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Picture grammar for *Santal* floor design as radial language

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Abstract

In this paper the application of two dimensional picture grammars for generating traditional *Sohrai* floor design of *Santal* is explored. Context-free picture grammars are able to yield most of the floor designs. Based on the concept of radial languages, a Q-i (quadrant-i) radial language model is proposed in this work. Further, the tiling operators are used to generate the composite two-dimensional words.

Keywords

Radial language, Q-i radial language, picture grammar, floor designs.

AMS Subject Classification

03E75, 47A05, 97A40, 01A13, 01A32.

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1. Introduction

Mathematics is a subject of logic which is considered as culture free. Some of the mathematicians like Bishop [1], an internalist, believe that mathematics is a cultural product which has developed as a result of various activities such as– counting, measuring, designing, playing and ability to explain, are part of that cultural product. However, the externalists deny this statement and advocate that mathematics is virtually culture free. Bodies of anthropological research focus on various developments related to intuitive mathematical thinking among illiterate and under-educated societies. These bodies of research have written on the importance of using culturally specific context in teaching and learning mathematics, including – (i) use of relevant examples from the students' own culture, and (ii) exposing students to varieties of cultural contexts. The knowledge of mathematical operation is situated cognitively by the manipulation of cultural activities.

In *Santal*[1] tribal community, *Sohrai*[2] and *Sakrat*[3] are observed very enthusiastically. The houses are cleaned and decorated with colorful traditional designs. Almost every part of the residential spaces like the walls, the floor, the gates, the sitting space outside the houses, in the veranda called *Pïndä*, doors, pillars, are decorated. The *Santal* women plas-

[1] *Santal* are one of the ethnic indigenous community, native to India, Bangladesh, Nepal and Bhutan. They are most prominent indigenous community in some of the Indian States like Odisha, Jharkhand, Bihar, West Bengal and Assam. They belong to proto-Austroloid racial group and speak *Santali*, it belong to Austroasiatic language family.*Ol Chiki*is the script used as formal medium of instruction and communication, approved as one of the official languages in 8th schedule of Indian constitution by the Indian government.

[2] It is their harvest festival, mainly celebrated at the beginning of the winter harvest. During this time, ritualistic mural *Sohrai*arts are done on mud walls to welcome the harvest and their cattle to celebrate.

[3] It is the last day of the harvest festival, celebrated with rituals, applying mustered oil on different parts of body and having feast with meat and bread made of rice flour.

ter and re-plaster the walls, floors etc. with mud and carve out the designs. Later they daub with handmade colors. Colors are usually prepared by using specific varieties of soil such as limestone, red soil and burnt ashes of rice straw, husks mixed with cow dung solution. Men folks help to collect all the basic requirements and participate in processing. Geometric patterns, basic floral patterns are mostly used in their wall paintings and floor designs. It is culturally practiced for ages. Usually they start this process of cleaning, plastering in the later half of December till early January, before *Sakrat*. They have been using their indigenous measurement system to generate triangles, rectangles, square, circles, rhombus etc. even before they were made familiar with the formal curriculum based mathematics education. They display their spatial knowledge, knowledge of balance, proportion, basic operation of mathematics in their everyday life through many cultural practices. They are totally unaware that by practicing this activities, they actually use their cognitive faculties to generate amazing patterns which emerge as aesthetically beautiful designs. They initially draw a rough sketches before making the design by dividing the available space in different categories i.e. rectangular or square. This expresses their application of proportions to resuscitate the abstract ideas into concrete designs that representing their ability to conceptualize the abstract thinking. The designs include perfect square, angular and circular pictures in proportion.

Like the global search of mathematical ideas in *Kolam* of South India [2], *Sona*, the sand art of Angola, South West Africa [3], an effort has been made to search for the relationship of mathematical ideas and culture in *Sohraï* floor designs.

2. Theoretical framework

Inspired by the seminal work in early 1970s by Siromoney and Siromoney on *Kolam*, the South India folk art, the grammar generation of various patterns, picture languages and tiling operation, an attempt has been done to generate some of the floor designs picture grammar by using the radial language in this paper. The paper explores to generate the patterns, alternately the picture words using context-free picture grammar where the terminals are chosen to be few primitive symbols. The Radial grammar and *L*- system by Siromoney et al.[4] provides the fundamental theoretical background for this work. Siromoney et al. [5] explained the designs of *Kolam*, the South Indian folk art by writing Array grammar. They expounded how the art was treated as two dimensional picture languages and generating finite designs. The picture transformation was done by translation, reflection, half-turn, magnification and conjugation. Array automata were defined corresponding to array language and characterization of context free and regular matrices. Later, Siromoney et al. explor how the Array model [6] could be worked with single grammar with a finite number of rules and a finite number of terminals or a finite number of instructions. They proposed that the model can be generate an infinite set consisting of the same kind of *Kolam* pattern of the different sizes. Further, the authors [4] proposed the generation of circular developmental patterns. Growth takes place in parallel, restricted by a table, and the radial and tangential generations are controlled by a control set. The new interpretation was given to the terminals where the terminals represented distances. The radial grammar and *L*-system model was able to describe the growth of terminals in two dimensions and symmetry of conical spiral in three dimensions.

3. Basic Concepts

3.1 Radial Language

Consider a radial pattern in the two dimensional plane. A picture pattern may be radially produced using a compound non-terminal. Based on the number of components of the compound non-terminal symbol the space of 360° may be equally divided. Consider an example where a compound non-terminal containing two components $\langle A_1A_2\rangle$ may produce two radials 180° apart. Similarly, the use of three components in the non-terminal $\langle A_1A_2A_3\rangle$ would cause three radials 120° apart. Figure 1 shows a Radial Patterns.

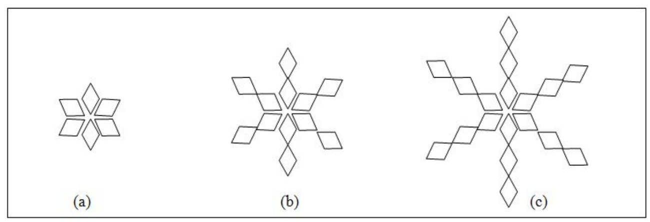


Figure 1. Radial Patterns.

3.1.1 Example

Consider the patterns given in Figure 1 with six radial patterns emerging from a point in a two dimensional space. Let $G = (N,T,P,S)$, where $N = \{S,\langle A_1...A_6\rangle\}$, $T = \{\diamond\}$ P :

(i) $S \to \langle A_1...A_6\rangle$

(ii) $\langle A_1...A_6\rangle \to \langle\diamond...\diamond\rangle\langle A_1...A_6\rangle$

(iii) $\langle A_1...A_6\rangle \to \langle A_1...A_6\rangle$

The three patterns in figure 1 are generated by application of the radial rules in the above grammar G. The pattern (a) may be generated by application of production rules (i) and (iii). The pattern (b) may be generated by the use of production rules (i), (ii), and (iii) consecutively. The pattern (c) in figure 1 may be produced by the application of rules, (i), (ii), (ii) and (iii). The three patterns are the radial languages $\{ M_n \mid n = 1,2and3\}$ depending on length of the radial substring. This may be achieved by applying the production rule ii the desired number of times.

3.1.2 Example

$G = (N,T,P,S)$. Where let $N = \{S,\langle B_1B_2\rangle\}$ and S the start symbol of a context-free grammar. Let P contains the following two production rules:

(i) $S \rightarrow \langle B_1B_2 \rangle$

(ii) $\langle B_1B_2 \rangle \rightarrow \langle || \rangle$

Then the string —can be derived applying the production rules (i) and (ii).

$S \underset{(i)}{\Rightarrow} \langle B_1B_2 \rangle$

$\underset{(ii)}{\Longrightarrow} \langle || \rangle$

However, pictorially $S \overset{*}{\Rightarrow}$ — is derived by the above grammar G.

3.1.3 Example

The Eight-Ray Design Let L_2 be an eight-ray design. Then consider the following grammar with terminal symbol '$|$'.

L_2 = a radical language {}

$G_2 = (N_2, T_2, P_2, S_2)$

$N_2 = \{S_2, \langle A_1A_2A_3A_4A_5A_6A_7A_8 \rangle$

$T_2 = \{|\}$

P_2 :

(i) $S_2 \rightarrow \langle A_1A_2A_3A_4A_5A_6A_7A_8 \rangle$

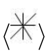
(ii) $\langle A_1A_2A_3A_4A_5A_6A_7A_8 \rangle \rightarrow \langle |||||||| \rangle$

To derive  use production rules (i) and (ii)

Syntactically $S_2 \underset{(i)}{\Rightarrow} \langle A_1A_2...A_8 \rangle$

$\underset{(ii)}{\Longrightarrow} \langle ||......| \rangle$

Pictorially $S_2 \underset{(i)}{\Rightarrow} \langle A_1A_2...A_8 \rangle$

$\underset{(ii)}{\Longrightarrow}$ ⟨⟩

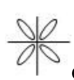
3.1.4 Example

For various other terminal symbol the following picture words or patterns may be derived by a context-free grammar with compound non-terminal and here, $k = 8$

(i) $S \rightarrow \langle B_1B_2...B_8 \rangle$

(ii) $\langle B_1B_2...B_8 \rangle \rightarrow \langle 00......0 \rangle$

$S \overset{*}{\Rightarrow}$ 

Using the above production rules,  can be derived,

$\langle B_1B_2...B_8 \rangle \rightarrow \langle 1010......0 \rangle$

$S \overset{*}{\Rightarrow}$ 

3.2 Quadrant Radial Language

Based on the basic concepts of Ray-languages or radial languages,operators for tiling the picture languages are utilized. we consider to divide the two dimensional space of 360° around a point of reference into four quadrants, I, II, III and IV. Thus the space around a point (origin) may be referred as Q-i, with the value of i ranging from 1 to 4 to specify the quadrant. The proposed Q-i radial language explores to consider the space of 90° to emanate the rays or the direction in which the terminal symbols from the alphabet of the language may be placed. Further, the principle of compound non-terminals is followed in the grammar for the Quadrant Radial languages. The angle of the rays is determined by the number of components say, k by dividing a quadrant, i.e. 90°, by k.

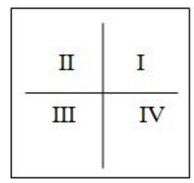
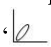
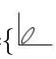


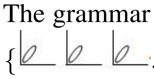
Figure 2. Quadrant division.

Consider a pattern in quadrant-I. Q-I radial language: When the Quadrant-I is used to yield the patterns, a sequence of non-terminal of k primitives can be used to generate patterns only in the quadrant-I. Similarly, it is applied to Quadrant-II, III IV.

Therefore, for a Q-I radial language the terminal symbol may be used to generate pattern/string only in the quadrant-I. Consider the pattern ''in quadrant-I. The terminal used are '0'and '$|$'.Here the space of90° angle, three rays are emanating at 0°, 45° and at 90°. Therefore, it is possible to write a context-free radial grammar for quadrant-I and also produce infinite string juxtaposing each quadrant-I word.

3.2.1 Example

Q-I radial language says L_1 ={}

The grammar for L_1 that may generate the infinite string of {...}, by the following grammar G_1.

$G_1 = (N_1, T_1, P_1, S_1)$

$N_1 = \{S_1, \langle B_1B_2B_3 \rangle$

$T_1 = \{0, |\}$

P_2 :

(i) $S_1 \rightarrow \langle B_1B_2B_3 \rangle$

(ii) $\langle B_1B_2B_3 \rangle \rightarrow \langle |0| \rangle$

To generate a string using grammar G_1

Syntactically $S_1 \underset{1}{\Rightarrow} \langle B_1B_2B_3 \rangle$

$\underset{2}{\Rightarrow} \langle |0| \rangle$

3.3 Conjugation/Concatenation

Picture transformation through conjugation was elaborated by [4]. Through the concatenation operation the interesting circular patterns are generated. Row and column catenation define rectangular array and for radial concatenation '$\odot$' has been applied. Conjugation is juxtaposing the picture words to derive patterns.

3.3.1 Conjugation Operators

For the picture transformation two operators are used to derive a two dimensional array of a basic picture or pattern. A horizontal conjugation has been observed in two dimensional array grammars to repeat the pattern horizontally for a desired number of times or even infinitely. Similarly, the vertical conjugation to generate the pattern vertically. However, considering a *Santhal* design a new operator - Concentric operator is proposed in this paper to position one pattern within the other at a center or origin. We use 'Φ' to denote a horizontal conjugation operator and '$\ominus$' used for vertical conjugation between patterns. For concentric operation the sign '$\odot$' is used in this paper. We illustrate these operators through following examples.

3.3.2 Horizontal concatenation

Let C be a Pattern and V a non-terminal then the production rules pertaining to horizontal concatenation would yield picture pattern of finite and infinite horizontal array.
Consider the production rules with following on the right hand side $C \Phi C$, $C \Phi V$ and $V \Phi V$.
Let $C_1 =$ $and C_2 =$
If $C_1 \Phi C_2 =$ $and C_2 \Phi C_1 =$

3.3.3 Vertical concatenation with the binary conjugation operator '$\ominus$'

If $C_1 \ominus C_2 =$ $and C_{12} =$

3.3.4 Concentric operation using '$\odot$'

If $D_x =$, $D_y =$ $and D_z =$ then the concentric operation will be $D = (D_{xy})_z$
$D =$

The horizontal and vertical conjugation operators Φ and $\ominus$ are associative.Therefore, they can derive a picture pattern either along X-axis or Y-axis respectively. In case of binary operator $\odot$ to position two patterns on the same origin, based on the radius of the pattern or the radius of the circle within the pattern with the smaller radius can be situated within the pattern with the larger radius. Thus the concentric operator $\odot$ can be proved to be commutative and associative. As the purpose of this paper is to investigate whether such patterns are radial languages and whether application of a context-free array grammar can derive some of the floor patterns. The properties of the conjugation operators are not being presented here in detail for the simple picture words.

4. Floor design a context free language

A sample of the simple floor design from the *Santal* floor patterns has been presented in figure 3. The pattern appears to be a concatenation of two basic designs a square with a petal pattern at the corners and an six petal pattern seem to be situated at the center of the square. Therefore, we explore to design the radial grammar with conjugation operators that can derive an array of pattern as given in the figure 3. For this purpose we consider two languages, first, the language is consisting the floral pattern, a radial language and the next is square with petals in the corner as a Q-i radial language.Figure 3 shows a conjugation of petal patterns.

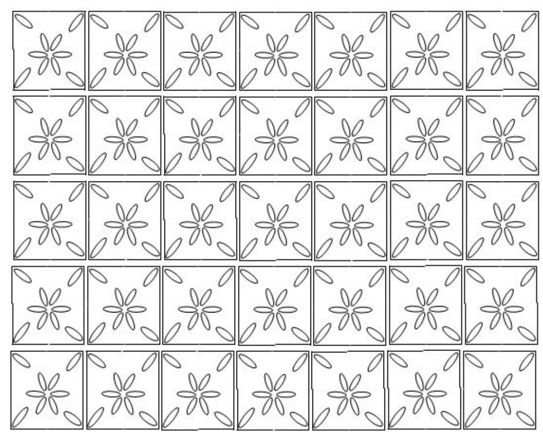


Figure 3. A Floor Design.

Consider the language L_3={ } , a radial language using the terminal symbol' '. The following 6-ray radial grammar can generate the words in L_3,
$G_3 = (N_3, T_3, P_3, S_3)$
$N_3 = \{S_3, \langle A_1A_2A_3A_4A_5A_6 \rangle\}$
$T_3 = \{ \}$
P_3 :

(i) $S_3 \rightarrow \langle A_1A_2A_3A_4A_5A_6 \rangle$

(ii) $\langle A_1A_2A_3A_4A_5A_6 \rangle \rightarrow \langle \rangle$

Then to generate the string using the production rules (i) and (ii) in P_3 as below:
Syntactically $S_3 \underset{(i)}{\Longrightarrow} \langle A_1A_2A_3A_4A_5A_6 \rangle$
$\underset{(ii)}{\Longrightarrow} \langle \rangle$
Visual view $S_3 \underset{(i)}{\Longrightarrow} \langle A_1A_2A_3A_4A_5A_6 \rangle$

$\underset{(ii)}{\Rightarrow}$ ⟨  ⟩

Consider the Q-I radial language says L_1={ }
The grammar for L_1 that may generate the infinitely many strings{ ...}.
$G_1 = (N_1, T_1, P_1, S_1)$
$N_1 = \{S_1, \langle B_1 B_2 B_3\rangle$
$T_1 = \{$0,1$\}$
P_2 :

(i) $S_1 \rightarrow \langle B_1 B_2 B_3\rangle$

(ii) $\langle B_1 B_2 B_3\rangle \rightarrow \langle$ 101 $\rangle$

To generate a string using grammar G_1
Syntactically $S_1 \underset{1}{\Rightarrow} \langle B_1 B_2 B_3\rangle$

$\underset{2}{\Rightarrow} \langle$ 101 $\rangle$
Visually $S_1 \underset{1}{\Rightarrow} \langle B_1 B_2 B_3\rangle$

$\underset{2}{\Rightarrow} \langle \rangle$

Similarly the language $L_2 = \{ \}$ can either be generated as a Q-II language as a quadrant II picture word or by applying the reverse operator. Incorporating the horizontal concatenation in the grammar to concatenate the strings of L_1 and L_2 another picture pattern can be derived. To illustrate this, consider the following two picture words P_1 and P_2. $P_1 =$
and $P_2 =$
Then, $P_2 \Phi P_1 =$
$P_1 \Phi P_2 =$
Consider Ⓡ to denote the unary operator to reverse the picture word as a mirror image. Then the pattern is the reverse (mirror image) of .
$P_1 = P_2^{®}$
$\therefore P_1 =$.
$(P_1)^{®} \rightarrow$ —
$\rightarrow$
$\rightarrow$

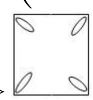
The pattern can alternately be produced by applying the reverse operator.
$P_1 \Phi P_2^{®}$
$\Rightarrow \ \Phi \ (\ )^{®}$

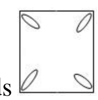
$\Rightarrow \ \Phi$
$\Rightarrow$
Let us consider the binary conjugate operator '⊖' for deriving vertical array.
P_{12}

$\Rightarrow \ \ominus$

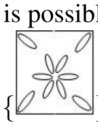
$\Rightarrow$
It may be observed that the reverse operator Ⓡ if applied on an argument of horizontal conjugate would reverse from quadrant I to quadrant II and vice versa while the reverse operator applied with vertical operator would cause a vertical reversal from quadrant I to quadrant IV and vice versa and from quadrant II to quadrant III and vice versa. Therefore, vertical reversal of pattern P_1 is .
$P_1^{®} \ominus P_1$
$\Rightarrow (\ )^{®} \ominus$

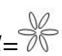
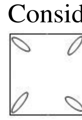
$\Rightarrow$
$\left(P_1 \Phi P_1^{®}\right)^{®} \ominus \left(P_1 \Phi P_1^{®}\right)$
$\Rightarrow \left(\ \Phi\ (\ )^{®}\right)^{®} \ominus \left(\ \Phi\ (\ )^{®}\right)$
$\Rightarrow (\ )^{®} \ominus (\ )$

$\Rightarrow$ 

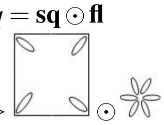
Applying the concentric operator '⊙' on picture words 

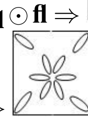
and  it is possible to derive the L_sq as given below,

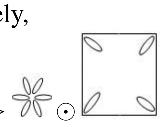
Let $L_sq = \{$  $\}$

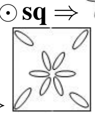
Consider the picture word fl=  and picture word sq =


Then applying the concentric operator **fl** ⊙ **sq** or **sq** ⊙ **fl** would derive the resultant picture word the desired pattern
Then $L_sq =$ **sq** ⊙ **fl**

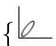
sq ⊙ **fl** ⇒ 

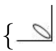
⇒ 
Alternately,

fl ⊙ **sq** ⇒ 

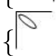
⇒ 

4.1 Q-i radial languages

Using the Q-i radial grammar the picture words in the quadrants I, II, III and IV can be generated to obtain the following languages. Further applying the horizontal and vertical conjugate operators the same picture word can be derived as illustrated in this section. $L_1 = \{$$\}$Quadrant-I radial language

$L_2 = \{$$\}$Quadrant-II radial language

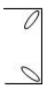
$L_3 = \{$$\}$Quadrant-III radial language

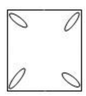
$L_4 = \{$$\}$Quadrant-IV radial language

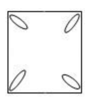
$L_1 \oplus L_2 \Rightarrow$ 

$L_4 \ominus L_1 \Rightarrow$ 

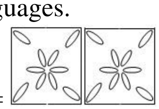
$L_4 \oplus L_3 \Rightarrow$ 

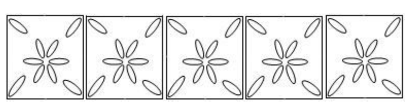
$L_3 \ominus L_2 \Rightarrow$ 

$(L_4 \ominus L_1) \oplus (L_3 \ominus L_2) =$ 

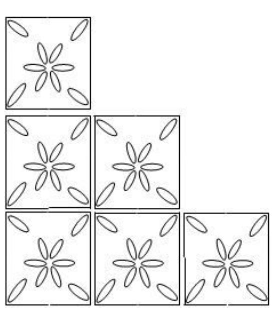
$\left(L_4 \oplus L_3\right) \ominus \left(L_1 \oplus L_2\right) =$ 

The operators '' and '$\ominus$' are also applied on the languages to yield picture words by concatenating picture words of the two languages horizontally and vertically and so to derive the two-dimensional array of the patterns resulting in a third picture languages.

$L_sq \oplus L_sq =$ 

$L_sq \oplus {}^5 L_sq =$ 

$\left(\left(\left(L_sq \oplus {}^3 L_sq\right) \ominus {}^2 L_sq\right) \ominus L_sq\right) =$



5. Conclusion

There is no doubt that mathematical ideas are embedded in various cultural practices. Cultures contain evidences of these cognitive competencies to inculcate and preserve mathematical thinking. A context free radial grammar through the introduction of quadrant radial grammar for a simple *Sohraï* floor design is successfully presented in this paper. There exist complex and intricate designs as *Kolam* patterns, *Santhali* women too draw more complex floor designs by manipulating their spatial and geometrical imagination, attempt to elaborate them through picture grammar may be carried out as future work.

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