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On the anti fuzzy subsemirings under t-norms

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Abstract

In this paper we introduce anti T-fuzzy subsemirings and anti T-product of two fuzzy sets which can be regarded as a generalization of anti fuzzy subgroups under t-norms.

Keywords

Anti-T-fuzzy subsemiring, anti-T-product and homomorphism.

AMS Subject Classification

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Contents

1. Introduction

After an introduction of fuzzy sets by L.A. Zadeh [4] several researchers explored on the generalization of the notion of fuzzy set. Since the concept of fuzzy subgroups was introduced by Rosenfeld [1], it has been studying by several authors in [2, 5-10]. Recently, Biswas [8] has proposed the concept of anti fuzzy subgroups. We will generalize this concept to that of anti fuzzy subsemirings under t-norms and investigate some of their properties. We will also study the problems of the anti products of anti fuzzy subsemirings under t-norms.

Throughout this paper, let R be a ring, $I = [0,1]$. We will denote a t-norm by T and refer for its properties to $[7,10]$.

2. Preliminaries

Definition 2.1. *Let T_1 and T_2 be t-norms and $f : I \to I$ an order-preserving bijection. We say that T_2 is the conjugate of T_1, written as $\overline{T}_1$, if*
$\overline{T}_1(a,b) = T_2(a,b) = 1 - T_1(1-a, 1-b), \quad \forall a,b \in I.$
and that T_2 dominates T_1, written as $T_2 >> T_1$ or $T_1 << T_2$, if
$T_2(T_1(a,b), T_1(c,d)) >> T_1(T_2(a,c), T_2(b,d)) \quad \forall a,b,c,d \in I.$

Definition 2.2. *Let X be an ordinary set. By a fuzzy subsets u of X, we mean a function $u : X \to I$ with $u(x)$ as the grade of membership for $\forall x \in X$.*

Definition 2.3. *Let R be a semiring. A fuzzy subset A of R is said to be an anti-fuzzy subsemiring of R if it satisfies the following conditions.*
(i) $\mu_A(x+y) \le \max\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \le \max\{\mu_A(x), \mu_A(y)\}$, *for all x and y in R.*

Definition 2.4. *A fuzzy subsemiring of R under a t-norm T (called T-fuzzy subsemiring of R, for short) is a fuzzy subset u of R satisfying*
(i) $u(x+y) \ge T(u(x), u(y)), \quad \forall x,y \in R.$
(ii) $u(xy) \ge T(u(x), u(y)), \quad \forall x,y \in R.$

3. Anti Fuzzy Subsemiring under t-norms

Definition 3.1. *A fuzzy subset u of R is called anti-fuzzy subsemiring of R under a t-norm T (called anti-T-fuzzy subsemiring of R, for short) if*
(i) $u(x+y) \le \overline{T}(u(x), u(y))$
(ii) $u(xy) \le \overline{T}(u(x), u(y))$
where $\overline{T}$ is the conjugate of $T, \forall x,y \in R$.

Proposition 3.2. *A fuzzy subset u of R is an anti-T-fuzzy subsemiring of R iff its complement u^c, defined by $u^c(x) = 1 - u(x), \forall x \in R$, is a T-fuzzy subsemiring of R.*

Proof. Let u be an anti-T-fuzzy subsemiring of R.
Then (i) $u(x+y) \le \overline{T}(u(x), u(y))$
(ii) $u(xy) \le \overline{T}(u(x), u(y))$
Now $u^c(x+y) = 1 - u(x+y)$
$\ge 1 - \overline{T}(u(x), u(y))$

$= 1-[1-T(1-u(x),1-u(y))]$
$= T(u^c(x),u^c(y))$
and $u^c(xy) = 1-u(xy)$
$\geq 1-\overline{T}(u(x),u(y))$
$= 1-[1-T(1-u(x),1-u(y))]$
$= T(u^c(x),u^c(y))$
Hence u^c is a T-fuzzy subsemiring of R. □

Proposition 3.3. *Let T be a t-norm satisfying $T(a,b) < 1$, $\forall a,b \in (0,1)$. If u is an anti T-fuzzy subsemiring of R, then $L(u) = \{x \in R; u(x) < 1\}$ is a subsemiring of R.*

Proof. Let u be an anti-T-fuzzy subsemiring of R and $L(u) = \{x \in R; u(x) < 1\}$.
Let $x,y \in L(u)$. $\therefore u(x) < 1, u(y) < 1$.
$\therefore u(x+y) \leq \overline{T}(u(x),u(y))$
$= 1-T(1-u(x);1-u(y))$
< 1 $(\because T(1-u(x),1-u(y)) < 1)$.
$\therefore x+y \in L(u), \forall x,y \in L(u)$.
$u(xy) \leq \overline{T}(u(x),u(y))$
$= 1-T(1-u(x);1-u(y))$
< 1 $(\because T(1-u(x),1-u(y)) < 1)$.
$\therefore xy \in L(u), \forall x,y \in L(u)$.
$\therefore L(u)$ is a subsemiring of R. □

Definition 3.4. *Let X and Y be ordinary sets and $h : X \to Y$ be a mapping. If u is a fuzzy subsets of X, then the fuzzy subset $h(u)$ of Y defined by*

$$[h(u)](y) = \begin{cases} \inf\limits_{x \in h^{-1}(y)} & u(x)/h(x) = y \text{ if } y \in h(x) \\ 0 & \text{otherwise} \end{cases}$$

is called the image of u under h.

Definition 3.5. *If u is a fuzzy subset of Y, then the fuzzy subset $h^{-1}(u)$ of X defined by $[h^{-1}(u)](x) = u(h(x)), \forall x \in X$ is called the pre-image of u under h.*

Proposition 3.6. *Let h be a homomorphism of semiring R_1 into semiring R_2. If T is a continuous t-norm and u is an anti T-fuzzy subsemiring of R_1 then $h(u)$, the image of u under h, is an anti T-fuzzy subsemiring of R_2.*

$$[h(u)](y_1+y_2) = \inf_{x_1,x_2 \in R_1}\{u(x_1+x_2)/h(x_1+x_2) = y_1+y_2\}$$
$$\leq \inf_{x_1,x_2 \in R_1}\{\overline{T}(u(x_1),u(x_2))/h(x_1) = y_1; h(x_2) = y_2\}$$
$$= \overline{T}((\inf_{x_1 \in R_1} u(x_1)/h(x_1) = y_1),(\inf_{x_2 \in R_1} u(x_2)/h(x_2) = y_2))$$
$$= \overline{T}([h(u)](y_1),[h(u)](y_2))$$
$$[h(u)](y_1y_2) = \inf_{x_1,x_2 \in R_1}\{u(x_1x_2)/h(x_1x_2) = y_1y_2\}$$
$$\leq \inf_{x_1,x_2 \in R_1}\{\overline{T}(u(x_1),u(x_2))/h(x_1) = y_1; h(x_2) = y_2\}$$
$$= \overline{T}((\inf_{x_1 \in R_1} u(x_1)/h(x_1) = y_1),(\inf_{x_2 \in R_1} u(x_2)/h(x_2) = y_2))$$
$$= \overline{T}([h(u)](y_1),[h(u)](y_2)).$$
Hence $h(u)$ is anti T-fuzzy subsemiring.

Proposition 3.7. *Let h be a homomorphism of semiring R_1 into semiring R_2. If u is an anti-T-fuzzy subsemiring of R_2, then $h^{-1}(u)$, the pre-image of u under h, is an anti T-fuzzy subsemiring of R_1.*

Proof. Let $h : R_1 \to R_2$ be a homomorphism and u be an anti T-fuzzy subsemiring of R_2.
$[h^{-1}(u)](x_1+x_2) = u[h(x_1+x_2)]$
$= u[h(x_1)+h(x_2)]$
$\leq \overline{T}(u(h(x_1)),u(h(x_2)))$
$= \overline{T}([h^{-1}(u)](x_1),[h^{-1}(u)](x_2))$
$[h^{-1}(u)](x_1x_2) = u[h(x_1+x_2)]$
$= u[h(x_1)h(x_2)]$
$\leq \overline{T}(u(h(x_1)),u(h(x_2)))$
$= \overline{T}([h^{-1}(u)](x_1),[h^{-1}(u)](x_2))$.
Hence $h^{-1}(u)$ is an anti T-fuzzy subsemiring of R_1. □

4. Anti Products under t-norms of Anti T-fuzzy Subsemirings

Definition 4.1. *Let u and v be fuzzy subsets of R. The anti product of u and v under a t-norm T (called anti T-product of u and v for short), written as $[u.v]_{\overline{T}}$, is a fuzzy subset of R defined by*
$[u.v]_{\overline{T}}(x) = \overline{T}(u(x),v(x)), \forall x \in R$. Based on the properties of anti T-fuzzy subsemirings, we have the following properties of anti T-products.

Lemma 4.2. *If $T_2 >> T_1$, then $\overline{T}_2 << \overline{T}_1$.*

Proof. It is obvious. □

Proposition 4.3. *Let T_1 and T_2 be t-norms and $T_2 >> T_1$. If u and v are anti T_1-fuzzy subsemirings of R then $[u.v]_{\overline{T}_2}$, the anti T_2-product of u and v is also an anti T_1-fuzzy subsemiring of R_1.*

Proof.
$[u.v]_{\overline{T}_2}(x+y) = \overline{T}_2(u(x+y),v(x+y))$
$\leq \overline{T}_2(\overline{T}_1(u(x),u(y)),\overline{T}_1(v(x),v(y)))$
$\leq \overline{T}_1(\overline{T}_2(u(x),v(x)),\overline{T}_2(u(y),v(y)))$
$= \overline{T}_1((u.v)_{\overline{T}_2}(x),(u.v)_{\overline{T}_2}(y))$
$[u.v]_{\overline{T}_2}(xy) = \overline{T}_2(u(xy),v(xy))$
$\leq \overline{T}_2(\overline{T}_1(u(x),u(y)),\overline{T}_1(v(x),v(y)))$
$\leq \overline{T}_1(\overline{T}_2(u(x),v(x)),\overline{T}_2(u(y),v(y)))$
$= \overline{T}_1((u.v)_{\overline{T}_2}(x),(u.v)_{\overline{T}_2}(y))$.
Hence T_2 product of u and v is also anti T_1-fuzzy subsemiring of R. □

Proposition 4.4. *Let T_1, T_2 be t-norms and $T_2 >> T_1$. u and v are anti T_1-fuzzy subsemirings of R_1. If h is homomorphism of $R_1 \to R_2$ then $h^{-1}([u.v]_{\overline{T}_2})$ is also an anti T_1-fuzzy subsemiring of R_1.*

Proof.
$h^{-1}([u.v]_{\overline{T}_2})(x+y) = [u.v]_{\overline{T}_2}[h(x+y)]$
$= [u.v]_{\overline{T}_2}[h(x)+h(y)]$
$= \overline{T}_2(u(h(x)+h(y)),v(h(x)+h(y)))$
$\leq \overline{T}_2(\overline{T}_1(u(h(x)),u(h(y))),\overline{T}_1(v(h(x)),v(h(y))))$

$$\begin{aligned}
&\leq \overline{T}_1(\overline{T}_2(u(h(x)),v(h(x))),\overline{T}_2(u(h(y)),v(h(y))))\\
&= \overline{T}_1((u.v)_{\overline{T}_2}h(x),(u.v)_{\overline{T}_2}h(y))\\
&= \overline{T}_1(h^{-1}([u.v]_{\overline{T}_2}(x)),h^{-1}([u.v]_{\overline{T}_1}(y)))
\end{aligned}$$

$$\begin{aligned}
h^{-1}([u.v]_{\overline{T}_2})(xy) &= [u.v]_{\overline{T}_2}[h(xy)]\\
&= [u.v]_{\overline{T}_2}[h(x).h(y)]\\
&= \overline{T}_2(u(h(x).h(y)),v(h(x).h(y)))\\
&\leq \overline{T}_2(\overline{T}_1(u(h(x)),u(h(y))),\overline{T}_1(v(h(x)),v(h(y))))\\
&\leq \overline{T}_1(\overline{T}_2(u(h(x)),v(h(x))),\overline{T}_2(u(h(y)),v(h(y))))\\
&= \overline{T}_1((u.v)_{\overline{T}_2}h(x),(u.v)_{\overline{T}_2}h(y))\\
&= \overline{T}_1(h^{-1}([u.v]_{\overline{T}_2}(x)),h^{-1}([u.v]_{\overline{T}_2}(y))).
\end{aligned}$$

Hence $h^{-1}([u.v]_{\overline{T}_2})$ is also an anti-T_1-fuzzy subsemiring of R_1. □

Proposition 4.5. *Let T_1 be a continuous t-norm and the t-norm T_2 dominates T_1, u and v be anti T_1-fuzzy subsemiring of R_2. If h is a homomorphism of R_1 into R_2, then the image of $[u.v]_{\overline{T}_2}$ under $h, h([u.v])_{\overline{T}_2}$, is also an anti-$T_1$-fuzzy subsemiring of R_2.*

Proof.

$$\begin{aligned}
h([u.v]_{\overline{T}_2})(y_1+y_2) &= \inf_{x_1,x_2\in R_1}\{[u.v]_{\overline{T}_2}(x_1+x_2)/h(x_1+x_2)\\
&\qquad = y_1+y_2\}\\
&= \inf_{x_1,x_2\in R_1}\{\overline{T}_2(u(x_1+x_2),v(x_1+x_2))/h(x_1+x_2)\\
&\qquad = y_1+y_2\}\\
&\leq \inf_{x_1,x_2\in R_1}\{\overline{T}_2(\overline{T}_1(u(x_1),u(x_2)),\overline{T}_1(v(x_1),v(x_2)))/h(x_1)\\
&\qquad = y_1; h(x_2)=y_2\}\\
&\leq \inf_{x_1,x_2\in R_1}\{\overline{T}_1(\overline{T}_2(u(x_1),v(x_1)),\overline{T}_2(u(x_2),v(x_2)))/h(x_1)\\
&\qquad = y_1; h(x_2)=y_2\}\\
&= \inf_{x_1,x_2\in R_1}\{\overline{T}_1([u.v]_{\overline{T}_2}(x_1),[u.v]_{\overline{T}_2}(x_2))/h(x_1)\\
&\qquad = y_1; h(x_2)=y_2\}\\
&= \overline{T}_1((\inf_{x_1\in R_1}[u.v]_{\overline{T}_2}(x_1)/h(x_1)=y_1),\\
&\qquad (\inf_{x_2\in R_2}[u.v]_{\overline{T}_2}(x_2)/h(x_2)=y_2))\\
&= \overline{T}_1(h([u.v])_{\overline{T}_2}(y_1),h([u.v]_{\overline{T}_2})(y_2)).
\end{aligned}$$

$$\begin{aligned}
h([u.v]_{\overline{T}_2})(y_1y_2) &= \inf_{x_1,x_2\in R_1}\{[u.v]_{\overline{T}_2}(x_1x_2)/h(x_1x_2)=y_1y_2\}\\
&= \inf_{x_1,x_2\in R_1}\{\overline{T}_2(u(x_1x_2),v(x_1x_2))/h(x_1x_2)=y_1y_2\}\\
&\leq \inf_{x_1,x_2\in R_1}\{\overline{T}_2(\overline{T}_1(u(x_1),u(x_2)),\overline{T}_1(v(x_1),v(x_2)))\\
&\qquad /h(x_1)=y_1; h(x_2)=y_2\}\\
&\leq \inf_{x_1,x_2\in R_1}\{\overline{T}_1(\overline{T}_2(u(x_1),v(x_1)),\overline{T}_2(u(x_2),v(x_2)))\\
&\qquad /h(x_1)=y_1; h(x_2)=y_2\}\\
&= \inf_{x_1,x_2\in R_1}\{\overline{T}_1([u.v]_{\overline{T}_2}(x_1),[u.v]_{\overline{T}_2}(x_2))\\
&\qquad /h(x_1)=y_1; h(x_2)=y_2\}\\
&= \overline{T}_1((\inf_{x_1\in R_1}[u.v]_{\overline{T}_2}(x_1)/h(x_1)=y_1),\\
&\qquad (\inf_{x_2\in R_2}[u.v]_{\overline{T}_2}(x_2)/h(x_2)=y_2))\\
&= \overline{T}_1(h([u.v])_{\overline{T}_2}(y_1),h([u.v]_{\overline{T}_2})(y_2)).
\end{aligned}$$

Hence $h([u.v]_{\overline{T}_2})$ is also an anti T_1-fuzzy subsemiring of R_2. □

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