Malaya Journal of Matematik, Vol. 7, No. 2, 353-365, 2019

https://doi.org/10.26637/MJM0702/0031

LMI conditions for delay probability distribution dependent robust stability analysis of markovian jump stochastic neural networks with time-varying delays

N. Mala [1]*, A.R. Sudamani Ramaswamy [2] and A. Vinodkumar[3]

Abstract

This paper investigates the robust stability analysis for a class of uncertain stochastic neural networks (SNNs)with markovian jump and time-varying delays. Based on the stochastic analysis approach & Lyapunov-Krasovskii functional, a delay probability distribution dependent sufficient condition is obtained in the linear matrix inequality (LMI) form such that delayed markovian jump SNNs are robustly globally asymptotically stable in the mean square for all admissible uncertainties. An important feature of the result is that the stability conditions are dependent on the probability distribution of delays and upper bound of the derivative is allowed to be greater than or equal to 1. Numerical examples are given for the comparison to illustrate the effectiveness of our results.

Keywords

Delay probability distribution dependent, Linear matrix inequality, Lyapunov-Krasovskii functional, Markovian jump stochastic neural networks.

AMS Subject Classification 90B15, 35B35, 93E15.

[1] *Department of Mathematics, Kovai Kalaimagal College of Arts and Science, Coimbatore-641 109, Tamil Nadu, India.*
[2] *Department of Mathematics, Avinashilingam Deemed University for Women, Coimbatore-641 043, Tamil Nadu, India.*
[3] *Department of Mathematics, Amrita Vishwa Vidyapeetham University, Coimbatore-641 112, Tamil Nadu, India.*
***Corresponding author**: [1] mala.adu@gmail.com; [2]arsudamani@hotmail.com; [3]vinod026@gmail.com,a_vinodkumar@cb.amrita.edu
Article History: Received **24** March **2019**; Accepted **09** May **2019**

Contents

1. Introduction

In recent years, dynamics of neural networks have been widely studied due to their extensive applications in aerospace, defense, robotic, telecommunications, signal processing, pattern recognition, static image processing, associative memory and combinatorial optimization [1]. During the implementation of artificial neural networks, time delays often arise in the processing of information storage and transmission. Some of these applications require the equilibrium points of designed networks to be stable, see for example [2]-[9] and references therein. Furthermore, time delay is frequently a source of oscillation, divergence, or even instability and deterioration of neural networks. Generally speaking, the so-far obtained stability results for delayed neural networks can be classified into two types; that is, delay-independent stability [10]-[12] and delay-dependent stability [13]-[15]. In addition, the problem of neural networks with probability-distribution delay is investigated in [16, 17] and the references therein.

As time delays, there are two types of disturbances that is, parametric uncertainty and stochastic perturbations. First, uncertainties are frequently encountered in various engineering and communication systems. The characteristics of dynamic systems are significantly affected by the presence of the uncertainty, even to the extend of instability in extreme situation. The desired stability properties of neural networks are customarily based on imposing constraint conditions on the network parameters neural system. It is desired that the stability properties of neural networks should be affected by the small deviations in the values of the parameters. In other words, the neural networks must be globally robust stable (see [18] and references therein). Next, in real nervous systems, the synaptic transmission with the noisy process give the random fluctuations in the neurotransmitters, see [1, 19–21]. Practically, in the design of electrical circuit for neural networks will cause the stochastic phenomenon [1]. For further study on stability of SNNs with time varying delays [22]-[37]. To the best of authors knowledge, very few authors have studied the delay-probability-distribution-dependent robust stability analysis of SNNs with time-varying delays, which is very important in both theories and applications and also is a very challenging problem.

Modern industrial applications are come upon with numerous hybrid behavior of the processes. For example, any malfunction of sensors or actuators can cause a jumping behaviour in process performance. This type of jumping behavior may be modelled as a Markov jump systems. In other words, the neural networks may have finite modes and the mode may jump from one to another at different times. The jumping between different modes can be governed by a Markov chain [38]-[41]. Thus, Markovian jump systems correspond to an important class of systems that are subject to abrupt process changes. The abrupt changes in the systems are discrete events and are assumed to be modelled by a Markov chain taking values in a finite value set. Practical motivations as well as many theoretical results for Markovian jump system can be found, for instance, in [42]-[44]. More recently, Luo et al. [45] studied the Robust fault detection of Markovian jump systems with different system modes. Wang et al. [46] investigated the Delay-dependent H_∞ control for singular Markovain jump systems with time delay. Therefore, neural networks with Markovian jump parameters have received a great deal of attention. So studies of the stability criteria and the performance for Markovian jump systems with delays are more important to theoretical and practical applications.

By the above discussions, the LMI conditions for delay-probability-distribution-dependent robust stability analysis of SNNs with time-varying delays are considered in this paper. By constructing a novel Lyapunov-Krasovskii functional, employing some analysis techniques and introducing some free-weighting matrices, sufficient conditions are derived for the considered SNNs in terms of LMIs, which can be easily calculated by MATLAB LMI control Toolbox. Numerical examples are given to illustrate the effectiveness and less conservativeness of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times n}$ denote the set of all $n\times n$ real matrices. The transposition denoted by superscript T. When X and Y are symmetric matrices $X \geq Y$ (respectively, $X > Y$), means that $X - Y$ is positive semi-definite (respectively, positive definite). I_n be the identity $n\times n$ matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Further, the complete probability space be $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. The notation $*$ always denotes the symmetric block in one symmetric matrix. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Problem description and preliminaries

Let $\{r(t), t\geq 0\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathfrak{F}, \mathscr{P})$ taking values in a finite state space $\mathscr{S} = \{1, 2, \ldots, N\}$ with generator $\mathscr{Q} = (q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} q_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + q_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0$, $q_{ij} \geq 0$ is the transition rate from i to j, if $i \neq j$ while $q_{ii} = -\sum_{j\neq i} q_{ij}$.

In this paper, we consider the following uncertainmarkovian jump stochastic Hopfield neural networks with time-varying delays:

$$\begin{aligned} dx(t) &= \Big[-A_i(t)x(t) + B_i(t)f(x(t)) \\ &\quad + W_i(t)f(x(t-\tau(t)))\Big]dt \\ &\quad + \Big[H_{0i}x(t) + H_{1i}x(t-\tau(t))\Big]dw(t) \\ x(t) &= \phi(t), \quad \forall t \in [-\bar{\tau}, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the neural state vector,

$$f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$$

is the neuron activation function vector with initial condition $f(0) = 0$. $w(t) = [w_1(t), \ldots, w_n(t)] \in \mathbb{R}^n$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}_t, \{\mathscr{F}_t\}_{t\geq 0}, P)$.

The time-varying delays $\tau(t)$ satisfies

$$0 \leq \tau(t) \leq \bar{\tau}, \quad \dot{\tau}(t) \leq \mu, \tag{2.2}$$

where $\bar{\tau}$ and μ are constants.In (2.1), $A_i(t) = A_i + \Delta A_i(t)$, $B_i(t) = B_i + \Delta B_i(t), W_i(t) = W_i + \Delta W_i(t)$ and where the diagonal matrices $A_i > 0 (i = 1, 2, \ldots, N)$ and B_i, W_i, H_{0i}, H_{1i} are connection weight matrices with appropriate dimensions. Further $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta W_i(t)$, denote the time-varying and norm-bounded uncertainties.

Assumption 2.1 Considering the information of probability distribution of the time delay $\tau(t)$, two sets and functions are defined $\Omega_1 = \{t : \tau(t) \in [0, \tau_0)\}$ and $\Omega_2 = \{t : \tau(t) \in [\tau_0, \bar{\tau}]\}$

$$\tau_1(t) = \begin{cases} \tau(t), & for \quad t \in \Omega_1 \\ \bar{\tau}_1, & for \quad t \in \Omega_2, \end{cases}$$

$$\text{and} \quad \tau_2(t) = \begin{cases} \tau(t), & for \quad t \in \Omega_2 \\ \bar{\tau}_2, & for \quad t \in \Omega_1, \end{cases} \tag{2.3}$$

$$\dot{\tau}_1(t) \le \mu_1 < \infty, \quad \dot{\tau}_2(t) \le \mu_2 < \infty \tag{2.4}$$

where $\tau_0 \in [0, \bar{\tau}]$, $\bar{\tau}_1 \in [0, \tau_0)$ and $\bar{\tau}_2 \in [\tau_0, \bar{\tau}]$. It is easy to know $t \in \Omega_1$ means the event $\tau(t) \in [0, \tau_0)$ occurs and $t \in \Omega_2$ means the event $\tau(t) \in [\tau_0, \bar{\tau}]$ occurs. Therefore, a stochastic variable $\alpha(t)$ can be defined as

$$\alpha(t) = \begin{cases} 1, & for \quad t \in \Omega_1 \\ 0, & for \quad t \in \Omega_2. \end{cases} \tag{2.5}$$

Assumption 2.2 $\alpha(t)$ is a Bernoulli distributed sequence with Prob$\{\alpha(t) = 1\} = \mathbb{E}\{\alpha(t)\} = \alpha_0$, Prob$\{\alpha(t) = 0\} = 1 - \mathbb{E}\{\alpha(t)\} = 1 - \alpha_0$, where $0 \le \alpha_0 \le 1$ is a constant and $\mathbb{E}\{\alpha(t)\}$ is the expectation of $\alpha(t)$.

Remark 2.1. *From Assumption 2.2, it is easy to know that* $\mathbb{E}\{\alpha(t) - \alpha_0\} = 0, \mathbb{E}\{(\alpha(t) - \alpha_0)^2\} = \alpha_0(1 - \alpha_0)$.

By Assumptions 2.1 and 2.2, the system (2.1) can be rewritten as

$$\begin{aligned} dx(t) = & [-A_i(t)x(t) + B_i(t)f(x(t)) \\ & + \alpha(t)W_i(t)f(x(t - \tau_1(t))) \\ & + (1 - \alpha(t))W_i(t)f(x(t - \tau_2(t)))]dt \\ & + [H_{0i}x(t) + \alpha(t)H_{1i}x(t - \tau_1(t)) \\ & + (1 - \alpha(t))H_{1i}x(t - \tau_2(t))]dw(t) \\ x(t) = & \xi(t), \quad t \in [-\bar{\tau}, 0], \end{aligned} \tag{2.6}$$

which is equivalent to

$$\begin{aligned} dx(t) = & [-A_i(t)x(t) + B_i(t)f(x(t)) \\ & + \alpha_0 W_i(t)f(x(t - \tau_1(t))) \\ & + (1 - \alpha_0)W_i(t)f(x(t - \tau_2(t))) \\ & + (\alpha(t) - \alpha_0)(W_i(t)f(x(t - \tau_1(t))) \\ & - W_i(t)f(x(t - \tau_2(t))))]dt \\ & + [H_{0i}x(t) + \alpha_0 H_{1i}x(t - \tau_1(t)) \\ & + (1 - \alpha_0)H_{1i}x(t - \tau_2(t)) \\ & + (\alpha(t) - \alpha_0)(H_{1i}x(t - \tau_1(t)) \\ & - H_{1i}x(t - \tau_2(t)))]dw(t) \\ x(t) = & \xi(t), \quad t \in [-\bar{\tau}, 0]. \end{aligned} \tag{2.7}$$

Remark 2.2. *The probability distribution of the delay taking values in some interval is assumed to be known in advance in this paper, and then a new model of the markovian jump SNNs (2.7) has been derived, which can be seen as an extension of the common markovian jump SNNs (2.1). Specially, when $\alpha(t) \equiv 1$, system (2.7) becomes system (2.1). When the probability of time delay taking values is known a priori, the possible values that the delay takes may be larger than those obtained based on the traditional methods, which will be illustrated via example in section* 4.

Assumption 2.3 The neural activation function $f_i(x_i)$ satisfies

$$l_i^- \le \frac{f_i(x_i) - f_i(y_i)}{x_i - y_i} \le l_i^+ \quad \forall x_i, y_i \in \mathbb{R}, \quad x_i \ne y_i, i = 1, \ldots, n \tag{2.8}$$

which implies that

$$(f_i(x_i) - l_i^+ x_i)(f_i(x_i) - l_i^- x_i) \le 0, \tag{2.9}$$

where l_i^-, l_i^+ are some constant.

The parameter uncertainties $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta W_i(t)$ are of the forms

$$\begin{bmatrix} \Delta A_i(t) & \Delta B_i(t) & \Delta W_i(t) \end{bmatrix} = H_i F_i(t) \begin{bmatrix} E_{1i} & E_{2i} & E_{3i} \end{bmatrix}, \tag{2.10}$$

where H_i, E_{1i}, E_{2i} and E_{3i} are given known matrices. $F_i(t)$ is an uncertain matrix satisfying

$$F_i^T(t)F_i(t) \le I. \tag{2.11}$$

Definition 2.3. *For system (2.7) and any $\xi \in L^2_{\mathscr{F}_0}([-\bar{\tau}, 0]; \mathbb{R}^n)$, the trivial solution is robustly, globally, asymptotically stable in the mean-square sense for all admissible uncertainties, if*

$$\lim_{t \longrightarrow \infty} \mathbb{E}|x(t, \xi)|^2 = 0.$$

Lemma 2.4. *[47](Schur Complement) Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, the inequality*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0,$$

holds, if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Lemma 2.5. *[48] For any constant matrix $M > 0$, any scalars a and b with $a < b$ and a vector function $x(t) : [a, b] \longrightarrow \mathbb{R}^n$ such that the integrals concerned are well defined, the following holds*

$$\Big[\int_a^b x(s)ds\Big]^T M \Big[\int_a^b x(s)ds\Big] \le (b - a)\Big[\int_a^b x(s)^T M x(s)ds\Big].$$

Lemma 2.6. *[49] Let $U, V(t), W$ and Z be real matrices of appropriate dimensions with Z satisfying $Z = Z^T$, then*

$$Z + UV(t)W + W^T V^T(t) U^T < 0, \quad V^T(t)V(t) \le I$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$Z + \varepsilon^{-1} U U^T + \varepsilon W^T W < 0,$$

3. MAIN RESULTS

Defining two new state variables for the markovian jump SNNs (2.7),

$$\begin{aligned} y(t) = & -A_i(t)x(t) + B_i(t)f(x(t)) + \alpha_0 W_i(t) \\ & \times f(x(t - \tau_1(t))) + (1 - \alpha_0)W_i(t)f(x(t - \tau_2(t))) \\ & + (\alpha(t) - \alpha_0) \\ & \times [W_i(t)f(x(t - \tau_1(t))) - W_i(t)f(x(t - \tau_2(t)))], \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} g(t) = {} & H_{0i}x(t) + \alpha_0 H_{1i}(x(t-\tau_1(t))) \\ & + (1-\alpha_0)H_{1i}x(t-\tau_2(t)) \\ & +(\alpha(t)-\alpha_0)[H_{1i}x(t-\tau_1(t)) - H_{1i}x(t-\tau_2(t))], \end{aligned} \tag{3.2}$$

the SNNs (2.7) can be written as

$$dx(t) = y(t)dt + g(t)d\omega(t). \tag{3.3}$$

Moreover, the following equality holds,

$$x(t) - x(t-\tau(t)) = \int_{t-\tau(t)}^{t} y(s)ds + \int_{t-\tau(t)}^{t} g(s)d\omega(s) \tag{3.4}$$

Theorem 3.1. *For given scalars $\tau_0 \geq 0$, $\bar{\tau}_0 > 0$, μ_1, $0 < \alpha_0 < 1$ satisfying $\alpha_0\mu_1 < 1$, the markovian jump SNNs (2.7) without uncertain parameters is asymptotically stable in the mean square if there exist matrices $P_i > 0$, $Q_i > 0, i = 1,2,3$, $R_l > 0$, $Z_l > 0$, $l = 1,2$, for any matrices $N_k, M_k, S_k, U_k, V_k, Y_k (k = 1,2)$ and there exist positive diagonal matrices $K_1 > 0$, $K_2 > 0$ and $K_3 > 0$ such that the following LMIs are feasible*

$$\Xi_1 = \begin{bmatrix} \Pi & -\alpha_0\tau_0 M \\ * & -\alpha_0\tau_0 Z_1 \end{bmatrix} < 0, \tag{3.5}$$

$$\Xi_2 = \begin{bmatrix} \Pi & -\tau_0(1-\alpha_0)N \\ * & -\tau_0(1-\alpha_0)Z_1 \end{bmatrix} < 0, \tag{3.6}$$

$$\Xi_3 = \begin{bmatrix} \Pi & -\tau_0 S \\ * & -\tau_0 Z_1 \end{bmatrix} < 0, \tag{3.7}$$

$$\Xi_4 = \begin{bmatrix} \Pi & -(\bar{\tau}-\tau_0)U \\ * & -(\bar{\tau}-\tau_0)Z_2 \end{bmatrix} < 0, \tag{3.8}$$

$$\Xi_5 = \begin{bmatrix} \Pi & -(\bar{\tau}-\tau_0)V \\ * & -(\bar{\tau}-\tau_0)Z_2 \end{bmatrix} < 0, \tag{3.9}$$

where

$$\Pi = \begin{bmatrix} \widetilde{\Omega} & M & N & S & U & V & \Gamma \\ * & -Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & -Z_1 & 0 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & * & -Z_2 & 0 \\ * & * & * & * & * & * & -\bar{P} \end{bmatrix} \tag{3.10}$$

and $\widetilde{\Omega} = (\widetilde{\Omega}_{i,j})_{10\times 10}$ *with*

$$\begin{aligned} \widetilde{\Omega}_{1,1} = {} & Q_1 + Q_2 + Q_3 + \sum_{j=1}^{N} q_{ij}P_j + M_1 + M_1^T - Y_1 A_i \\ & - A_i^T Y_1^T - K_1 L_1, \widetilde{\Omega}_{1,2} = -M_1 + M_2^T, \widetilde{\Omega}_{1,3} = 0, \\ & \widetilde{\Omega}_{1,4} = 0, \widetilde{\Omega}_{1,5} = 0, \widetilde{\Omega}_{1,6} = 0, \\ & \widetilde{\Omega}_{1,7} = -Y_1 - A_i^T Y_2^T + P_i, \widetilde{\Omega}_{1,8} = Y_1 B_i + K_1 L_2, \\ & \widetilde{\Omega}_{1,9} = \alpha_0 Y_1 W_i, \widetilde{\Omega}_{1,10} = (1-\alpha_0)Y_1 W_i, \\ & \widetilde{\Omega}_{2,2} = -(1-\alpha_0\mu_1)Q_1 - M_2 - M_2^T + N_1 + N_1^T, \\ & \widetilde{\Omega}_{2,3} = -N_1 + N_2^T, \widetilde{\Omega}_{2,4} = 0, \widetilde{\Omega}_{2,5} = 0, \widetilde{\Omega}_{2,6} = 0, \\ & \widetilde{\Omega}_{2,7} = 0, \widetilde{\Omega}_{2,8} = 0, \widetilde{\Omega}_{2,9} = 0, \widetilde{\Omega}_{2,10} = 0, \\ & \widetilde{\Omega}_{3,3} = -N_2 - N_2^T + S_1 + S_1^T - K_2 L_1 \\ & + \alpha_0(1-\alpha_0)\, H_{1i}^T \bar{P}\, H_{1i}, \widetilde{\Omega}_{3,4} = -S_1 + S_2^T, \\ & \widetilde{\Omega}_{3,5} = -\alpha_0(1-\alpha_0)\, {H_{1i}}^T \bar{P}\, H_{1i}, \\ & \widetilde{\Omega}_{3,6} = 0, \widetilde{\Omega}_{3,7} = 0, \widetilde{\Omega}_{3,8} = 0, \widetilde{\Omega}_{3,9} = K_2 L_2, \\ & \widetilde{\Omega}_{3,10} = 0, \widetilde{\Omega}_{4,4} = -Q_2 - S_2 - S_2^T + U_1 + U_1^T, \\ & \widetilde{\Omega}_{4,5} = -U_1 + U_2^T, \widetilde{\Omega}_{4,6} = 0, \widetilde{\Omega}_{4,7} = 0, \widetilde{\Omega}_{4,8} = 0, \\ & \widetilde{\Omega}_{4,9} = 0, \widetilde{\Omega}_{4,10} = 0, \widetilde{\Omega}_{5,5} = -U_2 - U_2^T \\ & + V_1 + V_1^T - K_3 L_1 + \alpha_0(1-\alpha_0)\, {H_{1i}}^T \bar{P}\, H_{1i}, \\ & \widetilde{\Omega}_{5,6} = -V_1 + V_2^T, \widetilde{\Omega}_{5,7} = 0, \\ & \widetilde{\Omega}_{5,8} = 0, \quad \widetilde{\Omega}_{5,9} = 0, \widetilde{\Omega}_{5,10} = K_3 L_2, \\ & \widetilde{\Omega}_{6,6} = -Q_3 - V_2 - V_2^T, \quad \widetilde{\Omega}_{6,7} = 0, \widetilde{\Omega}_{6,8} = 0, \\ & \widetilde{\Omega}_{6,9} = 0, \widetilde{\Omega}_{6,10} = 0, \\ & \widetilde{\Omega}_{7,7} = \tau_0 R_1 + (\bar{\tau}-\tau_0)R_2 - Y_2 - Y_2^T, \\ & \widetilde{\Omega}_{7,9} = \alpha_0 Y_2 W_i, \widetilde{\Omega}_{7,8} = Y_2 B_i, \\ & \widetilde{\Omega}_{7,10} = (1-\alpha_0)Y_2 W_i, \widetilde{\Omega}_{8,8} = -K_1 I, \\ & \widetilde{\Omega}_{8,9} = \widetilde{\Omega}_{8,10} = 0, \\ & \widetilde{\Omega}_{9,9} = -K_2 I, \widetilde{\Omega}_{10,10} = -K_3 I, \end{aligned}$$

$$M = \begin{bmatrix} M_1^T & M_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$N = \begin{bmatrix} 0 & N_1^T & N_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$S = \begin{bmatrix} 0 & 0 & S_1^T & S_2^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$U = \begin{bmatrix} 0 & 0 & 0 & U_1^T & U_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & V_1^T & V_2^T & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$Y = \begin{bmatrix} Y_1^T & 0 & 0 & 0 & 0 & 0 & Y_2^T & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\begin{aligned} \Gamma^T = {} & [\bar{P}\, H_{0i} \quad 0 \quad \alpha_0 \bar{P}\, H_{1i} \quad 0 \quad (1-\alpha_0)\bar{P}\, H_{1i} \\ & \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\ \bar{P} = {} & \tau_0 Z_1 + (\bar{\tau}-\tau_0)Z_2 + P_i. \end{aligned}$$

Proof. Consider the Lyapunov-Krasovskii functional

$$V(x_t,t) = V_1(x_t,t) + V_2(x_t,t) + V_3(x_t,t),$$

where

$$\begin{aligned}
V_1(x_t,t) &= x^T(t)\, P_i x(t),\\
V_2(x_t,t) &= \int_{t-\alpha_0\tau_1(t)}^{t} x^T(s)Q_1x(s)ds\\
&\quad + \int_{t-\tau_0}^{t} x^T(s)Q_2x(s)ds\\
&\quad + \int_{t-\bar{\tau}}^{t} x^T(s)Q_3x(s)ds,\\
V_3(x_t,t) &= \int_{-\tau_0}^{0}\int_{t+\theta}^{t} y^T(s)R_1y(s)dsd\theta\\
&\quad + \int_{-\bar{\tau}}^{-\tau_0}\int_{t+\theta}^{t} y^T(s)R_2y(s)dsd\theta,\\
&\quad + \int_{-\tau_0}^{0}\int_{t+\theta}^{t} g^T(s)Z_1g(s)dsd\theta\\
&\quad + \int_{-\bar{\tau}}^{-\tau_0}\int_{t+\theta}^{t} g^T(s)Z_2g(s)dsd\theta,
\end{aligned}$$

where $x_t = \{x(t+\theta) : -\bar{\tau} \le \theta \le 0\}$. Then, it can be obtained by Ito's formula that

$$dV(x_t,t) = LV(x_t,t)dt + 2x^T(t)\, P_i g(t)d\omega(t), \qquad (3.11)$$

where

$$\begin{aligned}
LV_1(x_t,t) &= 2x^T(t)\, P_i y(t) + g^T(t)\, P_i g(t)\\
&+ \sum_{j=1}^{N} q_{ij}x^T(t)P_jx(t),\\
LV_2(x_t,t) &\le x^T(t)Q_1x(t) - (1-\alpha_0\mu_1)x^T(t-\alpha_0\tau_1(t))\\
&\times Q_1x(t-\alpha_0\tau_1(t)) + x^T(t)Q_2x(t)\\
&- x^T(t-\tau_0)Q_2x(t-\tau_0) + x^T(t)Q_3x(t)\\
&- x^T(t-\bar{\tau})Q_3x(t-\bar{\tau})
\end{aligned}$$

$$\begin{aligned}
LV_3(x_t,t) &= \tau_0 y^T(t)R_1y(t) - \int_{t-\tau_0}^{t} y^T(s)R_1y(s)ds\\
&+ (\bar{\tau}-\tau_0)y^T(t)R_2y(t) - \int_{t-\bar{\tau}}^{t-\tau_0} y^T(s)R_2y(s)ds\\
&+ \tau_0 g^T(t)Z_1g(t) - \int_{t-\tau_0}^{t} g^T(s)Z_1g(s)ds\\
&+ (\bar{\tau}-\tau_0)g^T(t)Z_2g(t) - \int_{t-\bar{\tau}}^{t-\tau_0} g^T(s)Z_2g(s)ds.
\end{aligned}$$

$$\begin{aligned}
LV_3(x_t,t) &\le y^T(t)(\tau_0R_1 + (\bar{\tau}-\tau_0)R_2)y(t)\\
&- \int_{t-\alpha_0\tau_1(t)}^{t} y^T(s)R_1y(s)ds\\
&- \int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} y^T(s)R_1y(s)ds\\
&- \int_{t-\tau_0}^{t-\tau_1(t)} y^T(s)R_1y(s)ds - \int_{t-\tau_2(t)}^{t-\tau_0} y^T(s)R_2y(s)ds\\
&- \int_{t-\bar{\tau}}^{t-\tau_2(t)} y^T(s)R_2y(s)ds + g^T(t)(\tau_0Z_1 + (\bar{\tau}-\tau_0Z_2)g(t)\\
&- \int_{t-\alpha_0\tau_1(t)}^{t} g^T(s)Z_1g(s)ds\\
&- \int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} g^T(s)Z_1g(s)ds - \int_{t-\tau_0}^{t-\tau_1(t)} g^T(s)Z_1g(s)ds\\
&- \int_{t-\tau_2(t)}^{t-\tau_0} g^T(s)Z_2g(s)ds\\
&- \int_{t-\bar{\tau}}^{t-\tau_2(t)} g^T(s)Z_2g(s)ds.
\end{aligned}$$

From (2.9), for any matrices $K_i = diag(k_{i1}, k_{i2}, \ldots, k_{in}) \ge 0, i = 1,2,3$, it is easy to obtain

$$\begin{aligned}
&= \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -K_1L_1 & K_1L_2 \\ K_1L_2 & -K_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}\\
&+ \sum_{i=1}^{2} \begin{bmatrix} x(t-\tau_i(t)) \\ f(x(t-\tau_i(t))) \end{bmatrix}^T\\
&\begin{bmatrix} -K_{i+1}L_1 & K_{i+1}L_2 \\ K_{i+1}L_2 & -K_{i+1} \end{bmatrix} \begin{bmatrix} x(t-\tau_i(t)) \\ f(x(t-\tau_i(t))) \end{bmatrix} \ge 0,
\end{aligned} \qquad (3.12)$$

where $L_1 = diag\left(l_1^+l_1^-, \ldots, l_n^+l_n^-\right)$ and $L_2 = diag\left(\frac{l_1^++l_1^-}{2}, \ldots, \frac{l_n^++l_n^-}{2}\right)$ are matrices of appropriate dimensions. Now, we define the new vector

$$\begin{aligned}
\xi^T(t) &= \Big[x^T(t)\; x^T(t-\alpha_0\tau_1(t))x^T(t-\tau_1(t))\\
&\times x^T(t-\tau_0)\; x^T(t-\tau_2(t))x^T(t-\bar{\tau})\\
&\times y^T(t)f^T(x(t))\; f^T(x(t-\tau_1(t)))\; f^T(x(t-\tau_2(t)))\Big].
\end{aligned}$$

From (3.1), (3.2) and (3.4), we can see that the following equations hold for any matrices M, N, S, U, V and Y with appropriate dimensions,

$$\begin{aligned}
&2\xi^T(t)M\Big[x(t) - x(t-\alpha_0\tau_1(t)) - \int_{t-\alpha_0\tau_1(t)}^{t} y(s)ds\\
&- \int_{t-\alpha_0\tau_1(t)}^{t} g(s)d\omega(s)\Big] = 0,
\end{aligned} \qquad (3.13)$$

$$\begin{aligned}
&2\xi^T(t)N\Big[x(t-\alpha_0\tau_1(t)) - x(t-\tau_1(t)) -\\
&\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} y(s)ds - \int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)} g(s)d\omega(s)\Big] = 0,
\end{aligned} \qquad (3.14)$$

$$2\xi^T(t)S\Big[x(t-\tau_1(t))-x(t-\tau_0)-\int_{t-\tau_0}^{t-\tau_1(t)}y(s)ds - \int_{t-\tau_0}^{t-\tau_1(t)}g(s)d\omega(s)\Big]=0, \quad (3.15)$$

$$2\xi^T(t)U\Big[x(t-\tau_0)-x(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_0}y(s)ds - \int_{t-\tau_2(t)}^{t-\tau_0}g(s)d\omega(s)\Big]=0, \quad (3.16)$$

$$2\xi^T(t)V\Big[x(t-\tau_2(t))-x(t-\bar{\tau})-\int_{t-\bar{\tau}}^{t-\tau_2(t)}y(s)ds - \int_{t-\bar{\tau}}^{t-\tau_2(t)}g(s)d\omega(s)\Big]=0, \quad (3.17)$$

$$2\xi^T(t)Y\Big[-A_i(t)x(t)+B_i(t)f(x(t)) + \alpha_0 W_i(t)f(x(t-\tau_1(t)))+(1-\alpha_0)W_i(t)f(x(t-\tau_2(t))) + (\alpha(t)-\alpha_0)[W_i(t)f(x(t-\tau_1(t))) - W_i(t)f(x(t-\tau_2(t)))]-y(t)\Big]. \quad (3.18)$$

From the above equations (3.13)-(3.17), we have

$$\begin{aligned}
&-2\xi^T(t)M\int_{t-\alpha_0\tau_1(t)}^{t}g(s)d\omega(s)\le \xi^T(t)MZ_1^{-1}M^T\xi(t)\\
&+\int_{t-\alpha_0\tau(t)}^{t}g^T(s)d\omega(s)Z_1\int_{t-\alpha_0\tau(t)}^{t}g(s)d\omega(s),\\
&-2\xi^T(t)N\int_{t-\tau_1}^{t-\alpha_0\tau_1(t)}g(s)d\omega(s)\le \xi^T(t)NZ_1^{-1}N^T\xi(t)\\
&+\int_{t-\tau_1}^{t-\alpha_0\tau_1(t)}g^T(s)d\omega(s),Z_1\int_{t-\tau_1}^{t-\alpha_0\tau_1(t)}g(s)d\omega(s)\\
&-2\xi^T(t)S\int_{t-\tau_0}^{t-\tau_1(t)}g(s)d\omega(s)\le \xi^T(t)SZ_1^{-1}S^T\xi(t)\\
&+\int_{t-\tau_0}^{t-\tau_1(t)}g^T(s)d\omega(s)Z_1\int_{t-\tau_0}^{t-\tau_1(t)}g(s)d\omega(s),\\
&-2\xi^T(t)U\int_{t-\tau_2(t)}^{t-\tau_0}g(s)d\omega(s)\le \xi^T(t)UZ_2^{-1}U^T\xi(t)\\
&+\int_{t-\tau_2(t)}^{t-\tau_0}g^T(s)d\omega(s)Z_2\int_{t-\tau_2(t)}^{t-\tau_0}g(s)d\omega(s),\\
&-2\xi^T(t)V\int_{t-\bar{\tau}}^{t-\tau_2(t)}g(s)d\omega(s)\le \xi^T(t)VZ_2^{-1}V^T\xi(t)\\
&+\int_{t-\bar{\tau}}^{t-\tau_2(t)}g^T(s)d\omega(s)Z_2\int_{t-\bar{\tau}}^{t-\tau_2(t)}g(s)d\omega(s). \quad (3.19)
\end{aligned}$$

By Remark 2.1, it is easy to derive the following equality

$$\begin{aligned}
&\mathbb{E}\{g^T(t)\Big(P_i+\tau_0 Z_1+(\bar{\tau}-\tau_0)Z_2\Big)g(t)\}=\\
&\mathbb{E}\Big\{[H_{0i}x(t)+\alpha_0 H_{1i}x(t-\tau_1(t))+(1-\alpha_0)\\
&\times H_{1i}x(t-\tau_2(t))]^T\\
&\times \bar{P}[H_{0i}x(t)+\alpha_0 H_{1i}x(t-\tau_1(t))+(1-\alpha_0)\\
&\times H_{1i}x(t-\tau_2(t))]\\
&+2(\alpha(t)-\alpha_0)[H_{0i}x(t)+\alpha_0 H_{1i}x(t-\tau_1(t))\\
&+(1-\alpha_0)H_{1i}x(t-\tau_2(t))]^T\\
&\times \bar{P}[H_{1i}x(t-\tau_1(t))-Dx(t-\tau_2(t))]+(\alpha(t)-\alpha_0)^2\\
&\times [H_{1i}x(t-\tau_1(t))-Dx(t-\tau_2(t))]^T\bar{P}[H_{1i}x(t-\tau_1(t))\\
&-H_{1i}x(t-\tau_2(t))]\Big\}\\
&=[H_{0i}x(t)\\
&+\alpha_0 H_{1i}x(t-\tau_1(t))+(1-\alpha_0)H_{1i}x(t-\tau_2(t))]^T\\
&\times \bar{P}[H_{0i}x(t)+\alpha_0 H_{1i}x(t-\tau_1(t))+(1-\alpha_0)\\
&\times H_{1i}x(t-\tau_2(t))]+\alpha_0(1-\alpha_0)[H_{1i}x(t-\tau_1(t))\\
&-H_{1i}x(t-\tau_2(t))]^T\bar{P}[H_{1i}x(t-\tau_1(t)-H_{1i}x(t-\tau_2(t))]. \quad (3.20)
\end{aligned}$$

Since,

$$\mathbb{E}\Big\{\int_{t-\alpha_0\tau_1(t)}^{t}g^T(s)d\omega(s)Z_1\int_{t-\alpha_0\tau_1(t)}^{t}g(s)d\omega(s)\Big\} = \mathbb{E}\Big\{\int_{t-\alpha_0\tau_1(t)}^{t}g^T(s)Z_1g(s)ds\Big\}, \quad (3.21)$$

$$\mathbb{E}\Big\{\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}g^T(s)d\omega(s)Z_1\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}g^T(s)d\omega(s)\Big\} = \mathbb{E}\Big\{\int_{t-\alpha_0\tau_1(t)}^{t}g^T(s)Z_1g(s)ds\Big\}, \quad (3.22)$$

$$\mathbb{E}\Big\{\int_{t-\tau_0}^{t-\tau_1(t)}g^T(s)d\omega(s)Z_2\int_{t-\tau_0}^{t-\tau_1(t)}g^T(s)d\omega(s)\Big\} = \mathbb{E}\Big\{\int_{t-\tau_0}^{t-\tau_1(t)}g^T(s)Z_2g(s)ds\Big\}, \quad (3.23)$$

$$\mathbb{E}\Big\{\int_{t-\tau_2(t)}^{t-\tau_0}g^T(s)d\omega(s)Z_1\int_{t-\tau_2(t)}^{t-\tau_0}g^T(s)d\omega(s)\Big\} = \mathbb{E}\Big\{\int_{t-\tau_2(t)}^{t-\tau_0}g^T(s)Z_2g(s)ds\Big\}, \quad (3.24)$$

$$\mathbb{E}\Big\{\int_{t-\bar{\tau}}^{t-\tau_2(t)}g^T(s)d\omega(s)Z_2\int_{t-\bar{\tau}}^{t-\tau_2(t)}g^T(s)d\omega(s)\Big\} = \mathbb{E}\Big\{\int_{t-\bar{\tau}}^{t-\tau_2(t)}g^T(s)Z_2g(s)ds\Big\}. \quad (3.25)$$

Then, substituting inequalities (3.12)-(3.25) into (3.11), it is obtained that

$$\begin{aligned}
&LV(x_t,t)\\
&\le -\frac{1}{\alpha_0\tau_0}\int_{t-\alpha_0\tau_1(t)}^{t}\\
&\times \eta^T(t,s)\begin{bmatrix}\Pi & -\alpha_0\tau_0 M\\ * & -\alpha_0\tau_0 Z_1\end{bmatrix}\eta(t,s)ds\\
&-\frac{1}{\tau_0(1-\alpha_0)}\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}\\
&\times \eta^T(t,s)\begin{bmatrix}\Pi & -\tau_0(1-\alpha_0)N\\ * & -\tau_0(1-\alpha_0)Z_1\end{bmatrix}\eta(t,s)ds\\
&-\frac{1}{\tau_0}\int_{t-\tau_0}^{t-\tau_1(t)}\eta^T(t,s)\begin{bmatrix}\Pi & -\tau_0 S\\ * & -\tau_0 Z_1\end{bmatrix}\eta(t,s)ds\\
&-\frac{1}{\bar\tau-\tau_0}\int_{t-\tau_2(t)}^{t-\tau_0}\\
&\times \eta^T(t,s)\begin{bmatrix}\Pi & -(\bar\tau-\tau_0)U\\ * & -(\bar\tau-\tau_0)Z_2\end{bmatrix}\eta(t,s)ds\\
&-\frac{1}{\bar\tau-\tau_0}\int_{t-\bar\tau}^{t-\tau_2(t)}\\
&\times \eta^T(t,s)\begin{bmatrix}\Pi & -(\bar\tau-\tau_0)V\\ * & -(\bar\tau-\tau_0)Z_2\end{bmatrix}\eta(t,s)ds, \qquad (3.26)
\end{aligned}$$

where $\eta(t,s)=\begin{bmatrix}\xi^T(s) & y^T(s)\end{bmatrix}$ and $\Pi=\bar\Omega+MZ_1^{-1}M^T+NZ_1^{-1}N^T+SZ_1^{-1}S^T+UZ_2^{-1}U^T+VZ_2^{-1}V^T+\Gamma\bar P^{-1}\Gamma^T$.

Therefore, if (3.5)-(3.9) are satisfied, (3.26) implies that

$$\begin{aligned}
&LV(x_t,t)\\
&\le -\frac{1}{\alpha_0\tau_0}\int_{t-\alpha_0\tau_1(t)}^{t}\lambda\|x(t)\|^2ds\\
&-\frac{1}{\tau_0(1-\alpha_0)}\int_{t-\tau_1(t)}^{t-\alpha_0\tau_1(t)}\lambda\|x(t)\|^2ds\\
&-\frac{1}{\tau_0}\int_{t-\tau_0}^{t-\tau_1(t)}\lambda\|x(t)\|^2ds\\
&-\frac{1}{\bar\tau-\tau_0}\int_{t-\tau_2(t)}^{t-\tau_0}\lambda\|x(t)\|^2ds\\
&\times -\frac{1}{\bar\tau-\tau_0}\int_{t-\bar\tau}^{t-\tau_2(t)}\lambda\|x(t)\|^2ds\\
&=-\lambda\|x(t)\|^2 \qquad (3.27)
\end{aligned}$$

where $\lambda=min\{\lambda_{min}(\Xi_i)\}, i=1,\ldots,5$. Taking the expectation of both sides of (3.27) yields

$$\mathbb{E}\{LV(x_t,t)\}\le -\lambda\mathbb{E}\|x(t)\|^2 \qquad (3.28)$$

which indicates from the Lyapunov stability theory that the SNNs (2.7) is asymptotically stable in the mean square. □

Remark 3.2. *When it is not considered Markov jump parameters, i.e, the Markov chain $\{r(t),t\ge 0\}$ only takes a unique value 1(i.e, $S=\{1\}$), the system (2.7) will be reduced to the following time-varying delayed neural networks:*

$$\begin{aligned}
&dx(t)=[-A(t)x(t)+B(t)f(x(t))+\alpha_0 W(t)\\
&\times f(x(t-\tau_1(t)))+(1-\alpha_0)W(t)f(x(t-\tau_2(t)))\\
&+(\alpha(t)-\alpha_0)\\
&\times(W(t)f(x(t-\tau_1(t)))-W(t)f(x(t-\tau_2(t))))]dt\\
&+[H_0x(t)+\alpha_0H_1x(t-\tau_1(t))\\
&+(1-\alpha_0)H_1x(t-\tau_2(t))+(\alpha(t)-\alpha_0)\\
&\times(H_1x(t-\tau_1(t))-H_1x(t-\tau_2(t)))]dw(t) \quad (3.29)\\
&x(t)=\xi(t),\quad t\in[-\bar\tau,0].
\end{aligned}$$

For system (3.29), we have the following result by Theorem 3.1.

Theorem 3.3. *For given scalars $\tau_0\ge 0$, $\bar\tau_0>0$, μ_1, $0<\alpha_0<1$ satisfying $\alpha_0\mu_1<1$, the SNNs (3.29) without uncertain parameters is asymptotically stable in the mean square if there exist matrices $P>0$, $Q_i>0, i=1,2,3$, $R_l>0$, $Z_l>0$, $l=1,2$, for any matrices $N_k,M_k,S_k,U_k,V_k,Y_k(k=1,2)$ and there exist positive diagonal matrices $K_1>0$, $K_2>0$ and $K_3>0$ such that the following LMIs are feasible*

$$\Xi_1 = \begin{bmatrix}\Pi & -\alpha_0\tau_0 M\\ * & -\alpha_0\tau_0 Z_1\end{bmatrix}<0, \qquad (3.30)$$

$$\Xi_2 = \begin{bmatrix}\Pi & -\tau_0(1-\alpha_0)N\\ * & -\tau_0(1-\alpha_0)Z_1\end{bmatrix}<0, \qquad (3.31)$$

$$\Xi_3 = \begin{bmatrix}\Pi & -\tau_0 S\\ * & -\tau_0 Z_1\end{bmatrix}<0, \qquad (3.32)$$

$$\Xi_4 = \begin{bmatrix}\Pi & -(\bar\tau-\tau_0)U\\ * & -(\bar\tau-\tau_0)Z_2\end{bmatrix}<0, \qquad (3.33)$$

$$\Xi_5 = \begin{bmatrix}\Pi & -(\bar\tau-\tau_0)V\\ * & -(\bar\tau-\tau_0)Z_2\end{bmatrix}<0, \qquad (3.34)$$

where

$$\Pi=\begin{bmatrix}\bar\Omega & M & N & S & U & V & \Gamma\\ * & -Z_1 & 0 & 0 & 0 & 0 & 0\\ * & * & -Z_1 & 0 & 0 & 0 & 0\\ * & * & * & -Z_1 & 0 & 0 & 0\\ * & * & * & * & -Z_2 & 0 & 0\\ * & * & * & * & * & -Z_2 & 0\\ * & * & * & * & * & * & -\bar P\end{bmatrix}, \qquad (3.35)$$

and $\bar{\Omega} = (\Omega_{i,j})_{10\times 10}$ *with*

$$\begin{aligned}
&\Omega_{1,1} = Q_1 + Q_2 \\
&\quad + Q_3 + M_1 + M_1^T - Y_1 A - A^T Y_1^T - K_1 L_1, \\
&\Omega_{1,2} = -M_1 + M_2^T \quad \Omega_{1,3} = 0, \\
&\Omega_{1,4} = 0, \quad \Omega_{1,5} = 0, \quad \Omega_{1,6} = 0, \\
&\Omega_{1,7} = -Y_1 - A^T Y_2^T + P, \Omega_{1,8} = Y_1 B + K_1 L_2, \\
&\Omega_{1,9} = \alpha_0 Y_1 W, \quad \Omega_{1,10} = (1-\alpha_0) Y_1 W, \\
&\Omega_{2,2} = -(1-\alpha_0\mu_1) Q_1 - M_2 - M_2^T + N_1 + N_1^T, \\
&\Omega_{2,3} = -N_1 + N_2^T, \Omega_{2,4} = 0, \Omega_{2,5} = 0, \Omega_{2,6} = 0, \\
&\Omega_{2,7} = 0, \Omega_{2,8} = 0, \Omega_{2,9} = 0, \Omega_{2,10} = 0, \\
&\Omega_{3,3} = -N_2 - N_2^T \\
&\quad + S_1 + S_1^T - K_2 L_1 + \alpha_0(1-\alpha_0) H_1^T \bar{P} H_1, \\
&\Omega_{3,5} = -\alpha_0(1-\alpha_0) H_1^T \bar{P} H_1, \quad \Omega_{3,6} = 0, \\
&\Omega_{3,7} = 0, \Omega_{3,8} = 0, \quad \Omega_{3,9} = K_2 L_2, \\
&\Omega_{3,10} = 0, \quad \Omega_{3,4} = -S_1 + S_2^T, \\
&\Omega_{4,4} = -Q_2 - S_2 - S_2^T + U_1 + U_1^T, \\
&\Omega_{4,5} = -U_1 + U_2^T, \quad \Omega_{4,6} = 0, \quad \Omega_{4,7} = 0, \\
&\Omega_{4,8} = 0, \quad \Omega_{4,9} = 0, \quad \Omega_{4,10} = 0, \\
&\Omega_{5,5} = -U_2 - U_2^T \\
&\quad + V_1 + V_1^T - K_3 L_1 + \alpha_0(1-\alpha_0) H_1^T \bar{P} H_1, \\
&\Omega_{5,6} = -V_1 + V_2^T, \quad \Omega_{5,7} = 0, \quad \Omega_{5,8} = 0, \\
&\Omega_{5,9} = 0, \\
&\Omega_{6,6} = -Q_3 - V_2 - V_2^T, \quad \Omega_{6,7} = 0, \\
&\Omega_{6,8} = 0, \quad \Omega_{6,9} = 0, \Omega_{5,10} = K_3 L_2, \\
&\Omega_{6,10} =, \quad \Omega_{7,7} = \tau_0 R_1 + (\bar{\tau} - \tau_0) R_2 - Y_2 - Y_2^T, \\
&\Omega_{7,8} = Y_2 B, \quad \Omega_{7,9} = \alpha_0 Y_2 W, \\
&\Omega_{7,10} = (1-\alpha_0) Y_2 W, \quad \Omega_{8,8} = -K_1 I, \quad \Omega_{8,9} = 0 \\
&\Omega_{8,10} = 0, \Omega_{9,9} = -K_2 I, \quad \Omega_{10,10} = -K_3 I,
\end{aligned}$$

$$\begin{aligned}
&M = \begin{bmatrix} M_1^T & M_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&N = \begin{bmatrix} 0 & N_1^T & N_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&S = \begin{bmatrix} 0 & 0 & S_1^T & S_2^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&U = \begin{bmatrix} 0 & 0 & 0 & U_1^T & U_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&V = \begin{bmatrix} 0 & 0 & 0 & 0 & V_1^T & V_2^T & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
&Y = \begin{bmatrix} Y_1^T & 0 & 0 & 0 & 0 & 0 & Y_2^T & 0 & 0 & 0 \end{bmatrix}^T, \\
&\Gamma^T = [\bar{P} H_0 \quad 0 \quad \alpha_0 \bar{P} H_1 \quad 0 \quad (1-\alpha_0) \bar{P} H_1 \\
&\quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\
&\bar{P} = \tau_0 Z_1 + (\bar{\tau} - \tau_0) Z_2 + P.
\end{aligned}$$

Proof: The proof is similar as to Theorem 3.1.

Remark 3.4. *In [29, 30], when $\mu \geq 1$, Q will no longer be helpful to improve the stability condition since $-(1-\mu)Q$ is nonnegative definite. When $\mu_1 \geq 1$, if $\alpha_0\mu_1 < 1$ is satisfied, then $-(1-\alpha_0\mu_1)Q_1$ is still negative definite. Therefore, the constraint on $\mu_1 < 1$ is eliminated.*

Theorem 3.5. *For given scalars $\tau_0 \geq 0$, $\bar{\tau}_0 > 0$, μ_1, $0 < \alpha_0 < 1$ satisfying $\alpha_0\mu_1 < 1$, the SNNs (3.29) is asymptotically stable in the mean square if there exist matrices $P > 0$, $Q_i > 0, i = 1,2,3$, $R_l > 0$, $Z_l > 0$, $l = 1,2$, for any matrices $M_k, N_k, S_k, U_k, V_k, Y_k (k = 1,2)$ and there exist positive diagonal matrices $K_1 > 0$, $K_2 > 0$ and $K_3 > 0$ and scalar $\varepsilon > 0$ such that the following LMIs are feasible*

$$\bar{\Xi}_1 = \begin{bmatrix} \hat{\Pi} & -\alpha_0\tau_0 M \\ * & -\alpha_0\tau_0 Z_1 \end{bmatrix} < 0, \tag{3.36}$$

$$\bar{\Xi}_2 = \begin{bmatrix} \hat{\Pi} & -\tau_0(1-\alpha_0) N \\ * & -\tau_0(1-\alpha_0) Z_1 \end{bmatrix} < 0, \tag{3.37}$$

$$\bar{\Xi}_3 = \begin{bmatrix} \hat{\Pi} & -\tau_0 S \\ * & -\tau_0 Z_1 \end{bmatrix} < 0, \tag{3.38}$$

$$\bar{\Xi}_4 = \begin{bmatrix} \hat{\Pi} & -(\bar{\tau}-\tau_0) U \\ * & -(\bar{\tau}-\tau_0) Z_2 \end{bmatrix} < 0, \tag{3.39}$$

$$\bar{\Xi}_5 = \begin{bmatrix} \hat{\Pi} & -(\bar{\tau}-\tau_0) V \\ * & -(\bar{\tau}-\tau_0) Z_2 \end{bmatrix} < 0, \tag{3.40}$$

where

$$\hat{\Pi} = \begin{bmatrix} \hat{\Omega} & M & N & S & U & V & \Gamma & \Sigma \\ * & -Z_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & * & * & -\bar{P} & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

with

$$\begin{aligned}
\hat{\Omega} &= \bar{\Omega} \\
&\quad + diag\left(\varepsilon E_1^T E_1, 0,0,0,0,0,0, \varepsilon E_2^T E_2, \varepsilon E_3^T E_3, \varepsilon E_3^T E_3\right), \\
\sigma &= \sqrt{2 + \alpha_0^2 + (1-\alpha_0)^2}, \\
&\quad \Sigma^T = [\sigma H^T Y_1^T, 0,0,0,0,0, \sigma H^T Y_2^T, 0,0,0],
\end{aligned}$$

M, N, S, U, V and Y are defined as in Theorem 3.3.

Proof. Replace A, B, W in the LMI (3.35) with $A + \Delta A(t)$, $B + \Delta B(t)$, $W + \Delta W(t)$, respectively, we have

$$\Xi_i + \Theta\Psi\Upsilon + \Upsilon^T\Psi^T\Theta^T < 0 \quad i = 1, \ldots, 5. \tag{3.41}$$

where

$$\Theta = \Big[\Theta_1, \underbrace{0,\ldots,0}_{5}, \Theta_2, \underbrace{0,\ldots,0}_{9}\Big],$$

$$\Theta_1 = \Big[Y_1H, \underbrace{0,\ldots,0}_{6}, Y_1H, \alpha_0 Y_1H, (1-\alpha_0)Y_1H, \underbrace{0,\ldots,0}_{6}\Big],$$

$$\Theta_2 = \Big[Y_2H, \underbrace{0,\ldots,0}_{6}, Y_2H, \alpha_0 Y_1H, (1-\alpha_0)Y_2H, \underbrace{0,\ldots,0}_{6}\Big],$$

$$\Psi = diag(F(t),\ldots,F(t)),$$
$$\Upsilon = diag\Big(-E_1 \underbrace{0,\ldots,0}_{6}, E_2, E_3, E_3, \underbrace{0,\ldots,0}_{6}\Big).$$

Using Lemma 2.6 that the matrix inequality (3.41) is equivalent to the following inequality.

$$\Xi_i + \varepsilon^{-1}\Theta\Theta^T + \varepsilon\Upsilon^T\Upsilon < 0. \tag{3.42}$$

Using Schur complement, (3.42) is equivalent to (3.36)-(3.40) for a scalar $\varepsilon > 0$. Then, similar to the proof of the Theorem 3.3, we obtain the results of Theorem 3.5. Hence the detailed proof are omitted. □

4. Numerical Examples

In this section, we will give four examples showing the effectiveness of established results.

4.1 Example.

Consider the Markovian jump SNNs (2.7) with the following matrices

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 2.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.3 & -0.5 \\ 0.1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 & -0.4 \\ 0.3 & 0 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.2 & -0.4 \\ 0.3 & -0.1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -0.1 & -0.5 \\ 0.4 & -0.2 \end{bmatrix},$$

$$H_{01} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad H_{02} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} 0.3 & -0.3 \\ -0.3 & 0 \end{bmatrix},$$

$$H_{12} = \begin{bmatrix} 0.4 & -0.4 \\ -0.4 & 0 \end{bmatrix}, \quad \mathscr{Q} = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}.$$

Solving the LMI in (3.5)-(3.10) by MATLAB LMI toolbox, then the feasible solution is obtained for the corresponding values $L_1 = 0$, $L_2 = 0.5I$. Meawhile, in order to confirm the obtained results with Markovain jump time-varying delay given in (2.7), we gives the values for $\alpha = 0.99, \tau_0 = 0.4, \overline{\tau} = 1.2, \mu_1 = 0.2$, to get the feasible solution. Therefore, it follows from Theorem 3.1, that the system (2.7) is mean square asymptotically stable.

4.2 Example.

Consider the SNNs (3.29) with the following matrices

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0 \end{bmatrix},$$
$$W = \begin{bmatrix} -0.2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix},$$
$$H_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0 \end{bmatrix},$$
$$H = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, E_1 = [0.20.3], E_2 = [0.2 - 0.3],$$
$$E_3 = [-0.2 - 0.3], \quad L_1 = 0.25I, L_1 = 0,$$

by Assumption 2.3, $L_1 = 0, L_2 = 0.25I$ equivalent to $L = 0.5I$ in [31]. For various μ_1, the computed upper bound $\overline{\tau}$, which guarantee the robust stability of system (3.29), are listed in Table 1. From Table 1, when the information of the delay-probability distribution is considered, for various α_0 and $\mu_1$the allowable upper bound $\overline{\tau}$ is larger comparing those in [17, 29–31].

4.3 Example.

Consider the SNNs (3.29) with the following matrices

$$A = \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -4 \\ 0.1 & 0.3 \end{bmatrix},$$
$$W = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}, \quad H_0 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0.5 & -0.1 \\ -0.5 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$H = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad E_1 = E_2 = E_3 = [1 \quad 1], \quad L_1 = 0.$$

For various μ_1, the computed upper bound $\overline{\tau}$, which guarantee the robust stability of system (3.29), are listed in Table 2. From Table 2, when the information of the delay-probability distribution is considered, for various α_0 and μ_1 the allowable upper bound $\overline{\tau}$ is larger than those result discussed in [17].

Table 1. Maximum allowable upper bound of $\bar{\tau}$ with different μ for fixed $\tau_0 = 0.6$

Methods		$\mu_1 = 0.97$	$\mu_1 = 1$	$\mu_1 = 1.5$	$\mu_1 = 2$	*unknown*
[31]		-	-	-	-	0.419
[29]		0.785	0.779	0.779	0.779	0.779
[30]		0.771	0.746	0.746	0.746	0.746
[17]	$\alpha_0 = 0.2$	1.294	1.294	1.292	1.291	1.279
Theorem 3.5	$\alpha_0 = 0.2$	2.7482	2.7481	2.7456	2.7429	2.7339
[17]	$\alpha_0 = 0.4$	1.338	1.337	1.324	1.299	1.281
Theorem 3.5	$\alpha_0 = 0.4$	3.1819	3.1809	3.1642	3.1567	3.1567
[17]	$\alpha_0 = 0.6$	1.430	1.426	1.303	1.292	1.292
Theorem 3.5	$\alpha_0 = 0.6$	3.9210	3.9177	3.8990	3.8990	3.8990
[17]	$\alpha_0 = 0.8$	1.615	1.579	1.323	1.323	1.323
Theorem 3.5	$\alpha_0 = 0.8$	5.6591	5.6591	5.6591	5.6591	5.6591

Table 2. Maximum allowable upper bound of $\bar{\tau}$ with different μ for fixed $\tau_0 = 0.4$

Methods		$\mu_1 = 0.2$	$\mu_1 = 0.6$	$\mu_1 = 1$	$\mu_1 = 1.5$	$\mu_1 = 2$	$\mu_1 = 2.5$
[17]	$\alpha_0 = 0.2$	0.972	0.972	0.971	0.970	0.968	0.967
Theorem 3.5	$\alpha_0 = 0.2$	2.2317	2.2297	2.2275	2.2245	2.2213	2.2179
[17]	$\alpha_0 = 0.5$	1.092	1.083	1.071	1.044	1.024	1.024
Theorem 3.5	$\alpha_0 = 0.5$	2.9122	2.8929	2.8706	2.8586	2.8586	2.8586
[17]	$\alpha_0 = 0.8$	1.545	1.490	1.342	1.242	1.242	1.242
Theorem 3.5	$\alpha_0 = 0.8$	4.9575	4.8599	4.8160	4.8160	4.8160	4.8160
[17]	$\alpha_0 = 0.99$	5.523	5.181	3.529	3.243	3.243	3.243
Theorem 3.5	$\alpha_0 = 0.99$	24.9910	24.0252	23.9266	23.9266	23.9266	23.9266

4.4 Example.

Consider the SNNs (3.29) with the following matrices

$$A = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3 & -0.19 & 0.3 \\ -0.15 & 0.2 & 0.36 \\ -0.17 & 0.29 & -0.3 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.19 & -0.13 & 0.2 \\ 0.16 & 0.09 & 0.1 \\ 0.02 & -0.15 & 0.07 \end{bmatrix},$$

$$H_0 = H_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$E_1 = E_2 = E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In order to compare results in this paper with those in [29] [34] and [37], we assume the activation functions satisfy Assumption 2.5 with $l_1^- = l_2^- = l_3^- = 0$, $l_1^+ = 1.2, l_2^+ = 0.5, l_3^+ = 1.3$. In this case, the LMI-based conditions obtained in [34] are not feasible when $\mu \geq 0.7$. When the time-varying delay is differentiable and $\mu = 0.85$, by using Theorem 1 in [37] and Theorem 1 in [29], it is found that the maximum allowable upper bound of $\tau(t)$ as $\bar{\tau} = 9.6876$, and $\bar{\tau} = 7.7377$, respectively. However, using Theorem 3.5 in this paper, we obtain maximum allowable upper bound $\bar{\tau} = 9.7325$. When the time delay may not be differentiable; that is, μ is unknown, by using Theorem 2 in [37] and Theorem 2 in [29], it is found that the maximum allowable upper bound of $\tau(t)$ as $\bar{\tau} = 2.3879$, and $\bar{\tau} = 2.314$, respectively. However, using Theorem 3.5 in this paper, we obtain the maximum allowable upper bound $\bar{\tau} = 9.7325(\alpha_0 = 0.7)$.

According to Theorem 3.5, the upper bounds are derived on the time-varying delay to guarantee the system is robustly stochastically stable in the mean square. From Table 3, when the information of the delay-probability distribution is considered, for various α_0 and μ_1 the allowable upper bound $\bar{\tau}$ is larger than those results discussed in the literature [29, 34, 37]. Hence the proposed method gives the conservative results.

5. Conclusion

In this paper, the sufficient conditions guaranteeing the mean square robust asymptotic stability for markovian jump SNNs with time-varying delays have been proposed. Based on LMI methods, robust stability condition for the markovian jump SNNs have been obtained in the form of LMIs. Probability

Table 3. Maximum allowable upper bound of $\bar{\tau}$ with different μ for fixed $\tau_0 = 0.9$

Methods		$\mu_1 = 0.7$	$\mu_1 = 0.85$	$\mu_1 = 1$	$\mu_1 = 2$	$\mu_1 = 3$	*unknown*
[29]		-	7.7377	-	-	-	2.3514
[37]		-	9.6876	-	-	-	2.3879
Theorem 3.5	$\alpha_0 = 0.7$	9.7325	9.7325	9.7325	9.7325	9.7325	9.7325
Theorem 3.5	$\alpha_0 = 0.8$	10.0598	10.0598	10.0598	10.0598	10.0598	10.0598

distribution of the time-varying delays is introduced into the stability criteria and the new method removes the constraint that the derivative of the delay must be smaller than 1. Finally, three numerical examples are demonstrated to prove less conservative results by comparison of numerical results in the existing literature.

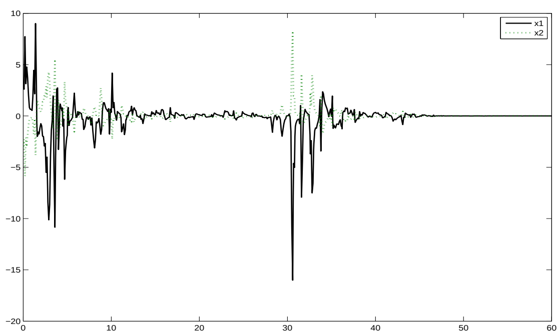

Figure 1. The state trajectories of Example 4.2 for $\bar{\tau} = 1$ with initial condition $(3, -3)$.

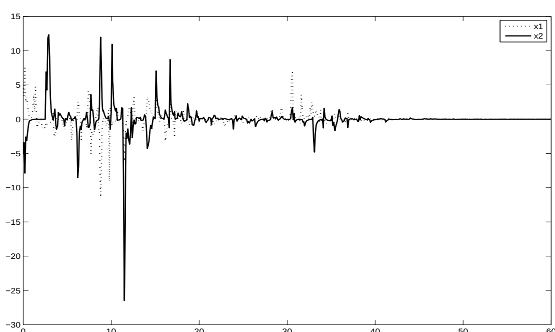

Figure 2. The state trajectories of Example 4.3 for $\bar{\tau} = 2.5$ with initial condition $(5, -5)$.

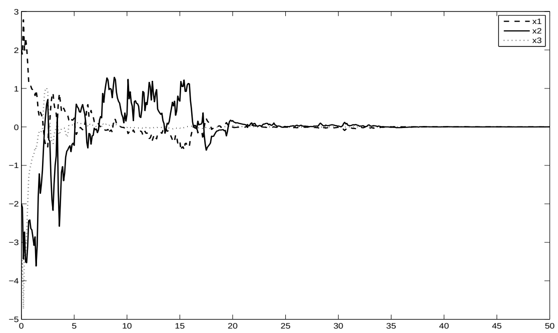

Figure 3. The state trajectories of Example 4.4 for $\bar{\tau} = 2$ with initial condition $(2, -2, -4)$.

References

[1] S. Haykin, 1998, "Neural Networks: A comprehensive foundation," Prentice Hall, NJ.

[2] X. Li 2009, "Global exponential stability for a class of neural networks," Applied Mathematics Letters **22**, 1235–1239.

[3] X. Li and Z. Chen 2009, "Stability properties for Hopfield neural networks with delays and impulsive perturbations," Nonlinear Analysis: Real World Applications **10**, 3253–3265.

[4] X. Li 2009, "Existence and global exponential stability of periodic solution for impulsive Cohen-Grossberg-type BAM neural networks with continuously distributed delays," Applied Mathematics and Computation **215**, 292–307.

[5] M. Gao, X. Lou and B. Cui 2008, "Robust exponential stability of markovian jumping neural networks with time-varying delay," International Journal of Neural Systems **18**, 207–218.

[6] T. Li, C. Sun, X. Zhao and C. Lin 2008, "LMI-Based asymptotic stability analysis of neural networks with time-varying delays," International Journal of Neural Systems **18**, 257–265.

[7] Y. Zhao, Y. Xia and Q. Lu 2009, "Stability analysis of a class of general periodic neural networks with delays and impulses," International Journal of Neural Systems **19**, 375-386.

[8] T. Li, A. Song and S. Fei, "Novel stability criteria on discrete-time neural networks with both time-varying and distributed delays," International Journal of Neural Systems **19**, 269–283.

[9] S. Arik 2005, "Global robust stability analysis of neural networks with discrete time delays," Chaos Solitons and Fractals **26**, 1407–1414.

[10] S. Arik 2004, "An analysis of exponential stability of delayed neural networks with time varying delays," Neural Networks **17**, 1027–1031.

[11] V. Singh 2005, "Global robust stability of delayed neural networks: an LMI approach," IEEE Transactions on

Circuits and Systems II Express Briefs **52**, 33–36.

[12] T. Chen and L. Rong 2003, "Delay-independent stability analysis of Cohen-Grossberg neural networks," Physics Letters A **317**, 436–449.

[13] H. J. Cho and J. H. Park 2007, "Novel delay-dependent robust stability criterion of delayed cellular neural networks," Chaos, Solitons and Fractals **32**, 1194–1200.

[14] X. F. Liao, G. Chen and E. N. Sanchez 2002, "Delay-dependent exponential stability analysis of delayed neural networks: an LMI approach," Neural Networks **15**, 855–866.

[15] Y. Chen and Y. Wu 2009, "Novel delay-dependent stability criteria of neural networks with time-varying delay," Neurocomputing **72**, 1065-1070.

[16] R. Yang, H. Gao, J. Lam and P. Shi 2009, "New stability criteria for neural networks with distributed and probabilistic delays," Circuits Systems and Signal Processing **28**, 505-522.

[17] J. Fu, H. G. Zhang and T. Ma 2009, Delay-probablity-distribution-dependent robust stability analysis for SNNs with time-varying delay, Progress Natural Science **19**, 1333–1340.

[18] Y. Wenwu and J. Cao 2007, "Robust control of uncertain stochastic recurrent neural networks with time-varying delay," Neural Processing Letters **26**, 101–119.

[19] S. Blythe, X. Mao and X. Liao 2001, "Stability of stochastic delay neural networks, Journal of the Franklin Institute **338**, 481–495.

[20] Y. Chen, A. Xue, X. Zhao, S. Zhou 2009, "Improved delay-dependent stability analysis for uncertain stochastic Hopfield neural networks with time-varying delays," IET Control Theory and Applications **3**, 88-97.

[21] P. Balasubramaniam, S. Lakshmanan 2011, LMI Conditions for Robust Stability Analysis of Stochastic Hopfield Neural Networks with Interval Time-Varying Delays and Linear Fractional Uncertainties, Circuits, Systems, and Signal Processing, **30**,1011-1028.

[22] W. Su and Y. Chen 2009, "Global asymptotic stability analysis for neutral stochastic neural networks with time-varying delays," Communications in Nonlinear Science and Numerical Simulation **14**, 1576–1581.

[23] J. Zhang, P. Shi and J. Qiu 2007, "Novel robust stability criteria for uncertain stochastic Hopfield neural networks with time-varying delays," Nonlinear Analysis: Real world Applications **8**, 1349–1357.

[24] J. Yu, K. Zhang and S. Fei 2009, "Further results on mean square exponential stability of uncertain stochastic delayed neural networks," Communication in Nonlinear Science and Numerical Simulation **14**, 1582-1589.

[25] W. Feng, S. X. Yang and H. Wu 2009, "On robust stability of uncertain stochastic neural networks with distributed and interval time-varying delays," Chaos, Solitons and Fractals **42**, 2095–2104.

[26] W. Feng, S. X. Yang, W. Fu, and H. Wu 2009, "Robust stability analysis of uncertain stochastic neural networks with interval time-varying delay," Chaos, Solitons and Fractals **41**, 414–424.

[27] Y. Wu, Y. Wu and Y. Chen 2009, "Mean square exponential stability of uncertain stochastic neural networks with time-varying delay," Neurocomputing **72**, 2379–2384.

[28] R. Yang, H. Gao and P. Shi 2009, "Novel robust stability criteria for stochastic Hopfield neural networks with time delays," IEEE Transactions on Systems, Man, and Cybernetics part B **39**, 467–474.

[29] W. H. Chen and X. M. Lu 2008, "Mean square exponential stability of uncertain stochastic delayed neural networks," Physics Letters A **372**, 1061–1069.

[30] H. Y. Li, B. Chen and Q. Zhou 2008, "Robust exponential stability for uncertain stochastic neural networks with discrete and distributed time-varying delays," Physics Letters A **372**, 3385–3894.

[31] H. Huang and G. Feng 2008, "Corrigendum to delay-dependent stability for uncertain SNNs with time-varying delay," [Physica A, 381 (2007) 93–103], Physica A **387**, 1431-1432.

[32] X. Li and J. Cao 2007, "Delay-independent exponential stability of stochastic Cohen-Grossberg neural networks with time-varying delays and reaction diffusion terms," Nonlinear Dynamics **50**, 363–371.

[33] Y. Liu, Z. Wang and X. Liu 2008, On delay-dependent robust exponential stability of SNNs with mixed time delays and Markovian switching, Nonlinear Dynamics **54**, 199–213.

[34] H. Huang and J. Cao 2007, Exponential stability analysis of uncertain SNNs with multiple delays, Nonlinear Analysis: Real World Applications **8**, 646–653.

[35] C. Huang, Y. He and L. Huang 2009, "Stability analysis of non-autonomous stochastic Cohen-Grossberg neural networks," Nonlinear Dynamics **57**, 469–478.

[36] J. Zhang, P. Shi, J. Qiu and H. Yang 2008, "A new criterion for exponential stability of uncertain SNNs with mixed delays," Mathematical and Computer Modelling **47**, 1042–1051.

[37] B. Zhang, S. Xu, G. Zong and Y. Zou 2009 , Delay-dependent exponential stability for uncertain stochastic Hopfield Neural Networks with time-varying delays, IEEE Transactions on Circuits and systems I **56**, 1241–1247.

[38] Q. Zhu and J. Cao 2012, Stability of Markovain jump neural networks with impulse control and time varying delays, Nonlinear Analysis: Real World Applications **13**, 2259–2270.

[39] H. Huang and Q. Du 2013, Global exponential stability of neutral high-order stochastic Hopfield neural networks with Markovian jump parameters and mixed time delays, ISA Transactions **52**, 759–767.

[40] J. W. Yi, Y. W. Wang, J. W. Xiao and Y. Huang 2013, Exponential synchronization of complex dynamical networks with markovian jump parameters and stochastic

delays and its application to multi-agent systems, Communications in Nonlinear Science and Numerical Simulation **18**, 1175–1192.

[41] D. Zhang and L. Yu 2012, Exponential state estimation for Markovian jumping neural networks with time-varying discrete and distributed delays, Neural Networks **35**, 103–111.

[42] C. D. Zheng, K. Qua and Z. Wang 2013, Robust convergence of Cohen Grossberg neural networks with mode-dependent time-varying delays and Markovian jump, Journal of the Franklin Institute **350**, 2166-2182.

[43] H. Li, Q. Zhou, B. Chen and H. Liu 2011, Parameter-dependent robust stability for uncertain Markovian jump systems with time delay, Journal of Franklin Institute **38**, 738–748.

[44] Z. G. Wu, J. H. Park, H. Su and J. Chu 2013, Delay dependent passivity for singular Markov jump systems with time-delays, Communications in Nonlinear Science and Numerical Simulation **18**, 669–681.

[45] M. Luo, G. Liu and S. Zhong 2013, Robust fault detection of Markovian jump systems with different system modes, Applied Mathematical Modelling **37**, 5001–5012.

[46] J. Wang, H. Wang, A. Xue and R. Lu 2013, Delay-dependent H_∞ control for singular Markovain jump systems with time delay, Nonlinear Analysis: Hybrid systems **8**, 1–12.

[47] B. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan 1994, ”Linear matrix inequalities in systems and control theory,” Philadelphia: SIAM.

[48] H. Zhang, Y. Wang and D. Liu 2008, Delay-dependent guaranteed cost control for uncertain stochastic fuzzy systems with multiple time delays, IEEE Transactions on Systems, Man, and Cybernetics Part B **38**, 126–40.

[49] L. Xie, M. Fu, and C. E. D. Souza 1992, H_∞ control and quadratic stabilization of systems with parameter uncetainty via output feedback. IEEE Transsctions on Automatic Control, **32**, 1253–1256.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆