with respect to parameters are stated in Table 4 and Table 5

The positive sign of sensitivity indices represents parameters are directly proportional to state variables and basic reproduction number i.e. if value of parameter is increased by 10% then the value of state variables and basic reproduction number will be increased by 10% of sensitivity indices. Similarly, negative sign of sensitivity indices represents parameters are inversely proportional to state variables and basic reproduction number i.e. if value of parameter is increased by 10% then the value of state variables and basic reproduction number will be decreased by 10% of sensitivity indices. From Table 4 and Table 5, it is cleared that (λ and ξ) are positively associated with basic reproduction number and (Π_1, a, d, σ and υ) are negatively associated with basic reproduction number. (Π_1, a, d, σ and υ) are positively associated with $\overline{S_1}$ and (λ and ξ) are negatively associated with $\overline{S_1}$. (λ) is positively associated with $\overline{E_1}$ and (Π_1, a, ξ, d, σ and υ) are negatively associated with $\overline{E_1}$. (λ and ξ) are positively associated with $\overline{I_1}$ and (Π_1, a, d, σ and υ) are negatively associated with $\overline{I_1}$. (λ, ξ and υ) are positively associated with $\overline{R_1}$ and (Π_1, a, d and σ) are negatively associated with $\overline{R_1}$. (a, d, σ and υ) are positively associated with $\overline{S_2}$ and (λ and ξ) are negatively associated with $\overline{S_2}$ and also $\left(i_{\Pi_1}^{\overline{S_2}} = 0\right)$ represents Π_1 is not associated with $\overline{S_2}$. (λ) is positively associated with $\overline{E_2}$ and (Π_1, a, ξ, d, σ and υ) are negatively associated with $\overline{E_2}$. (λ and ξ) are positively associated with $\overline{I_2}$ and (Π_1, a, d, σ and υ) are negatively associated with $\overline{I_2}$. (λ and ξ) are positively associated with $\overline{R_2}$ and (Π_1, a, d, σ and υ) are negatively associated with $\overline{R_2}$.

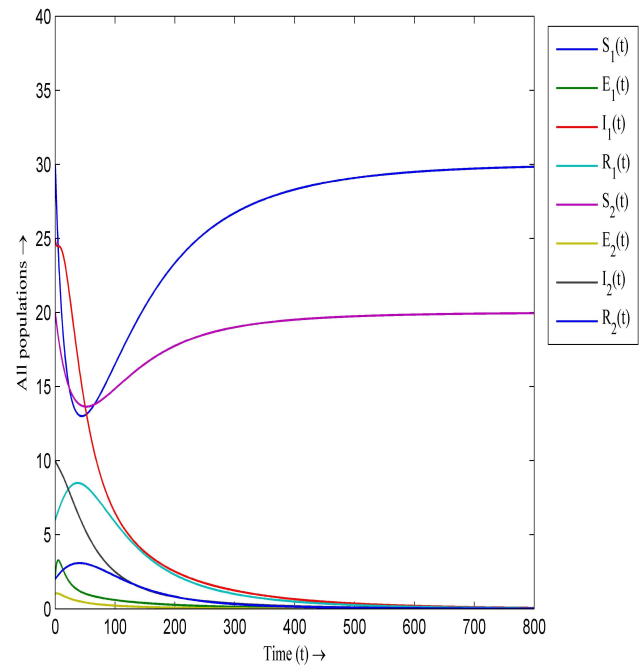

Figure 2. Growth of all Population with respect to time (t) for Disease Free State

8. Conclusion

Swine flu is a pandemic disease which effects the growth of population. In this paper, SEIR model with two patches is framed to study the impact of profession and surrounding on spread of swine flu. Reproduction number (R_0) is an important tool that indicates phase of any disease. $R_0 < 1$ represents stability of disease free endemic point whereas, $R_0 > 1$ depicts stability of endemic point, which are demonstrated in Fig.2 and Fig.3. Value of R_0 depends upon efficiency of mask used by infected one (a) and transmission rate of disease.

Keeping $\lambda = 0.0055$ fixed, it was concluded that as efficiency of cloth (a) increases, reproduction number (R_0) decreases (see Fig.4) and also infected population of both compartments decreases (see Figs.5). Keeping $a = 0.5$ fixed, it was found that as transmission rate of disease (λ) increases, reproduction number (R_0) increases (see Fig.6) and also infected population of both compartments increases (see Figs.7). Keeping $a = 0.5$ and $\lambda = 0.0055$ fixed, it was seen that as infected population who is associated with poultry farms (I_2) increases, infected population who is not associated with poultry farms also increases (see Fig.8). Threshold states are also obtained (see Figs.4 and 6).

Also it was seen that number of infected population is minimum with control and it is maximum without control

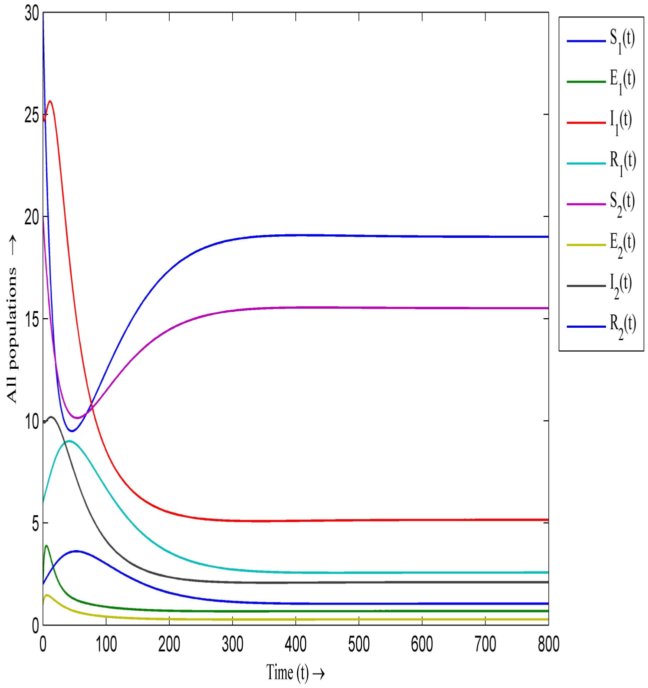

Figure 3. Growth of all Population with respect to time (t) for Endemic State

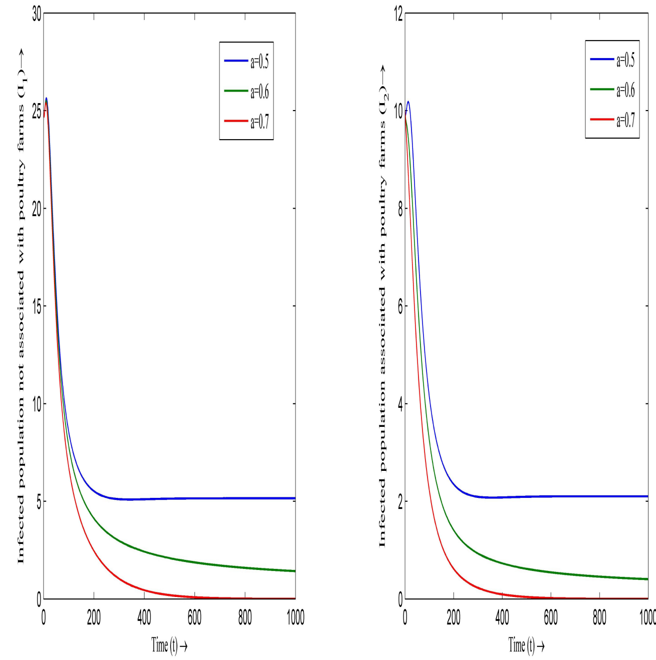

Figure 5. Plot between time (t) and infected population in both compartments for various values of a

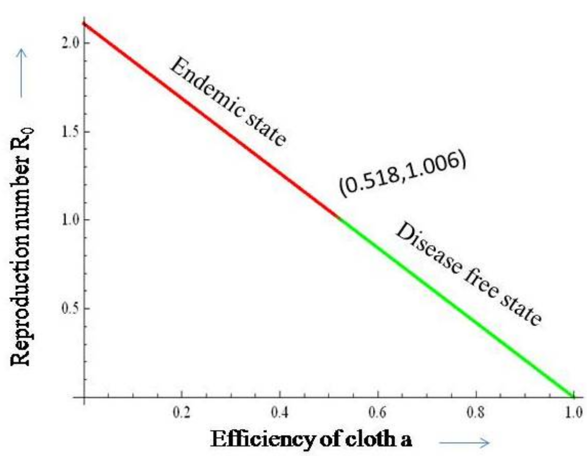

Figure 4. Plot between efficiency of cloth and Reproduction number

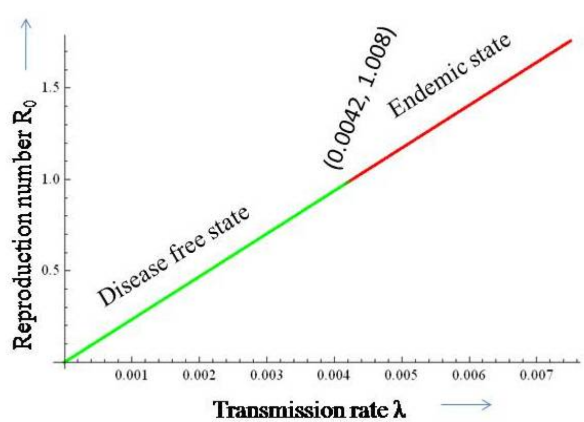

Figure 6. Plot between Transmission rate of swine flu and Reproduction number

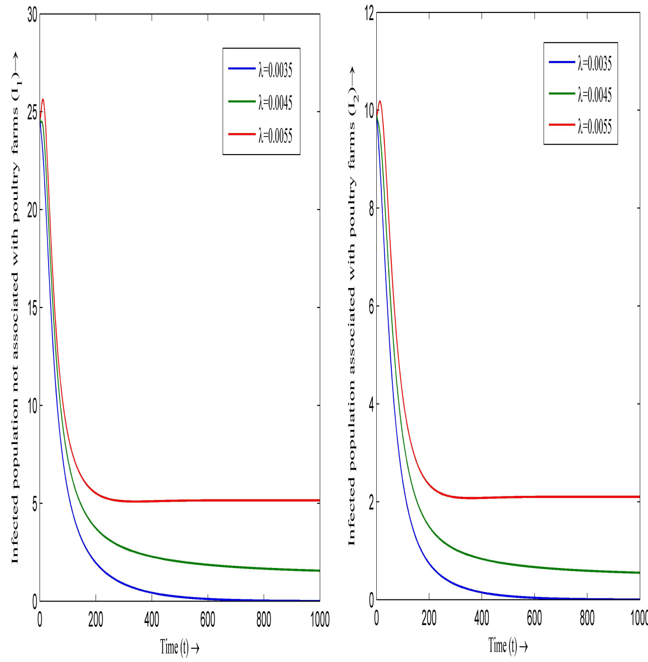

Figure 7. Plot between time (t) and infected population of both compartments for various values of λ

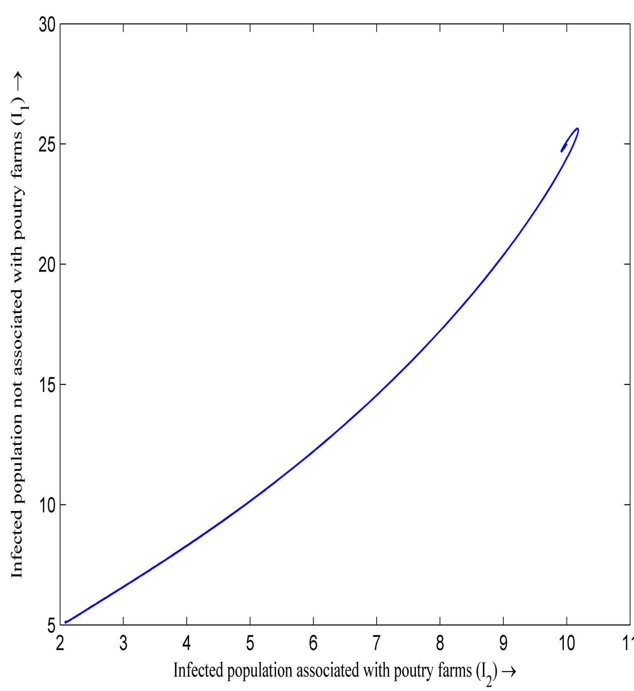

Figure 8. Plot between infected population (I_2)virus (I_1)

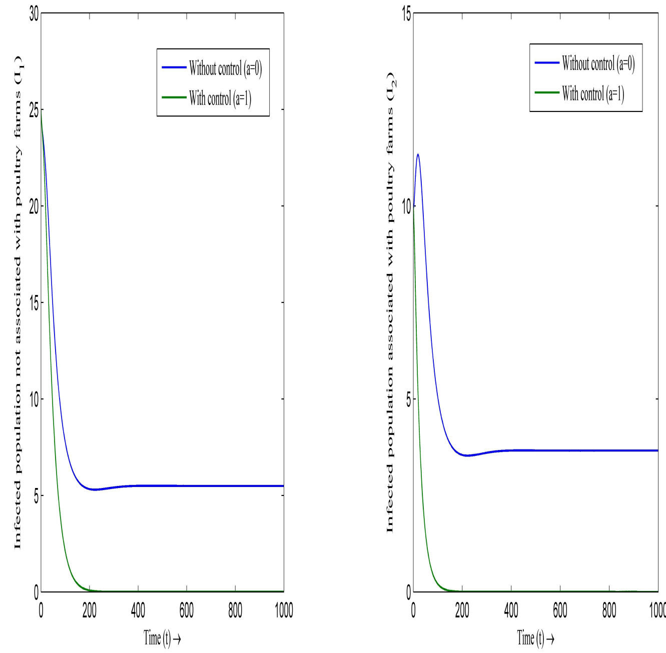

Figure 9. Plot between time $(t)$and infected population of both compartments for control parameter (a)

(see Fig.9).

The sign of sensitivity indices for parameters determines the impact of parameter on state variables and basic reproduction number and allow us that how to treat these parameters to restraint swine flu epidemic as follows:
1. Increase the recruitment rate Π_1 of susceptible population who is not associated with poultry farms.
2. Reduction in the transmission rate of disease λ by using mask, maintaining social distance and boosting hygiene.
3. Increase the efficiency of mask a to minimize the risk of infection.
4. Increase the recovery rate υ by sustaining strength and nutrition.
5. Decrease the conversion rate from exposed to infectious one ξ by giving awareness about initial treatment.
However, it is also argued that infected population may have positive or negative impact of profession and surrounding. It also effect community structure, depends upon infection severity in the surroundings. We can also conclude that people who deals with animals and associate with poultry farms should take care themselves to save society from disease.

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