Malaya Journal of Matematik, Vol. 7, No. 3, 408-411, 2019
https://doi.org/10.26637/MJM0703/0007

Soft almost regular spaces

Archana K. Prasad[1]* and S.S.Thakur[2]

Abstract
In this paper the concept of soft almost regular spaces have been introduced and studied.

Keywords
Soft sets, Soft topology, Soft regular, Soft weakly regular and Soft almost regular spaces.

AMS Subject Classification
Primary 54A40 , 54D10 ; Secondary 06D72

[1,2] *Department of Mathematics and Computer Science, R.D.V.V, Jabalpur (M.P) 482001, India and Department of Applied Mathematics, Jabalpur Engineering College, Jabalpur (M.P) 482011, India.*
***Corresponding author**: [1]kumari_archu @ yahoo.com; [2]samajh_singh @ yahoo.com
Article History: Received **24** March **2019**; Accepted **09** May **2019**

Contents

1. Introduction

The concept of soft set theory was introduced by Molodtsov [7] as a general mathematical tool for dealing with problems that contains uncertainty. In 2011, Shabir and Naz[11] initiated the study of soft topological spaces and derived their basic properties. Recently, Hussain and Ahmad [3] introduced the notion of soft regular spaces. The main aim of this paper is to introduce a new soft separation axiom called soft almost regularity which is a weak form of soft regularity and investigate some of their properties and characterizations.

2. Preliminaries

Throughout this paper X denotes a nonempty set, E denotes the set of parameters and S(X,E) denotes the family of soft sets over X. For definition and basic properties of soft sets, reader should refer [[1],[4],[6],[7],[9],[11],[13]].

Definition 2.1. *[11] A subfamily τ of* S(X,E) *is called a soft topology on* X *if :*

(a) $\widetilde{\phi},\widetilde{\mathrm{X}}$ *belongs to* τ.

(b) The union of any number of soft sets in τ *belongs to* τ.

(c) The intersection of any two soft sets in τ *belongs to* τ.

The triplet $(\mathrm{X},\tau,\mathrm{E})$ *is called a soft topological space. The members of* τ *are called soft open sets in* X *and their complements called soft closed sets in* X.

Lemma 2.2. *[11] Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space. Then the collection* $\tau_\alpha = \{\mathrm{F}(\alpha) : (\mathrm{F},\mathrm{E}) \in \tau\}$ *for each* $\alpha \in \mathrm{E}$, *defines a topology on* X.

Definition 2.3. *[11] In a soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *the intersection of all soft closed super sets of* (F,E) *is called the soft closure of* (F,E). *It is denoted by* $\mathrm{Cl}(\mathrm{F},\mathrm{E})$.

Definition 2.4. *[13] In a soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *the union of all soft open subsets of* (F,E) *is called soft interior of* (F,E). *It is denoted by* $\mathrm{Int}\ (\mathrm{F},\mathrm{E})$.

Definition 2.5 ([11],[13]). *Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space and let* $(\mathrm{F},\mathrm{E}),(\mathrm{G},\mathrm{E}) \in \mathrm{S}(\mathrm{X},\mathrm{E})$. *Then:*

(a) (F,E) *is soft closed if and only if* $(\mathrm{F},\mathrm{E}) = \mathrm{Cl}(\mathrm{F},\mathrm{E})$

(b) If $(\mathrm{F},\mathrm{E}) \subseteq (\mathrm{G},\mathrm{E})$, *then* $\mathrm{Cl}\ (\mathrm{F},\mathrm{E}) \subseteq \mathrm{Cl}(\mathrm{G},\mathrm{E})$.

(c) (F,E) *is soft open if and only if* $(\mathrm{F},\mathrm{E}) = \mathrm{Int}\ (\mathrm{F},\mathrm{E})$.

(d) If $(\mathrm{F},\mathrm{E}) \subseteq (\mathrm{G},\mathrm{E})$, *then* $\mathrm{Int}\ (\mathrm{F},\mathrm{E}) \subseteq \mathrm{Int}\ (\mathrm{G},\mathrm{E})$.

(e) $(\mathrm{Cl}\ (\mathrm{F},\mathrm{E}))^{\mathrm{C}} = \mathrm{Int}\ ((\mathrm{F},\mathrm{E})^{\mathrm{C}})$.

(f) $(\mathrm{Int}\ (\mathrm{F,E}))^{\mathrm{C}} = \mathrm{Cl}\ ((\mathrm{F,E})^{\mathrm{C}})$.

Lemma 2.6 ([3])**.** *Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space over* X *and* Y *be a nonempty subset of* X*. Then* $\tau_{\mathrm{Y}} = \{(\mathrm{F_Y, E}) : (\mathrm{F,E}) \in \tau\}$ *is said to be the soft relative topology on* Y *and* $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E})$ *is called a soft subspace of* $(\mathrm{X},\tau,\mathrm{E})$.

Lemma 2.7 ([3])**.** *Let* $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E})$ *be a soft subspace of a soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *and* $(\mathrm{F,E})$ *be a soft open set in* Y*. If* $\widetilde{\mathrm{Y}} \in \tau$ *then* $(\mathrm{F,E}) \in \tau$.

Definition 2.8 ([3])**.** *Let* $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E})$ *be a soft topological subspace of a soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *and* $(\mathrm{F,E})$ *be a soft set over* X *,then :*

(a) $(\mathrm{F,E})$ *is soft open in* Y *if and only if* $(\mathrm{F,E}) = \widetilde{\mathrm{Y}} \cap (\mathrm{G,E})$ *for some soft open set* $(\mathrm{G,E})$ *in* X.

(b) $(\mathrm{F,E})$ *is soft closed in* Y *if and only if* $(\mathrm{F,E}) = \widetilde{\mathrm{Y}} \cap (\mathrm{G,E})$ *for some soft closed set* $(\mathrm{G,E})$ *in* X.

Lemma 2.9 ([10])**.** *Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space and* $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E})$ *be a soft subspace of* $(\mathrm{X},\tau,\mathrm{E})$ *, then a soft closed set* $(\mathrm{F_Y, E})$ *of* Y *is soft closed in* X *if and only if* $\widetilde{\mathrm{Y}}$ *is soft closed in X.*

Definition 2.10 ([2],[5])**.** *The soft set* $(\mathrm{F,E}) \in \mathrm{S}$ *(X, E) is called a soft point if there exists* $x \in \mathrm{X}$ *and* $e \in E$ *such that* $F(e) = \{x\}$ *and* $F(\mathrm{e}^{\mathrm{C}}) = \phi$ *for each* $\mathrm{e}^{\mathrm{C}} \in \mathrm{E}-\{e\}$ *, and the soft point* $(\mathrm{F,E})$ *is denoted by* x_e*. We denote the family of all soft points over* X *by* $\mathrm{SP}(\mathrm{X,E})$*.*

Definition 2.11 ([13])**.** *The soft point* x_e *is said to be in the soft set* $(\mathrm{G,E})$*, denoted by* $x_e \in (\mathrm{G,E})$ *if* $x_e \subseteq (\mathrm{G,\ E})$.

Definition 2.12 ([2],[8])**.** *Let* $(\mathrm{F,E}),(\mathrm{G,E}) \in \mathrm{S}(\mathrm{X,E})$ *and* $x_e \in \mathrm{SP}(\mathrm{X,E})$*. Then we have:*

(a) $x_e \in (\mathrm{F,E})$ *if and only if* $x_e \notin (\mathrm{F,E})^{\mathrm{c}}$.

(b) $x_e \in (\mathrm{F,E}) \cup (\mathrm{G,\ E})$ *if and only if* $x_e \in (\mathrm{F,E})$ *or* $x_e \in (\mathrm{G,\ E})$.

(c) $x_e \in (\mathrm{F,E}) \cap (\mathrm{G,\ E})$ *if and only if* $x_e \in (\mathrm{F,E})$ *and* $x_e \in (\mathrm{G,E})$.

(d) $(\mathrm{F,E}) \subseteq (\mathrm{G,E})$ *if and only if* $x_e \in (\mathrm{F,E})$ *implies* $x_e \in (\mathrm{G,E})$*.*

Definition 2.13 ([3])**.** *Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space,* $(\mathrm{F,E})$ *be a soft set and* $\mathrm{x_e} \in \mathrm{X}$*. Then* $(\mathrm{F,E})$ *is called soft neighborhood of* $\mathrm{x_e}$*, if there exists soft open set* $(\mathrm{G,E})$ *such that* $\mathrm{x_e} \in (\mathrm{G,E}) \subseteq (\mathrm{F,E})$.

Definition 2.14 ([12])**.** *Let* (X,τ,E) *be a soft topological space and (F,E) and (G,E) be soft subsets of X. Then, (F,E) and (G,E) are said to be weakly seperated, if there exists soft open sets (U,E) and (V,E) of X such that* $(F,E) \subseteq (U,E)$, $(U,E) \cap (G,E) = \phi$ *and* $(G,E) \subseteq (V,E)$, $(V,E) \cap (F,E) = \phi$.

Definition 2.15 ([12])**.** *Let* (X,τ,E) *be a soft topological space and (F,E) and (G,E) be soft subsets of X. Then, (F,E) and (G,E) are said to be strongly seperated, if there exists soft open sets (U,E) and (V,E) of X such that* $(F,E) \subseteq (U,E)$, $(G,E) \subseteq (V,E)$ *and* $(U,E) \cap (V,E) = \phi$.

Definition 2.16 ([12])**.** *A soft topological space* (X,τ,E) *is said to be soft weakly regular, if every soft weakly seperated pair consisting of a soft regular closed set and a soft point can be strongly seperated.*

Definition 2.17 ([3])**.** *A soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *is said to be soft regular if for all soft closed sets* $(\mathrm{F,E})$ *in* X *and each soft point* x_e *such that* $x_e \notin (\mathrm{F,E})$*,there exists soft open sets* $(\mathrm{U,E})$ *and* $(\mathrm{V,E})$ *of* X *such that* $\mathrm{x_e} \in (\mathrm{U,E})$, $(\mathrm{F,E}) \subseteq (\mathrm{V,E})$ *and* $(\mathrm{U,E}) \cap (\mathrm{V,E}) = \phi$.

3. Main Results

Definition 3.1. *A soft topological space* $(\mathrm{X},\tau,\mathrm{E})$ *is said to be soft almost regular if for all soft regular closed sets* $(\mathrm{F,E})$ *in* X *and each soft point* x_e *such that* $x_e \notin (\mathrm{F,E})$*, there exists soft open sets* $(\mathrm{U,E})$ *and* $(\mathrm{V,E})$ *of* X *such that* $\mathrm{x_e} \in (\mathrm{U,E})$, $(\mathrm{F,E}) \subseteq (\mathrm{V,E})$ *and* $(\mathrm{U,E}) \cap (\mathrm{V,E}) = \phi$.

Remark 3.2. *Every soft regular space is soft almost regular but the converse may not be true. For,*

Example 3.3. *Let* (X,τ,E) *be soft topological spaces where* $X = \{a, b\}$, $E = \{e_1,e_2\}$ $\tau = \{\phi, X, \{(e_1,X),(e_2,\{b\})\},\{(e_1,\{a\}),(e_2,X)\},\{(e_1,\{a\}),(e_2,\{b\})\}\}$ *Then* (X,τ,E) *is soft almost regular but not soft regular.*

Theorem 3.4. *Let* $(\mathrm{X},\tau,\mathrm{E})$ *be a soft topological space. Then the following conditions are equivalent:*

(i) $(\mathrm{X},\tau,\mathrm{E})$ *is soft almost regular.*

(ii) *For each soft point* x_e *of* X *and each soft regular open set* $(\mathrm{V,E})$ *containing* x_e *,there exists a soft regular open set* $(\mathrm{U,E})$ *such that* $x_e \in (\mathrm{U,E}) \subseteq \mathrm{Cl}(\mathrm{U,E}) \subseteq (\mathrm{V,E})$.

(iii) *For each soft point* x_e *of* X *and each soft neighborhood* $(\mathrm{M,E})$ *of* x_e*, there exists a soft regular open neighborhood* $(\mathrm{V,E})$ *of* x_e *such that* $\mathrm{Cl}(\mathrm{V,E}) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{M,E}))$.

(iv) *For each soft point* x_e *of* X *and each soft neighborhood* $(\mathrm{M,E})$ *of* x_e*,there exists a soft open neighborhood* $(\mathrm{V,E})$ *of* x_e *such that* $\mathrm{Cl}(\mathrm{V,E}) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{M,E}))$.

(v) *For each soft regular closed set* $(\mathrm{A,E})$ *and each soft point* $x_e \notin (\mathrm{A,E})$*, there exists soft open sets* $(\mathrm{U,E})$ *and* $(\mathrm{V,E})$ *such that* $x_e \in (\mathrm{U,E})$, $(\mathrm{A,E}) \subseteq (\mathrm{V,E})$ *and* $\mathrm{Cl}(\mathrm{U,E}) \cap \mathrm{Cl}(\mathrm{V,E}) = \phi$.

(vi) *Each soft regular closed set* $(\mathrm{F,E})$ *is expressible as an intersection of some soft regular closed neighborhoods of*

(F,E).

(vii) *Each soft regular closed set* (F,E) *is identical with the intersection of all soft closed neighborhoods of* (F,E).

(viii) *For each soft set* (A,E) *and each soft regular open* (B,E) *such that* $(A,E)\cap(B,E)\neq\phi$*, there exists a soft open set* (G,E) *such that* $(A,E)\cap(G,E)\neq\phi$ *and* $Cl(G,E)\subseteq(B,E)$.

(ix) *For each nonempty soft set* (A,E) *and each soft regular closed set* (B,E) *satisfying* $(A,E)\cap(B,E)\neq\phi$*, there exists disjoint soft open sets* (G,E) *and* (H,E) *such that* $(A,E)\cap(G,E)\neq\phi$ *and* $(B,E)\subseteq(H,E)$.

Proof. **(i)** $\Rightarrow$**(ii)** If (V,E) is a soft regular open set of X containing x_e,then $(V,E)^c$ is a soft regular closed set such that $x_e\notin(V,E)^c$.Therefore there exists soft open sets (U_1,E) and (U_2,E) such that $x_e\in(U_1,E)$, $(V,E)^c\subseteq(U_2,E)$ and $(U_1,E)\widetilde{\cap}(U_2,E))=\phi$.Then $Cl(U_1,E)\widetilde{\cap}(U_2,E)=\phi$ because (U_2,E) is soft open and hence $Cl(U_1,E)\subseteq(U_2,E)^c\subseteq(V,E)$.Thus, $x_e\in(U_1,E)\subseteq Cl(U_1,E)\subseteq(V,E)$. Again $(U_1,E)\subseteq Int(Cl(U_1,E))$ $\subseteq Cl(U_1,E)\subseteq(V,E)$. Put $(U,E)=Int(Cl(U_1,E))$. Then, $(U_1,E)\subseteq(U,E)\subseteq Cl(U,E)=Cl(U_1,E)\subseteq(V,E)$. Hence , $x_e\in(U,E)\subseteq Cl(U,E)\subseteq(V,E)$ where (U,E) is soft regular open.

(ii) $\Rightarrow$**(iii)** Let (M,E) be a soft neighborhood of a soft point x_e of X, then there exists a soft open set (V,E) of X such that $x_e\in(V,E)\subseteq(M,E)$.Thus, $x_e\in Int(Cl(V,E))\subseteq Int(Cl(M,E))$. Put $Int(Cl(V,E))=(A,E)$. Then(A,E) is a soft regular open containing x_e.Therefore, there exists a soft regular open set (U,E) such that $x_e\in(U,E)\subseteq Cl(V,E)\subseteq(A,E)=Int(Cl(V,E))$ $\subseteq Int(Cl(M,E))$.

(iii) $\Rightarrow$**(iv)** The proof follows from the fact that every soft regular open neighborhood of a soft point x_e ,is a soft open neighborhood of x_e.

(iv) $\Rightarrow$**(v)** If (A,E) is a soft regular closed and $x_e\notin(A,E)$, then $(A,E)^c$ is a soft neighborhood of x_e. Therefore , there exists a soft open set (V,E) such that $x_e\in(V,E)\subseteq Cl(V,E)\subseteq(A,E)^c$. Again, since (V,E) is a soft neighborhood of x_e. Therefore there exists a soft open set (U,E) such that $x_e\in(U,E)$ $\subseteq Cl(U,E)\subseteq(V,E)$. Then (U,E) and $(Cl(V,E)^c)$ are soft open sets with disjoint closures containing x_e and (A,E) respectively.

(v) $\Rightarrow$**(vi)** If (F,E) is a soft regular closed set, then for each $x_e\notin(F,E)$, there exists soft open sets$(G,E)_{x_e}$and $(H,E)_{x_e}$ such that $(F,E)\subseteq(G,E)_{x_e}$, $x_e\in(H,E)_{x_e}$ and $Cl(H,E)_{x_e}\widetilde{\cap}Cl(G,E)_{x_e}=\phi$. Thus $(F,E)\subseteq(G,E)_{x_e}$, and $x_e\notin Cl(G,E)_{x_e}$. It can be seen easily that $(F,E)=\widetilde{\cap}(Cl(G,E)_{x_e})$. Also, each $Cl(G,E)_{x_e}$ is a soft regular closed neighborhood of (F,E).

(vi) $\Rightarrow$**(vii)** Obvious

(vii) $\Rightarrow$**(viii)** Let (A,E) be any soft set and let (B,E) be a soft regular open set such that $(A,E)\cap(B,E)\neq\phi$. Then, there exists a soft point $x_e\in(A,E)\cap(B,E)$. Therefore $(B,E)^c$ is a soft regular closed set and hence $(B,E)^c=\bigcap_{i\in I}(M_i,E)$ where $\{(M_i,E): i\in I\}$ is the family of soft closed neighborhoods of $(B,E)^c$. Since $x_e\in(B,E)$, therefore $x_e\notin\bigcap_{i\in I}(M_iE)$ and thus $x_e\notin(M_i,E)$ for some i. Since (M_i,E) is a soft neighborhood of $(B,E)^c$, therefore there exists a soft open set (H,E) such that $(B,E)^c\subseteq(H,E)\subseteq(M_i,E)$. Let $(P,E)=(M_i,E)^c$ .Then (P,E) is a soft open set containing x_e and also $x_e\in(A,E)$. Thus $x_e\in(P,E)\cap(A,E)$,that is $(P,E)\cap(A,E)\neq\phi$. Also $(H,E)^c$ is soft closed, therefore $(P,E)=Cl(M_i,E)^c\subseteq(H,E)^c\subseteq(B,E)$.

(viii) $\Rightarrow$**(ix)** Obvious.

(ix) $\Rightarrow$**(i)** If (A,E) is a soft regular closed and $x_e\notin(A,E)$, then $x_e\neq\phi$ and $(A,E)\cap x_e=\phi$. Therefore, there exists disjoint soft open sets (G,E) and (H,E) such that $x_e\cap(G,E)\neq\phi$ and $(A,E)\subseteq(H,E)$. Thus, $x_e\in(G,E)$, $(A,E)\subseteq(H,E)$ and $(G,E)\cap(H,E)\neq\phi$.Hence (X,τ,E) is soft almost regular. □

Lemma 3.5. *If* (Y,E) *is a soft dense subspace of a soft topological space* (X,τ,E) *then:* $Int_YCl_Y(A)=Int(Cl(A))\cap Y$.

Proof. Obvious. □

Theorem 3.6. *Let* (X,τ,E) *is a soft almost regular spaces and* (Y,τ_Y,E) *be a dense subspace of* X *then* (Y,τ_Y,E) *is a soft almost regular space.*

Proof. Let $y_e\in Y$ and (U,E) be a soft regular open set of Y containing y_e. Then by "**Lemma 3.5**", $(U,E)=Int_YCl_Y(U,E)$ $=Int(Cl(U,E))\cap Y$. Thus $Int(Cl(U,E))$ is a soft regular open set of X containing y_e. Since (X,τ,E) is soft almost regular there exists a soft open set (V,E) containing y_e such that $Cl(V,E)\subseteq(U,E)$. Consequently $(Cl(V,E)\cap Y)\subseteq(U,E)$. Hence (Y,τ_Y,E) is soft almost regular. □

Lemma 3.7. *If* Y *is a soft regular open subspace of* X *then every soft regular open subset of* X *is soft regular open in* X.

Theorem 3.8. *Let* (X,τ,E) *is a soft almost regular spaces and* Y *is a regular open subspace of* (X,τ,E) *then* (Y,τ_Y,E) *is soft almost regular.*

Proof. Let (X,τ,E) be an almost regular spaces and let (Y,τ_Y,E) be a regularly open subspace of (X,τ,E) .Let (U,E) be a soft regular open set of Y containing y_e. Then by "**Lemma 3.7**", (U,E) be a soft regular open set of X containing y_e.Since (X,τ,E) is soft almost regular by "**Theorem 3.4**", there exists a soft open set (V,E) containing y_e such that $Cl(V,E)\subseteq(U,E)$. Consequently $Cl_Y(V,E)\subseteq(U,E)$. Hence (Y,τ_Y,E) is soft almost regular. □

Remark 3.9. *Every soft almost regular space is soft weakly regular.*

4. Conclusion

In the present paper, we extended the concept of almost regularity to soft sets and presented its studies in soft topological spaces.

References

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, *Computers and Mathematics with Applications*, 57 (2009), 1547-1553.

[2] S. Das and S. K. Samanta, Soft metric, *Annals of Fuzzy Mathematics and Informatics,* 6(1) (2013), 77-94.

[3] S.Hussain and B.Ahmad, Soft separation axioms in soft topological spaces, *Hacettepe Journal of Mathematics and Statistics*, 44(3) (2015),559-568

[4] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft connectedness via soft ideals,*Journal of New Results in Science,* 4 (2014), 90-108.

[5] F. Lin, Soft connected spaces and soft paracompact spaces, *International Journal of Mathematical and Computational Sciences,* 7(2) (2013), 37-43.

[6] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, *Computers and Mathematics with Applications,* 45 (2003), 555-562.

[7] D. Molodtsov, Soft set theory-first results, *Computers and Mathematics with Applications*, 37 (1999) , 19-31.

[8] S. Nazmul and S. K. Samanta, Neighborhood properties of soft topological spaces, *Annals of Fuzzy Mathematics and Informatics ,* 6(1) (2013), 1-15.

[9] D. Pei and D. Miao, From soft sets to information systems, *in Proceedings of the IEEE International Conference on Granular Computing,* 2 (2005),617–621.

[10] G. Senel and N. Cagman , Soft topological subspaces, *Annals of Fuzzy Mathematics and Informatics* ,10 (4) (2015), 525–535.

[11] M. Shabir and M. Naz, On soft topological spaces, *Computers and Mathematics with Applications,* 61 (2011), 1786-1799.

[12] M.K.Singal and S.P.Arya, On almost-regular spaces, *Glasnik Mathematicki,* 4 (24) 1969, 89-99.

[13] I. Zorlutuna, N. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 3(2) (2012) ,171-185.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆