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Coincidence points for a pair of ordered F-contraction mappings in ordered partial metric spaces

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Abstract
The concept of ordered F-contraction in an ordered metric space was introduced by Durmaz et al. [9] and became a very important result in the existing metric fixed point theory. In this paper, we prove a fixed point theorem for a pair of compatible F-contraction maps in an ordered complete partial metric spaces. In particular, the main results generalize a fixed point theorem due to Durmaz et. al. [9] to partial metric spaces. An illustrative example is provided to support the theorem.

Keywords
Ordered partial metric spaces, F-contraction mappings, coincidence points.

AMS Subject Classification
47H10, 54H25.

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1. Introduction

In 2012, Wardowski introduced a new type of contraction known as F-contraction which generalizes the Banach Contraction Principle. In the results Wardowski defined an F-contraction map as follows:

Definition 1.1. [15] *Let (M,d) be a metric space, a mapping $T: M \longrightarrow M$ is said to be an F-contraction on M if there exists $\tau > 0$ such that, for all $x,y \in M$,*

$$d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(d(x,y)). \tag{1.1}$$

and $F: \mathbb{R}_+ \longrightarrow \mathbb{R}$, a mapping satisfying the following conditions:

F1: F is strictly increasing, that is for all $x,y \in \mathbb{R}_+$ such that $x < y \Rightarrow F(x) < F(y)$.

F2: For each sequence $\{\alpha_n\}_{n\geq 1}$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$, if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

F3: There exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

We denote by Δ_F the set of all functions satisfying the conditions $(F1)-(F3)$.

Moreover, Wardowski proved that every F-contraction mapping on a complete metric space has a unique fixed point. Futhermore, several contractions in the literature can be deduced by varying suitable elements of Δ_F.

The following example shows an F-contraction in metric spaces.

Example 1.2. *[15] Let $F: \mathbb{R}_+ \longrightarrow \mathbb{R}$ be defined by $F(\alpha) = \ln(\alpha)$. It is clear that F satisfies $(F1)-(F3)$ for any $k \in (0,1)$. Each mapping $T: M \longrightarrow M$ satisfying $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(d(x,y))$ is an F-contraction such that $d(Tx,Ty) \leq e^{-\tau} d(x,y)$ for all $x,y \in M, Tx \neq Ty$. Obviously, for all $x,y \in M$ such that $Tx = Ty$, the inequality $d(Tx,Ty) \leq e^{-\tau} d(x,y)$ holds and T is a Banach contraction. One can find more examples in [15].*

Recently, several researchers have shown interest in mappings satisfying the F-contraction condition. There exist numerous literatures on and around the notion of F-contractions; see ([3–5, 9, 13]).

In 1992, Matthews [11], introduced the notion of partial metric spaces and proved an analogue of Banach Contraction Principle on partial metric spaces. Matthews [11] provided the following definition:

Definition 1.3. [11] *Let X be a non-empty set. A partial metric space is a pair (X,p), where p is a function $p : X\times X \to \mathbb{R}^+$, called the partial metric, such that for all $x,y,z \in X$ the following axioms hold:*

(P1) $x = y \Leftrightarrow p(x,y) = p(x,x) = p(y,y)$;

(P2) $p(x,x) \leq p(x,y)$;

(P3) $p(x,y) = p(y,x)$; *and*

(P4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

Clearly, by (P1)-(P3), if $p(x,y) = 0$, then $x = y$. But, the converse is in general not true.

The most common example of partial metric spaces is a pair $([0,\infty), p)$ where $p(x,y) = \max\{x,y\}$ for all $x,y \in [0,\infty)$. More examples of partial metric spaces may be found in [7].

Each partial metric p on X generates a T_0 topology τ_p on X whose basis is the collection of all open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$ where
$B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x)+\varepsilon\}$ for all $x \in X$, and ε is a positive real number.

Definition 1.4. [2, 11] *Let (X,p) be a partial metric space. Then:*

(i) a sequence $\{x_n\}$ in (X,p) is said to be convergent to $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) a sequence $\{x_n\}$ in (X,p) is a Cauchy sequence if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.

(iii) a partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to the topology τ_p to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

(iv) a mapping $f : X \to X$ is said to be continuous at $x_0 \in X$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(B_p(x_0,\delta)) \subset B_p(f(x_0),\varepsilon)$.

2. Preliminaries

In this section, we recall some definitions and basic results of ordered partial metric spaces which will be used throughout the paper.

Following lemma was proved by Bukatin et al. [7] and will be useful in this paper.

Lemma 2.1. [7] *Let (X,p) be a partial metric space. Then the mapping $p^s : X\times X \to [0,\infty)$ given by*

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y),$$

for all $x,y \in X$ defines a metric on X.

Bukatin et al. [7] also proved the following lemma:

Lemma 2.2. [7] *Let (X,p) be a partial metric space. Then:*

(i) a sequence $\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).

(ii) a partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete.

Paesano and Vetro [13] provided the following definitions regarding partially ordered set, ordered partial metric space and regularity:

Definition 2.3. [13] *Let $(X,\preceq)$ be a partially ordered set. Let A and B be two non- empty subset of X. Two relations between A and B are denoted and defined as follows;*
$(r1)$ *$A \prec_1 B$ if for each $a \in A$ there exists $b \in B$ such that $a \preceq b$.*
$(r2)$ *$A \prec_2 B$ if for each $a \in A$ and $b \in B$, we have $a \preceq b$.*

Definition 2.4. [13] *If (X,p) be a partial metric space and $(X,\preceq)$ is partially ordered set, then $(X,p,\preceq)$ is called an ordered partial metric. We say that $x,y \in X$ are comparable if $x \preceq y$ or $y \preceq x$ holds. Further a self map $T : X \to X$ is called non-decreasing if $Tx \preceq Ty$ whenever $x \preceq y$ for all $x,y \in X$ and an ordered partial metric space $(X,p,\preceq)$ is regular if the following holds:*
For every non-decreasing sequence $\{x_n\}$ in X converging to some $x \in X$, we have $x_n \preceq x$ for all $n \in \mathbb{N}\cup\{0\}$.

First results on fixed point problems in partially ordered metric spaces were obtained by Ran and Reurings [14] and followed by Nieto and Rodriguez [12]. Abbas et al. [1] used the notion of the F-contraction to establish order-theoretic common fixed point results. Recently, Durmaz et al. [9] introduced the concept of ordered F-contraction in an ordered metric space using the results of Ran and Reurings [14] and proved the following fixed point theorem.

Theorem 2.5. [9] *Let $(X,d,\preceq)$ be an ordered complete metric space and $T : X \to X$ be an ordered F-contraction. Let T be a non-decreasing map and there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$. If T is continuous or X is regular then T has a fixed point.*

Durmaz et al. [9] generalized their results by fixing $f = I : X \to X$ in Theorem 2 given by Abbas et al. [1]. Moreover, they provided a condition that every pair of elements of X

should have a lower and upper bound to posses a unique fixed point. Following the results of Wardowski [15] and Durmaz et al. [9] this paper intends to prove a fixed point theorem for a pair of compatible F-contraction maps in an ordered complete partial metric spaces.

One can define a coincident point as follows:

Definition 2.6. *Let X be a non empty set, T and g are self maps of X. A point $x \in X$ is called a coincident point of T and g if $Tx = gx$.*

Kessy et al. [10] provided the following definition concerning a pair of compatible self maps in partial metric spaces:

Definition 2.7. [10] *Let $(X, \preceq)$ be partially ordered set and $f, g : X \to X$ be two mappings. A pair (f, g) are compatible if $\lim_{n\to\infty} p(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.*

Ćirić et al. [8] provided the following definition:

Definition 2.8. [8] *Let $(X, \preceq)$ be a partially ordered set and $F, g : X \to X$ be mappings of X into itself. F is g-non decreasing if for $x, y \in X$, we have $g(x) \preceq g(y) \Rightarrow F(x) \preceq F(y)$.*

Aryani *et al.* [6] proved the following theorem (see Theorem 2) for nonself mappings in partial metric spaces:

Theorem 2.9. [6] *Let (S_1, p) and(S_2, p) be partial metric spaces with $A \subseteq S_2$, $c \in A$ and function $f : A \to S_2$. The following statements are equivalent:*

(i) f continous at c

(ii) For any sequence $\{x_n\}$ at A convergent to $c \in A$, then the sequence$\{f(x_n)\}$ converges to $f(c)$.

If in Theorem 2.9 we assume that $f : X \to X$ is a self mapping then we get the following lemma:

Lemma 2.10. *If (X, p) is a partial metric space, $c \in X$ and a function $T : X \to X$, then the following statements are equivalent:*

(i) T is continuous

(ii) For any sequence x_n converging to $c \in X$, then Tx_n converges to $f(c)$.

Durmaz et al. [9] provided the following definition regarding an ordered F-contraction map:

Definition 2.11. [9] *Let $(X, \preceq, d)$ be an ordered metric space and $T : X \to X$ be a mapping. Let $Y = \{(x, y) \in X \times X : x \preceq y, d(Tx, Ty) > 0\}$ we say that T is an ordered F-contraction if $F \in \Delta_F$ and there exists $\tau > 0$ such that for all $(x, y) \in Y$, we have*

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)). \tag{2.1}$$

Durmaz et al. [9] proved the following theorem:

Theorem 2.12. [9] *Let $(X, d, \preceq)$ be an ordered complete metric space and $T : X \to X$ be an ordered F-contraction. Let T be a non-decreasing map and there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$. If T is continuous or X is regular then T has a fixed point.*

The purpose of this paper is to extend Theorem 2.12 to an ordered partial metric space in order to obtain a fixed point theorem for an ordered F-contraction map.

3. Main Results

In this section, we deal with the existence and uniqueness of fixed point of a F-contraction map in an ordered partial metric space. First we will provide the extension of Definition 2.11 in an ordered partial metric space which is as follows:

Definition 3.1. *Let $(X, \preceq, p)$ be an ordered partial metric space and $T : X \to X$ be a mapping. Also let $Y = \{(x, y) \in X \times X : x \preceq y, p(Tx, Ty) > 0\}$. We say that T is an ordered F-contraction if $F \in \Delta_F$ and there exists $\tau > 0$ such that for all $(x, y) \in Y$, we have*

$$\tau + F(p(Tx, Ty)) \leq F(p(x, y)). \tag{3.1}$$

Next, we prove a fixed point theorem for a pair of compartible ordered F-contraction mappings.

Theorem 3.2. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a partial metric space on X such that (X, p) is a complete partial metric space. Suppose T and g are continuous self F-contraction mappings on X, $T(X) \subseteq g(X)$, T is a monotone g-non decreasing map and*

$$\tau + F(p(Tx, Ty)) \leq F(\mathbb{M}(x, y)) \tag{3.2}$$

where
$\mathbb{M}(x, y) = \max\left\{p(gx, gy), p(gx, Tx), p(gy, Ty), \frac{1}{2}\left[p(gx, Ty) + p(gy, Tx)\right]\right\}$ *for all $x, y \in X$ for which gx and gy are comparable and $\tau > 0$. If there exists $x_0 \in X$ such that $gx_0 \preceq Tx_0$ and T and g are compatible, then T and g have a coincident point.*

Proof. Let x_0 be such that $gx_0 \preceq Tx_0$ since $T(X) \subseteq g(X)$, we can choose $x_1 \in X$ so that $gx_1 = Tx_0$. Since $Tx_1 \in g(X)$, there exists $x_2 \in X$ such that $gx_2 = Tx_1$. By induction, we can construct a sequence $\{x_n\}$ in X such that $gx_{n+1} = Tx_n$ for every $n \geq 0$. Since T is a monotone g-non decreasing mapping, $gx_0 \preceq Tx_0 = gx_1$ implies $Tx_0 \preceq Tx_1$. Similarly, since $gx_1 \preceq gx_2$ we obtain $Tx_1 \preceq Tx_2$ and $gx_2 \preceq gx_3$. Continuing with this process we obtain

$$Tx_0 \preceq Tx_1 \preceq Tx_2 \preceq \ldots \preceq Tx_n \preceq Tx_{n+1} \preceq \ldots$$

Suppose that $p(Tx_n, Tx_{n+1}) > 0$ for all $n = 0, 1, 2, \ldots$ If not then $Tx_{n+1} = Tx_n$ for some n, $Tx_{n+1} = gx_{n+1}$ that is T and

g have a coincident point x_{n+1} and so this will end the proof. Consider

$$\begin{aligned}\tau+F(p(gx_{n+1},gx_{n+2})) =&\tau+F(p(Tx_n,Tx_{n+1}))\\ \leq& F(\mathbb{M}(x_n,x_{n+1})).\end{aligned}$$

where $\mathbb{M}(x_n,x_{n+1})$

$$\begin{aligned}
&=\max\Big\{p(gx_n,gx_{n+1}),p(gx_n,Tx_n),p(gx_{n+1},Tx_{n+1}),\\
&\qquad\frac{p(gx_n,Tx_{n+1})+p(gx_{n+1},Tx_n)}{2}\Big\}\\
&=\max\Big\{p(Tx_{n-1},Tx_n),p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1}),\\
&\qquad\frac{p(Tx_{n-1},Tx_{n+1})+p(Tx_n,Tx_n)}{2}\Big\}\\
&=\max\Big\{p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1}),\\
&\qquad\frac{p(Tx_{n-1},Tx_{n+1})+p(Tx_n,Tx_n)}{2}\Big\}\\
&\leq\max\Big\{p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1}),\\
&\qquad\frac{p(Tx_{n-1},Tx_n)+p(Tx_n,Tx_{n+1})}{2}\Big\}\\
&=\max\Big\{p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1})\Big\}.
\end{aligned}$$

Suppose

$$\begin{aligned}&\max\Big\{p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1})\Big\}\\ &=p(Tx_n,Tx_{n+1})\end{aligned}$$

then

$$\begin{aligned}\tau+F(p(gx_{n+1},gx_{n+2}))&=\tau+F(p(Tx_n,Tx_{n+1}))\\ &\leq F(p(Tx_n,Tx_{n+1})),\end{aligned}$$

which is a contradiction. Hence $\max\{p(Tx_{n-1},Tx_n),p(Tx_n,Tx_{n+1})\}=p(Tx_{n-1},Tx_n)$. Then, for all $n\in\mathbb{N}$, we can write

$$\begin{aligned}F(p(Tx_n,Tx_{n+1}))&\leq F(p(Tx_{n-1},Tx_n))-\tau\leq\dots\\ &\leq F(p(Tx_0,Tx_1))-n\tau.\end{aligned}\tag{3.3}$$

From (3.3), we obtain $\lim\limits_{n\to\infty}F(p(Tx_n,Tx_{n+1}))=-\infty$. Since $F\in\Delta_F$ then by $(F2)$ we have,

$$\lim_{n\to\infty}p(Tx_n,Tx_{n+1})=0.\tag{3.4}$$

By $(F3)$ there exists $k\in(0,1)$ such that

$$\lim_{n\to\infty}(p(Tx_n,Tx_{n+1}))^kF(p(Tx_n,Tx_{n+1}))=0.\tag{3.5}$$

Following (3.3), for all $n\in\mathbb{N}$ we obtain

$$\begin{aligned}&(p(Tx_n,Tx_{n+1}))^k(F(p(Tx_n,Tx_{n+1}))-F(p(Tx_0,Tx_1)))\\ &\qquad\leq-(p(Tx_n,Tx_{n+1}))^kn\tau\leq0.\end{aligned}\tag{3.6}$$

Taking into account (3.4), (3.5) and letting $n\to\infty$ in (3.6) we get

$$\lim_{n\to\infty}n(p(Tx_n,Tx_{n+1}))^k=0.\tag{3.7}$$

Since (3.7) holds, there exists $n_1\in\mathbb{N}$ such that

$$n(p(Tx_n,Tx_{n+1}))^k\leq1,$$

for all $n\geq n_1$. This implies that

$$\big(p(Tx_n,Tx_{n+1})\big)^k\leq\frac{1}{n^{\frac{1}{k}}},\ \text{for all } n\geq n_1.\tag{3.8}$$

Next, we will show that $\{Tx_n\}$ is a Cauchy sequence. Consider $n,m\in\mathbb{N}$ such that $m>n\geq n_1$, then by (3.8) and axiom (P3) of Definition 1.3 we have

$$\begin{aligned}p(Tx_n,Tx_m)&\leq p(Tx_n,Tx_{n+1})+\dots+p(Tx_{m-1},Tx_m)\\ &\qquad-\sum_{j=n+1}^{m-1}p(Tx_j,Tx_j)\\ &\leq p(Tx_n,Tx_{n+1})+p(Tx_{n+1},Tx_{n+2})+\dots\\ &\qquad+p(Tx_{m-1},Tx_m)\\ &=\sum_{i=n}^{m-1}p(Tx_i,Tx_{i+1})\\ &\leq\sum_{i=n}^{\infty}p(Tx_i,Tx_{i+1})\\ &\leq\sum_{i=n}^{\infty}\frac{1}{i^{\frac{1}{k}}}.\end{aligned}$$

The convergence of the series $\sum_{i=1}^{\infty}\frac{1}{i^{\frac{1}{k}}}$ implies that

$$\lim_{n\to\infty}p(Tx_n,Tx_m)=0.$$

By Lemma 2.1 we get that, for any $n,m\in\mathbb{N}$,

$$p^s(Tx_n,Tx_m)\leq2p(Tx_n,Tx_m)\to0$$

as $n\to\infty$. This implies that, $\{Tx_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence with respect to p^s and hence converges by Lemma 2.2. Thus there exists $u\in X$ such that, $\lim\limits_{n\to\infty}Tx_n=u$. By the continuity of T, we have $\lim\limits_{n\to\infty}T(Tx_n)=Tu$. Since $gx_{n+1}=Tx_n\to u$ and the pair (T,g) is compatible, we have

$$\lim_{n\to\infty}p(g(Tx_n),T(gx_n))=0.\tag{3.9}$$

By axiom (P3) of Definition 1.3 we have

$$\begin{aligned} p(Tu,gu) \leq\ & p(Tu,T(gx_n)) + p(T(gx_n),g(Tx_n)) \\ & + p(g(Tx_n),gu)) - p(T(gx_n),T(gx_n)) \\ & - p(g(Tx_n),g(Tx_n)). \end{aligned} \tag{3.10}$$

Now we apply Lemma 2.10. Letting $n \to \infty$ in (3.10) and using the fact that T and g are continuous, we obtain that $p(Tu,gu) = 0$ that is $Tu = gu$ and u is a coincidence point of T and g.

One can deduce the following corollary from Theorem 3.2:

Corollary 3.3. *Let (X,p) be a complete partial metric space. Let $T,g : X \to X$ be continuous mapping satisfying*

$$\tau + F(p(Tx,Ty)) \leq F(p(gx,gy))$$

for all $x,y \in X$ where $F \in \Delta_F$ and $\tau > 0$, If $Tgx = gTx$ and the mappings T,g satisfy the condition $T(X) \subseteq g(X)$ of Theorem 3.2 then the mappings have a coincidence point.

□

4. Example

Example 4.1. *Let $M = [0,1]$ with the usual order and let (X,p) be a complete partial metric space defined by $p(x,y) = \max\{x,y\}$ for all $x,y \in M$. Let $T,g : M \to M$ be a pair of compatible F-contraction mappings given by $Tx = \frac{x^3}{3x+9}$, $gx = \frac{x^2}{x+3}$.*

Let $F : \mathbb{R}^+ \to \mathbb{R}$ be defined by $F(\alpha) = \ln(\alpha)$ for all $\alpha \in \mathbb{R}^+$, and also let $\tau = \ln(3)$. We show that the condition (3.2) of Theorem 3.2 is satisfied. If $x,y \in X$ is such that $p(Tx,Ty) > 0$, this implies that

$$\tau + F(p(Tx,Ty)) = \tau + \ln\left[\max\left\{\frac{x^3}{3x+9}, \frac{y^3}{3y+9}\right\}\right].$$

Now suppose that $y \geq x$, Without loss of generality, we obtain that,

$$\tau + \ln\left[\max\left\{\frac{x^3}{3x+9}, \frac{y^3}{3y+9}\right\}\right] \leq F(\mathbb{M}(x,y)),$$

where
$\mathbb{M}(x,y) = gy = \max\{gx,gy\} = p(gx,gy)$.
Therefore

$$\begin{aligned} \ln(3) + \ln\left[\max\left\{\frac{x^3}{3x+9}, \frac{y^3}{3y+9}\right\}\right] &= \ln(3) + \ln\left(\frac{y^3}{3y+9}\right) \\ &\leq \ln\left(\frac{y^2}{y+3}\right) \\ &= F(p(x,y)). \end{aligned}$$

Likewise, if $y \leq x$ we obtain that $\tau + F(p(Tx,Ty)) \leq F(p(gx,gy))$

Remark 4.2. *As we observe in Example 4.1, if the assumption that every pair of elements has a lower bound and upper bound is not satisfied then, a fixed point of T may not be unique.*

5. Conclusion

In this paper, an approach has been developed for existence and uniqueness of coincidence points for a pair of ordered F- contraction mappings in an ordered partial metric space. The results due to Durmaz et al. [9] are extended for a pair of compatible ordered F-Contraction mappings.

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References

[1] M. Abbas , B. Ali, and S. Romaguera, *Fixed and periodic points of generalized contractions in metric spaces,* Fixed Point Theory and Applications, 1 (2013): 243.

[2] T. Abedelljawad, E. Karapinar and K. Tas, *Existance and uniqueness of common fixed point on partial metric spaces,* Appl. Math. Lett., 24 (2011): 1894–1899.

[3] O. Acar, G. Durmaz, and G. Minak. *Generalized multivalued F -contractions on complete metric spaces,* Bulletin of the Iranian Mathematical Society, 40(6)(2014): 1469–1478.

[4] J. Ahmad, A. Al-Rawashdeh and A. Azam, *New fixed point theorems for generalized F-contractions in complete metric spaces,* Fixed Point Theory and Applications, 1 (2015): 80.

[5] I. Altun, G. Minak, and H. Dag, *Multivalued F-contractions on complete metric space,* J. Nonlinear Convex Anal., 16(4)(2015): 659–666.

[6] F. Aryani, H. Mahmud, C. C. Marzuki, M Soleh, R Yendra and A. Fudholi, *Continuity Function on Partial Metric Space,* Journal of Mathematics and Statistics, 12(4) (2016), 271–276. DOI: 10.3844/jmssp.2016.271.276.

[7] M. Bukatin, R. Kopperman and S. Matthews, *Partial metric spaces,* American Mathematical Monthly, 116 (2009): 708–718.

[8] L. Ćirić, N. Cakic, M. Rajovic and J. S. Ume, *Monotone generalized nonlinear contractions in partially ordered metric spaces,* Fixed Point Theory and Applications, (2009)(1) 2008: Art. ID 131294, 11 pp.

[9] G. Durmaz, G. Minak and I. Altun, *Fixed points of ordered F-contractions,* Hacettepe Journal of Mathematics and Statistics, 45(1)(2016): 15–21.

[10] J. Kessy, S. Kumar and G. Kakiko, *Fixed points for hybrid pair of compatible mappings in partial metric spaces,* Advances in Fixed Point Theory, 7(4) (2017): 489–499.

[11] S. Matthews, *Partial metric topology in Papers on General Topology and Applications, Eighth Summer Conference at Queens College. Eds. S. Andima et. al.,* Annals of the New York Academy of Sciences, 728 (1992): 183–197.

[12] J. Nieto and R. Rodríguez-López, *Contractive mapping theorems in partially ordered sets and applications to*

ordinary differential equations, A journal on the Theory of ordered sets and its application, 22(3)(2005): 223–239.
[13] D. Paesano and C. Vetro, *Multi-valued F-contractions in 0-complete partial metric spaces with application to Volterra type integral equation,* Revista de la Real Academia de Ciencias Exactas Fisicas Naturales, 108(2)(2014): 1005–1020.
[14] A. Ran and M. Reurings, *A fixed point theorem in partially ordered sets and some applications to matrix equations,* Proc. Amer. Math. Soc., 132 (2004): 1435–1443.
[15] D. Wardowski, *Fixed points of a new type of contractive mappings in complete metric spaces,* Fixed Point Theory and Applications, 94(2012).

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