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The generalized B curvature tensor on $(LCS)_n$-manifolds

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Abstract
The present paper deals with the study of generalized B curvature tensor on $(LCS)_n$-manifolds. Here we describe flatness, semisymmetry and recurrent properties on $(LCS)_n$-manifolds. Moreover we consider the conditions $B \cdot R = 0$, $B \cdot B = 0$ and $B \cdot S = 0$ and obtained interesting results

Keywords
Lorentzian metric, $(LCS)_n$-manifolds, semisymmetric, ϕ-recurrent, η-Einstein manifold.

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1. Introduction

The idea of Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) was introduced by Shaikh [6] and studied their existence, more applications to general theory of relativity and cosmology. Also, $(LCS)_n$-manifolds generalizes the concept of LP Sasakian manifolds, which is given by Matsumoto [4]. The notion of $(LCS)_n$-manifolds were weakened by several authors in different ways such as in [7–9, 11] and many others.

On the other hand Shaikh and Kundu [10] introduced and studied a type of tensor field, called generalized B curvature tensor on a Riemannian manifold. This includes the structures of Quasi-conformal, Weyl conformal, Conharmonic and Concircular curvature tensors.

In this paper we made an attempt to study certain properties of B curvature tensor on $(LCS)_n$-manifolds. The paper is organized as follows: After preliminaries, in Section 3 we study B flat, $\xi - B$ flat and $\phi - B$ flat $(LCS)_n$-manifolds and found that the manifold is Einstein or η-Einstein provided B curvature tensor is not an Weyl-conformal, concircular and conharmonic structures. Next we consider B semisymmetric and B-ϕ semi-symmetric $(LCS)_n$-manifolds and it is shown that manifold is Einstein or η-Einstein if B curvature tensor is not an Weyl-conformal, concircular and conharmonic curvature tensors. In Section 5 we proved that a B-ϕ-recurrent $(LCS)_n$-manifold with constant scalar curvature is B-ϕ-symmetric manifold. In the last Section we describe an $(LCS)_n$-manifold satisfying conditions $B \cdot R = 0$, $B \cdot B = 0$ and $B \cdot S = 0$.

2. Preliminaries

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \longrightarrow \Re$ is a non-degenerate inner product of signature $(-,+,....,+)$, where T_pM denotes the tangent vector space of M at p and R is the real number space. A non zero vector $v \in T_pM$ is said to be timelike (resp., non-spacelike, null, space like) if it satisfies $g_p(v,v) < 0)(resp. \leq= 0, = 0, > 0)$[5].

Definition 2.1. *In a Lorentzian manifold (M,g) a vector field P defined by*

$$g(X,P) = A(X),$$

for any $X \in T_pM$ is said to be a concircular vector field if

$$(\nabla_X A)(Y) = \alpha g(X,Y) + w(X)A(Y),$$

where α is a non-zero scalar and w is a closed 1-form.

Let M be a Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{2.1}$$

Since ξ is a unit concircular vector field, it follows that there exists a non-zero 1-form η such that for

$$g(X,\xi) = \eta(X), \tag{2.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha\{g(X,Y) + \eta(X)\eta(Y)\} \quad (\alpha \neq 0), \tag{2.3}$$

for all vector fields X,Y, where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{2.4}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$.
Let us put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi. \tag{2.5}$$

Then from (2.3) and (2.5), we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

which tell us that ϕ is a symmetric $(1,1)$ tensor. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, its associated 1-form η and $(1,1)$-type tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold) [6]. Especially, we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [4]. In a $(LCS)_n$-manifold, the following relations hold [6].

$$\eta(\xi) = -1,\ \phi\xi = 0,\ \eta(\phi X) = 0, \tag{2.7}$$

$$g(\phi X,\phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.8}$$

$$R(X,Y)Z = (\alpha^2 - \rho)[g(Y,Z)X - g(X,Z)Y], \tag{2.9}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.10}$$

$$(\nabla_X \phi)(Y) = \alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{2.11}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{2.12}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.13}$$

$$Q\xi = (n-1)(\alpha^2 - \rho)\xi, \tag{2.14}$$

for any vector fields X,Y,Z, where R,S denotes the curvature tensor, and the Ricci tensor of the manifold respectively. Recently Shaikh and Kundu introduced generalized B curvature tensor [10] given by

$$\begin{aligned} B(X,Y)Z = {} & a_0 R(X,Y)Z + a_1[S(Y,Z)X \\ & -S(X,Z)Y + g(Y,Z)QX \\ & -g(X,Z)QY] + 2a_2 r[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{2.15}$$

where R,S,Q and r are the curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature respectively.

In particular, the B-curvature tensor is reduced to:

1. The quasi-conformal curvature tensor C^* [14] if $a_0 = a,\ a_1 = b$ *and* $a_2 = \frac{-1}{2n}[\frac{a}{n-1} + 2b]$.
2. The weyl-conformal curvature tensor $\tilde{C}$ [13] if $a_0 = 1,\ a_1 = \frac{-1}{n-2}$ *and* $a_2 = \frac{-1}{2(n-1)(n-2)}$.
3. The concircular curvature tensor C [12] if $a_0 = 1, a_1 = 0$ *and* $a_2 = \frac{-1}{n(n-1)}$.
4. The conharmonic curvature tensor P [3] if $a_0 = 1,\ a_1 = \frac{-1}{n-2}$ *and* $a_2 = 0$.

3. Main Results

4. B flat, $\xi - B$ flat and $\phi - B$ flat $(LCS)_n$-manifold

First we consider B-flat $(LCS)_n$-manifold M, i.e., $B(X,Y)Z = 0$, for any vector fields $X,Y,Z \in T_pM$. It can be easily seen that

$$\begin{aligned} & a_0 R(X,Y)Z + a_1[S(Y,Z)X \\ & -S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \\ & +2a_2 r[g(Y,Z)X - g(X,Z)Y] = 0. \end{aligned} \tag{4.1}$$

Taking inner product of (4.1) with respect to W, we get

$$\begin{aligned} & a_0 g(R(X,Y)Z,W) + a_1[S(Y,Z)g(X,W) \\ & -S(X,Z)g(Y,W) + g(Y,Z)S(X,W) - g(X,Z) \\ & S(Y,W)] + 2a_2 r[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] = 0. \end{aligned} \tag{4.2}$$

On plugging $X = W = e_i$ in (4.2), where e_i is an orthonormal basis for the tangent space at each point of the manifold and taking summation over i, $i = 1,2,\ldots,n$, we have

$$S(Y,Z) = \frac{r[2a_2(1-n) - a_1]}{a_0 + a_1(n-2)} g(Y,Z). \tag{4.3}$$

This leads us to the following theorem:

Theorem 4.1. *A B-flat $(LCS)_n$-manifold is an Einstein manifold provided B-curvature tensor is neither a weyl-conformal curvature tensor [13] nor a conharmonic curvature tensor [3].*

Next we consider, $\xi - B$ flat $(LCS)_n$-manifold i.e., $B(X,Y)\xi = 0$. Then it follows from above condition that

$$a_0R(X,Y)\xi + a_1[S(Y,\xi)X - S(X,\xi)Y + g(Y,\xi)QX - g(X,\xi)QY] + 2a_2r[g(Y,\xi)X - g(X,\xi)Y] = 0. \tag{4.4}$$

Using (2.9) and (2.12) in (4.4) and then taking inner product with respect to W, we get

$$a_0[g(X,W)\eta(Y) - g(Y,W)\eta(X)] + a_1[(n-1)(\alpha^2-\rho)\{g(X,W)\eta(Y) - g(Y,W)\eta(X)\} + S(X,W)\eta(Y) - S(Y,W)\eta(X)] + 2a_2r[g(X,W)\eta(Y) - g(Y,W)\eta(X)] = 0. \tag{4.5}$$

On plugging $X = \xi$ in (4.5), gives

$$S(Y,W) = \frac{1}{a_1}[(-a_0-(n-1)a_1)(\alpha^2-\rho) + -2a_2r]g(Y,W)\frac{1}{a_1}[(-a_0-2(n-1)a_1)(\alpha^2-\rho) - 2a_2r]\eta(Y)\eta(W). \tag{4.6}$$

Hence we can state the following theorem:

Theorem 4.2. *A $\xi - B$ flat $(LCS)_n$-manifold is η-Einstein provided the B-curvature tensor is not a concircular curvature tensor [12].*

Finally we consider $\phi - B$ flat $(LCS)_n$-manifold, i.e.,

$$\phi^2(B(\phi X,\phi Y)\phi Z) = 0. \tag{4.7}$$

By using (2.15) in (4.7) and then taking inner product with respect to W and then contracting, we get

$$S(Y,W) = \frac{1}{3a_1}[a_0(n-2)(\alpha^2-\rho) + a_1(r-(n-1)(\alpha^2-\rho)) + 2a_2r(n-2)] g(X,W) + \frac{1}{3a_1}[a_0(n-2)(\alpha^2-\rho) + a_1\{r-4(n-1)(\alpha^2-\rho)+n(n-1)(\alpha^2-\rho)\} + 2a_2(n-2)r]\eta(X)\eta(W). \tag{4.8}$$

Hence we can state the following theorem:

Theorem 4.3. *A $\phi - B$ flat $(LCS)_n$-manifold is η-Einstein provided the B-curvature tensor is not a concircular curvature tensor [12].*

5. Semisymmetric properties on $(LCS)_n$-manifold

Definition 5.1. *An n-dimensional $(n>1)$ $(LCS)_n$-manifold M is said to be B-semisymmetric, if it satisfy the condition $R\cdot B = 0$.*

Let us suppose that $(LCS)_n$-manifold is B-semisymmetric. Thus it follows from above condition that

$$R(\xi,X)B(U,V)W - B(R(\xi,X)U,V)W - B(U,R(\xi,X)V)W - B(U,V)R(\xi,X)W = 0. \tag{5.1}$$

Using (2.9), (2.12) and (2.15) in (5.1) and then taking inner product with respect to ξ, we get

$$-a_0R(U,V,W,X) - a_1[S(V,W)g(U,X) - S(U,W)g(V,X) + S(U,X)g(V,W) - S(V,X)g(U,W)] - 2a_2r[g(V,W)g(U,X) - g(U,W)g(V,X)] + (a_0(\alpha^2-\rho) + a_1(n-1)(\alpha^2-\rho) + 2a_2r)[g(X,U)g(V,W) - g(X,V)g(U,W)] + a_1(n-1)(\alpha^2-\rho)[g(X,V)g(U,W) + g(V,W)\eta(U)\eta(X) - g(X,V)g(U,W) - g(U,W)\eta(V)\eta(X) + g(V,X)\eta(U)\eta(W) + g(U,X)\eta(V)\eta(W)] = 0. \tag{5.2}$$

On contracting above equation, gives

$$S(V,W) = K_1g(V,W) + K_2\eta(V)\eta(W). \tag{5.3}$$

Where

$$K_1 = \frac{(n-1)(\alpha^2-\rho)(a_0+(n-2)a_1) - r(a_1+4a_2(n-1))}{a_0+a_1(n-2)},$$

$$K_2 = \frac{(n-1)a_1(\alpha^2-\rho)}{a_0+a_1(n-2)}.$$

Hence we can state the following:

Theorem 5.2. *A B-semisymmetric $(LCS)_n$-manifold is η-Einstein provided B-curvature tensor is neither a weyl-conformal curvature tensor [13] nor a conharmonic curvature tensor [3].*

Next we consider $(LCS)_n$-manifold which is $B-\phi$-semisymmetric i.e, $B\cdot\phi = 0$. Then it follows

$$B(X,Y)\phi Z - \phi B(X,Y)Z = 0. \tag{5.4}$$

By virtue of (2.6) and (2.9) we have from (2.15), that

$$B(X,Y)\phi Z = a_0[g(Y,Z)X + \eta(Y)\eta(Z)X - g(X,Z)Y - \eta(X)\eta(Z)Y] + a_1[S(Y,Z)X + (n-1)(\alpha^2-\rho)\eta(Y)\eta(Z)X - S(X,Z)Y - (n-1)(\alpha^2-\rho)\eta(X)\eta(Z)Y + g(Y,Z)QX + \eta(Y)\eta(Z)QX - g(X,Z)QY - \eta(X)\eta(Z)QY] + 2a_2r[g(Y,Z)X + \eta(Y)\eta(Z)X - g(X,Z)Y - \eta(X)\eta(Z)Y], \tag{5.5}$$

and

$$\phi B(X,Y)Z = a_0(\alpha^2-\rho)[g(Y,Z)X + \eta(Y)\eta(Z)X - g(X,Z)Y - \eta(X)\eta(Z)Y] + a_1[S(Y,Z)X + \eta(X)S(Y,Z)\xi - S(X,Z)Y - \eta(Y)S(X,Z)\xi + g(Y,Z)QX + \eta(X)g(Y,Z)Q\xi - g(X,Z)QY - \eta(Y)g(X,Z)Q\xi] + 2a_2r[g(Y,Z)X + \eta(X)g(Y,Z)\xi - g(X,Z)Y - \eta(Y)g(X,Z)\xi]. \tag{5.6}$$

Substituting (5.5) and (5.6) in (5.4) and then taking inner product with respect to U, we get

$$a_1[(\alpha^2-\rho)(\eta(Y)\eta(Z)g(X,U)-\eta(X)\eta(Z)g(Y,U))+\eta(Y)\eta(Z)S(X,U)-\eta(X)\eta(Z)S(Y,U)-\eta(X)\eta(U)S(Y,Z)+\eta(Y)\eta(U)S(X,Z)-\eta(X)g(Y,Z)S(\xi,U)+\eta(Y)g(X,Z)S(\xi,U)]+2a_2r[\eta(Y)\eta(Z)g(X,U)-\eta(X)\eta(Z)g(Y,U)-\eta(X)\eta(U)g(Y,Z)+\eta(Y)\eta(U)g(X,Z)]=0. \tag{5.7}$$

On plugging $Y=U=e_i$ in (5.7), where e_i is an orthonormal basis for the tangent space at each point of the manifold and taking summation over i, $i=1,2,\dots,n$, we get

$$S(X,Z)=\frac{1}{a_1}[(n-1)(\alpha^2-\rho)a_1+2a_2r]g(X,Z)+\frac{1}{a_1}[-a_1((n-1)^2(\alpha^2-\rho)+r)-2na_2r]\eta(X)\eta(Z). \tag{5.8}$$

Hence we can state the following:

Theorem 5.3. *Let M be a $B-\phi$-semisymmetric $(LCS)_n$-manifold. Then the manifold is η-Einstein provided B-curvature tensor is not a concircular curvature tensor [12].*

6. $B-\phi$-recurrent $(LCS)_n$-manifold

Definition 6.1. *An $(LCS)_n$-manifold is said to be $B-\phi$-recurrent manifold if there exists a non-zero 1-form A such that*

$$\phi^2((\nabla_W B)(X,Y)Z)=A(W)B(X,Y)Z, \tag{6.1}$$

for any vector fields $X,Y,Z,W\in T_pM$. If $A(W)=0$ then $B-\phi$-recurrent manifold reduces to $B-\phi-$symmetric manifold.

Let us consider a $B-\phi$-recurrent $(LCS)_n$-manifold. Then by using (2.6) in (6.1), we have

$$(\nabla_W B)(X,Y)Z+\eta((\nabla_W B)(X,Y)Z)\xi=A(W)B(X,Y)Z, \tag{6.2}$$

from which it follows that

$$g((\nabla_W B)(X,Y)Z,U)+\eta((\nabla_W B)(X,Y)Z)\eta(U)=A(W)g(B(X,Y)Z,U). \tag{6.3}$$

By virtue of (2.9), (2.12) and (2.15), above equation becomes

$$a_0g((\nabla_W R)(X,Y)Z,U)+a_1[g((\nabla_W S)(Y,Z)X,U)-g((\nabla_W S)(X,Z)Y,U)+g(Y,Z)(\nabla_W S)(X,U)-g(X,Z)(\nabla_W S)(Y,U)]+2a_2d_rW[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]+\eta(U)\{a_0g((\nabla_W R)(X,Y)Z,\xi)+a_1[g((\nabla_W S)(Y,Z)X,\xi)-g((\nabla_W S)(X,Z)Y,\xi)+g(Y,Z)(\nabla_W S)(X,\xi)-g(X,Z)(\nabla_W S)(Y,\xi)]\}+2a_2drW[g(Y,Z)g(X,\xi)-g(X,Z)g(Y,\xi)]=A(W)\{g(R(X,Y)Z,U)+a_1[S(Y,Z)g(X,U)-S(X,Z)g(Y,U)+g(Y,Z)S(X,U)-g(X,Z)S(Y,U)]+2a_2r[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]\}. \tag{6.4}$$

On contracting (6.4), we get

$$a_1(n-2)(\nabla_W S)(Y,Z)+[a_1+2a_2(2n-1)]g(Y,Z)drW+a_1\alpha\eta(Z)S(Y,W)-\alpha a_1(n-1)(\alpha^2-\rho)\eta(Z)g(Y,W)-2a_2drW\eta(Y)\eta(Z)=A(W)\{(a_0-a_1(n-2))S(Y,Z)+(2a_2r(n-1)-a_1r)g(Y,Z)\}. \tag{6.5}$$

On plugging $Z=\xi$ in (6.5), gives

$$a_1\alpha(n-1)^2(\alpha^2-\rho)g(Y,W)+[a_1+4na_2]\eta(Y)drW=a_1\alpha(n-1)S(Y,W)+A(W)\{[a_0-a_1(n-2)](n-1)(\alpha^2-\rho)+(2a_2r(n-1)-a_1r)\}\eta(Y). \tag{6.6}$$

Again taking $Y=\xi$ in (6.6), we get

$$A(W)=\frac{[a_1+4na_2]drW}{[a_0-a_1(n-2)](n-1)(\alpha^2-\rho)+(2a_2r(n-1)-a_1r)}. \tag{6.7}$$

If the manifold has a constant scalar curvature r, then we have $drW=0$.
Hence the equation (6.7) turns into

$$A(W)=0. \tag{6.8}$$

By using (6.8) in (6.1), we get

$$\phi^2((\nabla_W B)(X,Y)Z)=0. \tag{6.9}$$

Hence we can state the following:

Theorem 6.2. *An n-dimensional $B-\phi$-recurrent $(LCS)_n$-manifold with constant scalar curvature r is a $B-\phi-$symmetric manifold.*

7. An $(LCS)_n$- manifold satisfying $B\cdot R=0$, $B\cdot B=0$ and $B\cdot S=0$

Let us consider an $(LCS)_n$- manifold satisfying the condition $B\cdot R=0$. Then we have

$$B(\xi,U)R(X,Y)Z-R(B(\xi,U)X,Y)Z-R(X,B(\xi,U)Y)Z-R(X,Y)B(\xi,U)Z=0. \tag{7.1}$$

By using (2.9), (2.12) and (2.15) in (7.1) and then plugging $Z=\xi$, we get

$$-2a_1[S(U,X)+(n-1)(\alpha^2-\rho)\eta(U)\eta(X)]Y-2a_1[S(U,Y)+(n-1)(\alpha^2-\rho)\eta(U)\eta(X)]X=0. \tag{7.2}$$

Taking inner product of above equation with respect to W, gives

$$a_1[S(U,X)g(Y,W)+S(U,Y)g(X,W)+(n-1)(\alpha^2-\rho)[\eta(U)\eta(X)g(Y,W)+\eta(U)\eta(Y)g(X,W)]]=0.$$

Which implies that either $a_1=0$ or

$$S(U,X)g(Y,W)+S(U,Y)g(X,W)+(n-1)(\alpha^2-\rho)[\eta(U)\eta(X)g(Y,W)+\eta(U)\eta(Y)g(X,W)]=0. \tag{7.3}$$

Putting $X = U = e_i$ in (7.3), where e_i is an orthonormal basis for the tangent space at each point of the manifold and taking summation over i, $i = 1, 2, \ldots, n$, we get

$$S(Y,W) = [(n-1)(\alpha^2-\rho)-r]g(Y,W) + [(1-n)(\alpha^2-\rho)]\eta(Y)\eta(W). \tag{7.4}$$

Hence we can state the following:

Theorem 7.1. *Let M be an $(LCS)_n$-manifold satisfying the condition $B \cdot R = 0$. Then M is η-Einstein or B-curvature tensor reduces to concircular curvature tensor [12].*

Next consider an $(LCS)_n$-manifold satisfying the condition $B \cdot B = 0$. Then it can be easily seen that

$$B(\xi,U)B(X,Y)\xi - B(B(\xi,U)X,Y)\xi - B(X,B(\xi,U)Y)\xi - B(X,Y)B(\xi,U)\xi = 0. \tag{7.5}$$

By using (2.9), (2.12) and (2.15) in (7.5) and then taking inner product with respect to ξ and finally plugging $X = \xi$, we get

$$S(U,Y) = \lambda_1 g(U,Y) + \lambda_2 \eta(U)\eta(Y). \tag{7.6}$$

Where

$$\lambda_1 = \frac{a_1(n-1)(\alpha^2-\rho)[a_1-1-A]-2Aa_2r-2A^2}{a_1(\alpha^2-\rho)[n-1+a_0]+2a_0(\alpha^2-\rho)+4a_2r+2a_1a_2r}$$
$$\lambda_2 = \frac{A_1-AB+2A^2-6Aa_2r-2Aa_0(\alpha^2-\rho)+2B[a_0(\alpha^2-\rho)+2a_2r]}{a_1(\alpha^2-\rho)[n-1+a_0]+2a_0(\alpha^2-\rho)+4a_2r+2a_1a_2r}$$
$$A_1 = (n-1)(\alpha^2-\rho)[-a_0a_1(\alpha^2-\rho) + 2A(a_1+1) - a_1(a_1-1)(n-1) - 2a_1a_2r - B],$$
$$A = a_0(\alpha^2-\rho) + a_1(n-1)(\alpha^2-\rho) + 2a_2r,$$
$$B = a_0(\alpha^2-\rho) + 2a_1(n-1)(\alpha^2-\rho) + 2a_2r.$$

Hence we can state the following:

Theorem 7.2. *An n-dimensional $(LCS)_n$-manifold satisfying $B \cdot B = 0$ is an η-Einstein manifold.*

Finally we consider an $(LCS)_n$- manifold satisfying the condition $B \cdot S = 0$. Then we have

$$S(B(\xi,X)U,V) + S(U,B(\xi,X)V) = 0, \tag{7.7}$$

using (2.12), (2.14) and (2.15) in (7.7) and then plugging $U = \xi$, we get

$$S(X,V) = (n-1)(\alpha^2-\rho)g(X,V). \tag{7.8}$$

Hence we can state the following:

Theorem 7.3. *An n-dimensional $(LCS)_n$-manifold satisfying $B \cdot S = 0$ is an Einstein manifold.*

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