



Properties of intuitionistic anti fuzzy normal subrings

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Abstract

In this paper we introduce the definition of intuitionistic anti fuzzy normal subrings. We also made an attempt to study the algebraic nature of intuitionistic anti fuzzy normal subrings of a ring.

Keywords

Intuitionistic fuzzy subsets, intuitionistic anti fuzzy subring, intuitionistic anti fuzzy normal subrings.

AMS Subject Classification

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1. Introduction

After an introduction of fuzzy sets by L.A. Zadeh [8] several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic fuzzy set was introduced by K.T. Atanassov [1] as a generalization of the notion of a fuzzy set. In this paper, we discuss algebraic nature of intuitionistic anti fuzzy normal subrings and prove some results on these.

2. Preliminaries

Definition 2.1. An intuitionistic fuzzy subset (IFS) A in χ is defined as an object of the form $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in \chi\}$, where $\mu_A : \chi \rightarrow [0, 1]$ and $\gamma_A : \chi \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in \chi$ respectively and for every $x \in \chi$ satisfying $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.2. Let $(R, +, \cdot)$ be a ring. An intuitionistic fuzzy A of R is said to be an intuitionistic anti fuzzy subring of R (IAFSR) if it satisfies the following axioms:

1. $\mu_A(x - y) \leq \max\{\mu_A(x), \mu_A(y)\}$
2. $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$
3. $\gamma_A(x - y) \geq \min\{\gamma_A(x), \gamma_A(y)\}$
4. $\gamma_A(xy) \geq \min\{\gamma_A(x), \gamma_A(y)\}$, for all $x, y \in R$.

Definition 2.3. Let R be a ring. An intuitionistic anti fuzzy subring A of R is said to be an intuitionistic anti fuzzy normal subring (IAFNSR) of R if it satisfies the following axioms:

1. $\mu_A(xy) = \mu_A(yx)$
2. $\gamma_A(xy) = \gamma_A(yx)$, for all $x, y \in R$.

Definition 2.4. Let A and B be intuitionistic fuzzy sets of the rings with identity R_1 and R_2 respectively and $A \times B$ is an intuitionistic anti fuzzy subring of $R_1 \times R_2$. Then the following are true.

1. If $\mu_A(x) \geq \mu_B(e')$ and $\gamma_A(x) \leq \gamma_B(e')$ then A is an intuitionistic anti fuzzy subring of R_1 .
2. If $\mu_B(x) \geq \mu_A(e)$ and $\gamma_B(x) \leq \gamma_A(e)$, then B is an intuitionistic anti fuzzy subring of R_2 .
3. Either A is an intuitionistic anti fuzzy subring of R_1 or B is an intuitionistic anti fuzzy subring of R_2 .

Definition 2.5. Let A and B be two intuitionistic anti fuzzy subrings R_1 and R_2 respectively. The product of A and B , denoted by $A \times B$, is defined as

$A \times B = \{ \langle (x, y), \mu_{A \times B}(x, y), \gamma_{A \times B}(x, y) \rangle / \text{for all } x \in R_1 \text{ and } y \in R_2 \}$, where $\mu_{A \times B}(x, y) = \max\{\mu_A(x), \mu_B(y)\}$ and $\gamma_{A \times B}(x, y) = \min\{\gamma_A(x), \gamma_B(y)\}$.

3. Properties of intuitionistic anti fuzzy normal subrings

Theorem 3.1. If A and B are two intuitionistic anti fuzzy normal subrings of a ring R , then their intersection $A \cap B$ is an intuitionistic anti fuzzy normal subring of R .

Proof. Let $x, y \in R$.

Let $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle / x \in R \}$ and

$B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle / x \in R \}$ be intuitionistic anti fuzzy normal subrings of a ring R .

Let $C = A \cap B$ and $C = \{ \langle x, \mu_C(x), \gamma_C(x) \rangle / x \in R \}$

where $\max\{\mu_A(x), \mu_B(x)\} = \mu_C(x)$ and

$\min\{\gamma_A(x), \gamma_B(x)\} = \gamma_C(x)$.

Clearly, C is an intuitionistic anti fuzzy subring of a ring R .

Since A and B are two intuitionistic anti fuzzy subring of a ring R . Now

$$\begin{aligned} \mu_C(xy) &= \max\{\mu_A(xy), \mu_B(xy)\} \\ &= \max\{\mu_A(yx), \mu_B(yx)\} && \text{by definition} \\ &= \mu_C(yx) \end{aligned}$$

Therefore $\mu_C(xy) = \mu_C(yx)$, for all $x, y \in R$.

Also

$$\begin{aligned} \gamma_C(xy) \min\{\gamma_A(xy), \gamma_B(xy)\} \\ &= \min\{\gamma_A(yx), \gamma_B(yx)\} && \text{by definition} \\ &= \gamma_C(yx) \end{aligned}$$

Therefore $\gamma_C(xy) = \gamma_C(yx)$, for all $x, y \in R$. Hence intersection of two intuitionistic anti fuzzy normal subring is an intuitionistic anti fuzzy normal subring of a ring R . \square

Theorem 3.2. If A is an intuitionistic anti fuzzy normal subring of a ring R , then $\square A$ is an intuitionistic anti fuzzy normal subring of a ring R .

Proof. Let $\square A = \{ \langle x, \mu_A(x), \mu_A^C(x) \rangle / x \in R \}$.

Since A is an intuitionistic anti fuzzy subring

$$\begin{aligned} \mu_A(x-y) &\leq \max\{\mu_A(x), \mu_A(y)\} \\ \mu_A(xy) &\leq \max\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

Now

$$\begin{aligned} \mu_A^C(x-y) &= 1 - \mu_A(x-y) \\ &\geq 1 - \max\{\mu_A(x), \mu_A(y)\} \\ &= \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \min\{\mu_A^C(x), \mu_A^C(y)\} \end{aligned}$$

$$\begin{aligned} \mu_A^C(xy) &= 1 - \mu_A(xy) \\ &\geq 1 - \max\{\mu_A(x), \mu_A(y)\} \\ &= \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \min\{\mu_A^C(x), \mu_A^C(y)\} \end{aligned}$$

Therefore $\square A$ is intuitionistic anti fuzzy subring of R .

Now

$$\begin{aligned} \mu^C(xy) &= 1 - \mu(xy) \\ &= 1 - \mu(yx) \\ &= \mu^C(yx) \end{aligned}$$

Therefore $\square A$ is intuitionistic anti fuzzy normal subring of R . \square

Theorem 3.3. If A is an intuitionistic anti fuzzy normal subring of a ring R , then $\diamond A$ is an intuitionistic anti fuzzy normal subring of a ring R .

Proof. Let $\diamond A = \{ \langle x, \gamma^C(x), \gamma(x) \rangle / x \in R \}$.

Since A is an intuitionistic anti fuzzy subring of R ,

$$\begin{aligned} \gamma_A(x-y) &\geq \min\{\gamma(x), \gamma(y)\} \\ \gamma(xy) &\geq \min\{\gamma(x), \gamma(y)\}, \quad \text{for all } x, y \in R. \end{aligned}$$

Now

$$\begin{aligned} \gamma_A^C(x-y) &= 1 - \gamma_A(x-y) \\ &\leq 1 - \min\{\gamma(x), \gamma(y)\} \\ &= \max\{1 - \gamma(x), 1 - \gamma(y)\} \\ &= \max\{\gamma^C(x), \gamma^C(y)\} \end{aligned}$$

$$\begin{aligned} \gamma_A^C(xy) &= 1 - \gamma_A(xy) \\ &\leq 1 - \min\{\gamma(x), \gamma(y)\} \\ &= \max\{1 - \gamma(x), 1 - \gamma(y)\} \\ &= \max\{\gamma^C(x), \gamma^C(y)\} \end{aligned}$$

Therefore $\diamond A$ is intuitionistic anti fuzzy subring of R .

Now

$$\begin{aligned} \gamma^C(xy) &= 1 - \gamma(xy) \\ &= 1 - \gamma(yx) \\ &= \gamma^C(yx) \end{aligned}$$

Therefore $\diamond A$ is intuitionistic anti fuzzy normal subring of R . \square

Theorem 3.4. An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ is an intuitionistic anti fuzzy normal subring of a ring R if and only if the fuzzy subsets μ_A and γ_A^C are anti fuzzy normal subring of R .

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic anti fuzzy normal subring of R . Then clearly μ_A is an anti fuzzy normal subring



of R .
Now

$$\begin{aligned}\gamma_A^C(x-y) &= 1 - \gamma_A(x-y) \\ &\leq 1 - \min\{\gamma_A(x), \gamma_A(y)\} \\ &= \max\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= \max\{\gamma_A^C(x), \gamma_A^C(y)\}\end{aligned}$$

$$\begin{aligned}\gamma_A^C(xy) &= 1 - \gamma_A(xy) \\ &\leq 1 - \min\{\gamma_A(x), \gamma_A(y)\} \\ &= \max\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= \max\{\gamma_A^C(x), \gamma_A^C(y)\}\end{aligned}$$

$$\begin{aligned}\gamma_A^C(xy) &= 1 - \gamma_A(xy) \\ &= 1 - \gamma_A(yx) \\ &= \gamma_A^C(yx)\end{aligned}$$

Thus γ_A^C is an anti fuzzy normal subring of R .

Conversely, μ_A and γ_A^C are anti fuzzy normal subring of R .

$$\begin{aligned}\gamma_A^C(x-y) &\leq \max\{\gamma_A^C(x), \gamma_A^C(y)\} \\ 1 - \gamma_A(x-y) &\leq \max\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= 1 - \min\{\gamma_A(x), \gamma_A(y)\} \\ \Rightarrow \gamma_A(x-y) &\geq \min\{\gamma_A(x), \gamma_A(y)\} \\ \gamma_A^C(xy) &\leq \max\{\gamma_A^C(x), \gamma_A^C(y)\} \\ 1 - \gamma_A(xy) &\leq \max\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= 1 - \min\{\gamma_A(x), \gamma_A(y)\} \\ \Rightarrow \gamma_A(xy) &\geq \min\{\gamma_A(x), \gamma_A(y)\} \\ \gamma_A^C(xy) &= \gamma_A^C(yx) \\ 1 - \gamma_A(xy) &= 1 - \gamma_A(yx) \\ \Rightarrow \gamma_A(xy) &= \gamma_A(yx)\end{aligned}$$

Thus $A = (\mu_A, \gamma_A)$ is an intuitionistic anti fuzzy normal subring of a ring R . \square

Theorem 3.5. An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ is an intuitionistic anti fuzzy normal subring of a ring R if and only if the fuzzy subsets μ_A^C and γ_A are fuzzy normal subrings of R .

Proof. Suppose $A = (\mu_A, \gamma_A)$ is an intuitionistic anti fuzzy normal subring of R . Clearly γ_A is a fuzzy normal subring of R . Now we have to show that μ_A^C is also a fuzzy normal subring of R .

Now

$$\begin{aligned}\gamma_A^C(x-y) &= 1 - \gamma_A(x-y) \\ &\geq 1 - \max\{\mu_A(x), \mu_A(y)\} \\ &= \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \min\{\mu_A^C(x), \mu_A^C(y)\}\end{aligned}$$

$$\begin{aligned}\mu_A^C(xy) &= 1 - \mu_A(xy) \\ &\geq 1 - \max\{\mu_A(x), \mu_A(y)\} \\ &= \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \min\{\mu_A^C(x), \mu_A^C(y)\}\end{aligned}$$

$$\begin{aligned}\mu_A^C(xy) &= 1 - \mu_A(xy) \\ &= 1 - \mu_A(yx) \\ &= \mu_A^C(yx)\end{aligned}$$

$\therefore \mu_A^C$ is fuzzy normal subring.

Conversely, μ_A^C, γ_A are fuzzy normal subring of R .

$$\begin{aligned}\mu_A^C(x-y) &\geq \min\{\mu_A^C(x), \mu_A^C(y)\} \\ 1 - \mu_A(x-y) &\geq \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= 1 - \max\{\mu_A(x), \mu_A(y)\}\end{aligned}$$

$\therefore \mu_A(x-y) \leq \max\{\mu_A(x), \mu_A(y)\}$

$$\begin{aligned}\mu_A^C(xy) &\geq \min\{\mu_A^C(x), \mu_A^C(y)\} \\ 1 - \mu_A(xy) &\geq \min\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= 1 - \max\{\mu_A(x), \mu_A(y)\}\end{aligned}$$

$\therefore \mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$

$$\begin{aligned}\mu_A^C(xy) &= \mu_A^C(yx) \\ 1 - \mu_A(xy) &= 1 - \mu_A(yx) \\ \mu_A(xy) &= \mu_A(yx)\end{aligned}$$

Thus $A = (\mu_A, \gamma_A)$ is an intuitionistic anti fuzzy normal subring of a ring R . \square

4. Direct product of intuitionistic anti fuzzy normal subrings

In this section we discuss direct product of intuitionistic anti fuzzy normal subrings. If R_1, R_2 are rings, then direct product $R_1 \times R_2$ of R_1 and R_2 is a ring with point wise addition '+' and multiplication 'o' defined as $(a, b) + (c, d) = (a + c, b + d)$ and $(a, b) \circ (c, d) = (ac, bd)$ respectively for every $(a, b), (c, d)$ in $R_1 \times R_2$.

Theorem 4.1. If A and B are two intuitionistic anti fuzzy normal subrings of rings R_1 and R_2 respectively, then $A \times B$ is an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$.

Proof. Let $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in R_1\}$ and $B = \{(y, \mu_B(y), \gamma_B(y)) / y \in R_2\}$ be intuitionistic anti fuzzy normal subrings of R_1 and R_2 respectively.

Now $A \times B = \{(x, y), \mu_{A \times B}(x, y), \gamma_{A \times B}(x, y)\} /$ for all $x \in R_1$ and $y \in R_2$, where $\mu_{A \times B}(x, y) = \max\{\mu_A(x), \mu_B(y)\}$ and $\gamma_{A \times B}(x, y) = \min\{\gamma_A(x), \gamma_B(y)\}$. We have to show that $A \times B$ is an intuitionistic anti fuzzy normal subrings of $R_1 \times R_2$. Let $(a, b), (c, d) \in R_1 \times R_2$.

Now,

$$\begin{aligned}\mu_{A \times B}((a, b) - (c, d)) &= \mu_{A \times B}(a - c, b - d) \\ &= \max\{\mu_A(a - c), \mu_B(b - d)\} \\ &= \mu_A(a - c) \gamma \mu_B(b - d) \\ &\leq \{\mu_A(a) \gamma \mu_A(c)\} \gamma \{\mu_B(b) \gamma \mu_B(d)\} \\ &= \mu_A(a) \gamma \{\mu_A(c) \gamma \mu_B(b)\} \gamma \mu_B(d) \\ &= \mu_A(a) \gamma \{\mu_B(b) \gamma \mu_A(c)\} \gamma \mu_B(d) \\ &= \{\mu_A \gamma \mu_B(b)\} \gamma \{\mu_A(c) \gamma \mu_B(d)\} \\ &= \mu_{A \times B}(a, b) \gamma \mu_{A \times B}(c, d)\end{aligned}$$



and

$$\begin{aligned}
 \mu_{A \times B}((a, b) \circ (c, d)) &= \mu_{A \times B}(ac, bd) \\
 &= \max\{\mu_A(ac), \mu_B(bd)\} \\
 &= \mu_A(ac) \gamma \mu_B(bd) \\
 &\leq \{\mu_A(a) \gamma \mu_A(c)\} \gamma \{\mu_B(b) \gamma \mu_B(d)\} \\
 &= \mu_A(a) \gamma \{\mu_A(c) \gamma \mu_B(b)\} \gamma \mu_B(d) \\
 &= \mu_A(a) \gamma \{\mu_B(b) \gamma \mu_A(c)\} \gamma \mu_B(d) \\
 &= \{\mu_A \gamma \mu_B(b)\} \gamma \{\mu_A(c) \gamma \mu_B(d)\} \\
 &= \mu_{A \times B}(a, b) \gamma \mu_{A \times B}(c, d)
 \end{aligned}$$

$$\begin{aligned}
 \mu_{A \times B}((a, b) \circ (c, d)) &= \mu_{A \times B}(ac, bd) \\
 &= \max\{\mu_A(ac), \mu_B(bd)\} \\
 &= \max\{\mu_A(ca), \mu_B(db)\}, \\
 &\text{since } A \text{ and } B \text{ are IAFNSRs} \\
 &= \mu_{A \times B}(ca, db) \\
 &= \mu_{A \times B}((c, d) \circ (a, b))
 \end{aligned}$$

Similarly, $\gamma_{A \times B}((a, b) - (c, d)) \geq \gamma_{A \times B}(a, b) \wedge \gamma_{A \times B}(c, d)$,
 $\gamma_{A \times B}((a, b) \circ (c, d)) \geq \gamma_{A \times B}(a, b) \wedge \gamma_{A \times B}(c, d)$ and
 $\gamma_{A \times B}((a, b) \circ (c, d)) = \gamma_{A \times B}((c, d) \circ (a, b))$.

Hence $A \times B$ is an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$. \square

Theorem 4.2. Let A and B be intuitionistic fuzzy sets of the rings R_1 and R_2 respectively. Suppose that e_1 and e_2 are identity element of R_1 and R_2 respectively. If $A \times B$ is an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$, then at least one of the following two statements must holds.

1. $\mu_B(e_2) \leq \mu_A(x)$ and $\gamma_B(e_2) \geq \gamma_A(x)$, for all $x \in R_1$.
2. $\mu_A(e_1) \leq \mu_B(y)$ and $\gamma_A(e_1) \geq \gamma_B(y)$, for all $y \in R_2$.

Proof. Let $A \times B$ is an IAFNSR of $R_1 \times R_2$. If possible, let the statements (i) and (ii) does not holds.

Then we can find $x \in R_1$ and y in R_2 such that $\mu_A(x) < \mu_B(e_2)$, $\gamma_A(x) > \gamma_B(e_2)$ and $\mu_B(y) < \mu_A(e_1)$, $\gamma_B(y) > \gamma_A(e_1)$. Thus we have

$$\begin{aligned}
 \mu_{A \times B}(x, y) &= \max\{\mu_A(x), \mu_B(y)\} \\
 &< \max\{\mu_A(e_1), \mu_B(e_2)\} \\
 &= \mu_{A \times B}\{e_1, e_2\}
 \end{aligned}$$

and

$$\begin{aligned}
 \gamma_{A \times B}(x, y) &= \min\{\gamma_A(x), \gamma_B(y)\} \\
 &> \min\{\gamma_A(e_1), \gamma_B(e_2)\} \\
 &= \gamma_{A \times B}\{e_1, e_2\}
 \end{aligned}$$

which implies that $A \times B$ is not an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$. A contradiction. Hence either $\mu_B(e_2) \leq \mu_A(x)$ and $\gamma_B(e_2) \geq \gamma_A(x)$ holds for all x in R_1 or $\mu_A(e_1) \leq \mu_B(y)$ and $\gamma_A(e_1) \geq \gamma_B(y)$, holds for all y in R_2 . \square

Theorem 4.3. Let A and B be intuitionistic fuzzy set of the subring R_1 and R_2 respectively such that $\mu_A(x) \geq \mu_B(e_2)$ and $\gamma_A(x) \leq \gamma_B(e_2)$ holds for all $x \in R_1$, e_2 being the identity element of R_2 . If $A \times B$ is an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$, then A is intuitionistic anti fuzzy normal subring of subring R_1 .

Proof. Let $\mu_A(x) \geq \mu_B(e_2)$ and $\gamma_A(x) \leq \gamma_B(e_2)$, for all $x \in R_1$. We have to show that A is intuitionistic anti fuzzy normal subring of R_1 .

Now

$$\begin{aligned}
 \mu_A(x - y) &= \mu_A(x + (-y)) \\
 &= \max\{\mu_A(x + (-y)), \mu_B(e_2 + (-e_2))\} \\
 &= \mu_{A \times B}((x + (-y)), e_2 + (-e_2)) \\
 &= \mu_{A \times B}((x, e_2) + (-y, -e_2)) \\
 &= \mu_{A \times B}((x, e_2) - (y, e_2)) \\
 &\leq \mu_{A \times B}(x, e_2) \vee \mu_{A \times B}(y, e_2), \\
 &\text{since } A \times B \text{ is IAFNSR} \\
 &= \max\{\mu_A(x), \mu_B(e_2)\} \vee \max\{\mu_A(y), \mu_B(e_2)\} \\
 &= \mu_A(x) \vee \mu_A(y)
 \end{aligned}$$

and

$$\begin{aligned}
 \mu_A(xy) &= \max\{\mu_A(xy), \mu_B(e_2 e_2)\} \\
 &= \mu_{A \times B}(xy, e_2 e_2) \\
 &= \mu_{A \times B}(x, e_2) \cdot (y, e_2) \\
 &\leq \mu_{A \times B}(x, e_2) \vee \mu_{A \times B}(y, e_2), \text{ since } A \times B \text{ is IAFNSR} \\
 &= \max\{\mu_A(x), \mu_B(e_2)\} \vee \max\{\mu_A(y), \mu_B(e_2)\} \\
 &= \mu_A(x) \vee \mu_A(y)
 \end{aligned}$$

Now

$$\begin{aligned}
 \mu_A(xy) &= \max\{\mu_A(xy), \mu_B(e_2 e_2)\} \\
 &= \mu_{A \times B}((xy, e_2 e_2)) \\
 &= \mu_{A \times B}((x, e_2) \cdot (y, e_2)) \\
 &= \mu_{A \times B}((y, e_2) \cdot (x, e_2)), \text{ since } A \times B \text{ is IAFNSR} \\
 &= \mu_{A \times B}(yx, e_2 e_2) \\
 &= \max\{\mu_A(yx), \mu_B(e_2 e_2)\} \\
 &= \mu_A(yx)
 \end{aligned}$$

Similarly, we can prove that $\gamma_A(x - y) \geq \min\{\gamma_A(x), \gamma_A(y)\}$, $\gamma(xy) \geq \min\{\gamma_A(x), \gamma_A(y)\}$ and $\gamma_A(xy) = \gamma_A(yx)$ for all $x, y \in R_1$. Thus A is an intuitionistic anti fuzzy normal subring of R_1 . \square

Theorem 4.4. Let A and B be intuitionistic fuzzy set of the subring R_1 and R_2 respectively such that $\mu_B(y) \leq \mu_B(e_1)$ and $\gamma_B(y) \leq \gamma_B(e_1)$ holds for all $y \in R_2$, e_1 being the identity element of R_1 . If $A \times B$ is an intuitionistic anti fuzzy normal subring of $R_1 \times R_2$, then B is intuitionistic anti fuzzy normal subring of subring R_2 .

Proof. The proof is similar to the above theorem. \square



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