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Properties and applications of soft digraphs

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Abstract

In this paper, some properties of a soft digraph are studied. In this connection some terminologies regarding soft digraphs are defined. Also, the relationship between digraphs and matrices are studied. Later on soft tournaments are introduced and some properties of soft tournaments are discussed. Finally applications of soft digraphs and tournaments in solving decision making problem, traffic flow problem and determination of the strongest team in a cricket championship are shown.

Keywords

Soft Set, Digraph, Tournaments, Soft digraph, Soft tournaments.

AMS Subject Classification

15A48.

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1. Introduction

Molodtsov [15] introduced the theory of soft sets as a new mathematical tool to deal with the uncertainties which traditional mathematical tools can not solve. Now soft set theory is rapidly used in many fields like engineering, medical science, sociology, economics etc to deal with the complexities of modeling with uncertain data. Many practical problem can be solved easily with the help of soft set theory rather than some well known theories viz. fuzzy set theory [20], probability theory, intuitionistic fuzzy sets theory [2, 3], vague sets theory [5], theory of rough sets [16] etc since these theories have certain limitations. The problem with the fuzzy set is that it lacks parameterization of tools. Thus many authors like Maji, Roy and Biswas [8, 9] have further studied the theory of soft sets and used this theory to solve some decision making problems. They have also introduced the concept of fuzzy soft set and intuitionistic fuzzy soft set [10–12], in a more generalized way. In 2009, Ali et al [1] has defined some new operations on soft sets.

Research in soft set theory (SST) has been done in many areas like algebra, topology, applications etc (see [1, 7, 13, 14, 19], for example). On the other hand several authors have recently studied Fuzzy graph theory, soft graph theory and studied the properties of fuzzy graphs, soft graphs and their properties [18] with the help of traditional graph theory [4, 6]. We have introduced soft digraph theory in the light of soft set theory and digraph theory [17].

In this paper, we will study the following: In Section 2, some preliminary definitions and example regarding soft set theory, soft digraph theory are given which will be used in the rest of the paper. In section 3, some properties of soft digraphs are investigated. The relation between soft digraph and matrices are discussed in section 4. Section 5 is devoted for soft tournament and its properties. Finally in the last section i.e. section 6, applications using soft tournament and

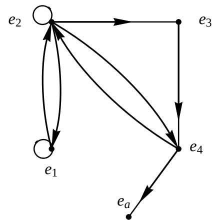

Figure 1. The Digraph D

soft digraphs are discussed.

2. Preliminaries

2.1 Soft Set Theory

The idea of soft sets was first given by Molodtsov. Later Maji and Roy [12] have defined operations on soft set and studied their properties.

Definition 2.1. *[15] Let U be an initial universal set and let $E = \{e_i; i = 1, \ldots, n\}$ be a set of parameters. Suppose $P(U)$ denote the power set of U and A be a subset of E. A pair (F,A) is called a soft set over U if and only if F is a mapping given by $F : A \to P(U)$.*

Example 2.2. *As an illustration, suppose a soft set (F,E) describes attractiveness of the shirts which the authors are going to wear.*

U = the set of all shirts under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$. E = colorful, bright, cheap, warm = $\{e_1, e_2, e_3, e_4\}$. Let $F(e_1) = \{x_1, x_2\}$, $F(e_2) = \{x_1, x_2, x_4, x_3\}$, $F(e_3) = \{x_4\}$, $F(e_4) = \{x_2, x_5\}$.

So, the soft set (F,E) is a family $\{F(e_i); i = 1, \ldots, 4\}$ of U.

2.2 Soft Digraph

Now we have defined the soft digraph corresponding to the soft set (F,E) in [17] as follows:

Definition 2.3. *[17] Suppose e_a be any arbitrary parameter such that $F(e_a) = \phi$. Consider $D = (V_D, A_D)$ be any digraph with vertex set V_D and arc set A_D such that, $V_D = E \cup \{e_a\}$ $A_D = \{(e_i, e_j) : h_j \in F(e_i) \text{ and } j \le |E|\} \cup \{(e_i, e_a) : h_j \in F(e_i) \text{ and } j > |E|\}$. Then D is called a soft digraph of the soft set (F,E). The vertex e_a is called the universal vertex for any soft digraph D.*

Example 2.4. *Consider the digraph D in the Figure 1 corresponding to the soft set in Example 2.2. It is clear that D is a soft digraph by Definition 2.3.*

3. Soft digraph and its properties

An arc $x = (e_i, e_i)$ in soft digraph D is called a *loop* in D, and we say that soft digraph D *includes* a loop at the vertex e_i. If $e_i \ne e_j$ and $x = (e_i, e_j)$ is an arc in soft digraph D, we say that x is *incident* with e_i and e_j; e_i is *adjacent to* e_j; and e_j is *adjacent from* e_i. The *outdegree* $od(e_j)$ (resp. *indegree* $id(e_j)$) of a vertex e_j in a soft digraph D is the number of vertices of D adjacent from (resp. to) e_j. Two arcs in a soft digraph are said to be parallel if they have same start and end vertices. A soft digraph without having any parallel arcs is said to be a simple soft digraph. It is customary to represent a digraph by a diagram with nodes representing the vertices and directed line segments (arcs) representing the arcs of the digraph.

A *soft directed walk* in a soft digraph is an alternating sequence of vertices and arcs $e_1(e_1, e_2)e_2, \ldots, e_n$. The length of such a soft directed walk is n, the number of arcs in it. A *closed soft walk* has same first and last vertices and a *spanning walk* contains all the vertices. A *soft path* is a walk in which all vertices are distinct. A *soft cycle* is a nontrivial closed walk with all vertices distinct. A soft digraph having no cycle is said to be *acyclic*. A soft digraph is *strongly connected* if for any two vertices are mutually reachable. The soft digraph is *unilaterally connected* if for any two vertices at least one is reachable from the other. Otherwise the soft digraph is called *weakly connected*.

Theorem 3.1. *Suppose D be a soft digraph with n-vertices and q-arcs corresponding to the soft set (F,E). If $\{e_1, e_2, \ldots, e_{n-1}, e_a\}$ is the set of vertices in D, then*

$$\sum id(e_i) = \sum od(e_i) = q$$

Proof. It is given that D be a soft digraph with n-vertices and q-arcs. When the out-degrees of the vertices are assumed, we are considering $\sum |F(e_i)|$. Now for each $x_j \in F(e_i)$ generates an arc (e_i, e_j) if $j \le |E|$ or (e_i, e_a) if $j > |E|$. Thus $\sum |F(e_i)|$ also gives us the total number of arcs (e_i, e_j) or (e_i, e_a). Hence, we have

$$\sum od(e_i) = \sum |F(e_i)| = q.$$

Similarly, the result follows also for in-degrees. ☐

Example 3.2. Consider the soft digraph D in Figure 1. Here, we have $i(e_1) = 2, i(e_2) = 3, i(e_3) = 1, i(e_4) = 2, i(e_a) = 1$ and $o(e_1) = 2, o(e_2) = 4, o(e_3) = 1, o(e_4) = 2, o(e_a) = 0$. Now

$$\sum id(e_i) = \sum od(e_i) = 9 = q,$$

where q is the number of arcs.

Theorem 3.3. *No soft digraph is strongly connected.*

Proof. For any soft digraph D corresponding to a soft set (F,A), we have an universal vertex e_a such that $F(e_a) = \phi$. This implies that outdegree of a universal vertex is zero. Hence the result follows. ☐

Corollary 3.4. *A soft digraph has at least one point of out-degree zero.*

Since a soft digraph cannot be strongly connected, thus a soft digraph may be unilaterally connected. In this case, we have the following:

Theorem 3.5. *Suppose D be a simple soft digraph without any loop corresponding to a soft set (F,A) which is unilaterally connected. Then the minimum and maximum possible number q of arcs among all p vertices are $p-1$ and $(p-1)^2$-respectively.*

Proof. we will proof this result by induction on p, the number of vertices. Now by definition, for any soft digraph D corresponding to a soft set (F,A), we have an universal vertex e_a of outdegree zero. Suppose $p=2$ i.e. D is a soft digraph containing two vertices say e_1, e_a. Since D is unilaterally connected , hence it contains an arc at most (e_1, e_a), which is the minimum as well as maximum no of arcs.

Let us assume that the result holds for $p=k$. Then we have the minimum and maximum number of arcs in D are $m_k = (k-1)$ and $M_k = (k-1)^2$ respectively. Now suppose $p = k+1$. In this case, we are adding a vertex say e_{k+1} with a soft digraph D of k-vertices. As a result, we can add at least 1 arc, say (e_{k+1}, e_a) to D minimally and at most $(2k-1)$ arcs, say $\forall i = 1,\dots,k$, $\{(e_i, e_{k+1}), (e_{k+1}, e_i)\} \setminus \{(e_a, e_{k+1})\}$ to D maximally. Thus we have,

$$\begin{aligned} m_{k+1} &= m_k + 1 = (k-1)+1 = k \\ M_{k+1} &= M_k + (2k-1) = (k-1)^2 + (2k-1) = k^2. \end{aligned}$$

Hence, the result follows. □

4. Soft Digraph and Matrices

The adjacency matrix $B(D) = [b_{ij}]$ of a soft digraph D is a $n \times n$ matrix with $b_{ij} = 1$ if $(e_i, e_j) \in D$, and 0 otherwise. Clearly the row sums of $B(D)$ are the outdegrees and the column sums are the indegrees of the vertices of D. Since for any soft digraph D corresponding to a soft set (F,A), we have $F(e_a) = \phi$, thus the outdegree of an universal vertex e_a is zero. Hence for any adjacency matrix $B(D)$, the row corresponding to the vertex e_a contains only the zero entries.

Example 4.1. The adjacency matrix B corresponding to the soft digraph D in Example 2.2 is as following:

$$B = \left[\begin{array}{cccccc} & e_1 & e_2 & e_3 & e_4 & e_a \\ e_1 & 1 & 1 & 0 & 0 & 0 \\ e_2 & 1 & 1 & 1 & 1 & 0 \\ e_3 & 0 & 0 & 0 & 1 & 0 \\ e_4 & 0 & 1 & 0 & 0 & 1 \\ e_a & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Theorem 4.2. *The (i,j)-th entry of b^n_{ij} of B^n is the number of walks of length n from the vertices e_i to e_j in a soft digraph D corresponding to the soft set (F,A).*

Proof. We will use mathematical induction technique on the length of walks to proof this result.

Suppose $n=1$. Then the (i,j)-th entry of B is the number of different $e_i - e_j$ walks in D of length 1 which can be easily seen since a length 1 walk from e_i to e_j is an arc from e_i to e_j.

Let us consider that the results holds for $n = k-1$. Choosing $B^{k-1} = [m_{ij}]$, we are assuming that m_{ij} is the number of different walks of length $k-1$ from e_i to e_j. Now we have,

$$\begin{aligned} B^k &= B^{k-1} \times B \\ &= \sum_{t=1}^{n} \left((i,t)\text{-th element of } B^{k-1}\right) \times \left((t,j)\text{-th element of B}\right) \\ &= \sum_{t=1}^{n} m_{it} b_{tj} \end{aligned}$$

Now every $e_i - e_j$ walk of length k consists of a $e_i - e_t$ walk of length $k-1$ followed by an arc $e_t e_j$. Since there are m_{it} such walks of length $k-1$ and b_{tj} such arcs for each vertex e_t, thus the total no of walks are $\sum_{t=1}^{n} m_{it} b_{tj}$. Hence the result holds. □

Example 4.3. Consider the adjacency matrix

$$B \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

specifying the soft digraph D in Figure 1. Now we obtain B^2 as follows:

$$B^2 = [m_{ij}] = \begin{bmatrix} 2 & 2 & 1 & 1 & 0 \\ 2 & 3 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since m_{ij} gives the number of walks of length 2 from vertex i to j, then m_{11} gives the number of walks of length 2 from vertex e_1 to e_1 is 2. The walks from e_1 to e_1 are (e_1, e_1, e_1) and (e_1, e_2, e_1). Hence the result follows.

5. Soft Tournament

Definition 5.1. *Suppose D be a absolute digraph corresponding to a absolute soft set (F,A). A soft tournament is a complete orientation of a absolute soft digraph D.*

Example 5.2. We consider the soft set (F,A) over the universal set $U = \{x_1, x_2, x_3\}$ where $A = \{e_1, e_2\}$, $F(e_1) = U = F(e_2)$. Now we draw the soft absolute digraph D_1 of the absolute soft set (F,A). We take an orientation subdigraph T_1 of the soft absolute digraph D_1. Clearly T_1 is a soft tournament.

Also T_1 is called a transitive triple. In any soft digraph, we have an universal vertex e_a such that $od(e_a) = 0$. Thus a soft cyclic triple tournament does not exist.

Theorem 5.3. *Every soft tournament has a spanning path.*

Proof. It can be easily seen that every tournament with 2,3 vertices has a spanning path. Assume that the result holds

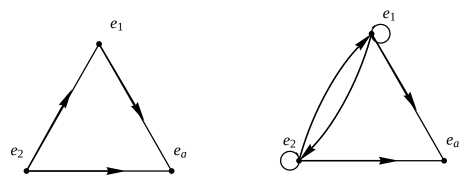

Figure 2. Absolute Digraph D_1 and Soft Tournament T_1

for all soft tournaments with k vertices. Suppose T be a tournament with $k+1$-vertices. Suppose e_0 be any point of T. Then $T-e_0$ is a tournament of k-vertices. So, it has a spanning path P, say $e_1e_2\dots e_{k-1}e_a$. If the arc e_0e_1 is in T, then we are done. If e_1e_0 is in T, suppose e_i be the first point of P for which the arc e_0e_i is in T, if any. Then $e_{i-1}e_0$ is in T, so that $e_1e_2\dots e_{i-1}e_0e_i\dots e_{k-1}e_a$ is a spanning path. If no such e_i exists, then $e_1e_2\dots e_{k-1}e_0e_a$ is a spanning path. □

Definition 5.4. *The score of a vertex e_i in a soft tournament T is defined to its outdegree. If soft tournament T has vertex set $\{e_a,e_1,\dots,e_n\}$ where $od(e_a)\le od(e_1)\le\dots od(e_n)$ then the sequence $\big(od(e_a),od(e_1),\dots,od(e_n)\big)$ is called a score sequence of soft tournament T.*

Example 5.5. Consider the tournament T_1 in Figure 2. The out degree of the vertices of T_1 are as follows:

$$o(e_a)=0, o(e_1)=1, o(e_2)=2$$

Then $(0,1,2)$ is a score sequence of T_1.

Theorem 5.6. *The distance from a vertex with maximum score in any soft tournament to any other vertex is at most 2.*

Proof. Suppose $od(e_i)=m$ and suppose the vertices joined by an arc from e_i be $e_1,e_2,\dots,e_{m-1},e_a$. If the soft tournament T has k vertices then each of the remaining $k-m-1$ vertices $\acute{e}_1,\acute{e}_2,\dots,\acute{e}_{k-m-1}$ are adjacent to e_i, since T is a soft tournament, i.e. for these remaining vertices $\acute{e}_j, 1\le j\le k-m-1$, there are arcs from $\acute{e}_j$ to e_i.

Then for each l, $1\le l\le m-1$, the arc from e_i to e_l gives a directed path of length 1 from e_i to e_l.

Now for the rest of the part, given a vertex $\acute{e}_j$, if there is an arc from e_l to $\acute{e}_j$ for some l then $e_ie_l\acute{e}_j$ gives a directed path of the desired type. Now suppose there is a $\acute{e}_p$, $1\le p\le k-m-1$, such that there is no vertex e_l, $1\le l\le m-1$ has an arc from e_l to $\acute{e}_p$. Since we also have an arc from $\acute{e}_p$ to e_i this gives $od(\acute{e}_p)\ge m+1$. This contradicts our assumption that e_i has maximum degree m. Thus each $\acute{e}_j$ must have an arc joining it from some e_l and this completes the proof. □

Theorem 5.7. *If $\{s_1,s_2,\dots,s_n\}$ is a score sequence of a soft tournament T, then*

$$\sum_{i=1}^{n} s_i = \frac{n(n-1)}{2}$$

Proof. Any complete digraph K_n^* with n-vertices contains n^2-arcs. Since any soft tournament with n-vertices is a completely oriented digraph with no loops, thus any soft tournament with score sequence $\{s_1,s_2,\dots,s_n\}$ has the following properties:

$$\sum_{i=1}^{n} s_i = \frac{n^2-n}{2} = \frac{n(n-1)}{2}.$$

□

Example 5.8. The sum of score sequence of T_1 in Figure 2 is $\frac{3(3-1)}{2}=3$.

Corollary 5.9. *For any soft tournament T with outdegree sequence $\{s_1,s_2,\dots,s_n\}$ and with n-vertices,*

$$\sum_{i=1}^{n} {s_i}^2 = \sum_{i=1}^{n}(n-1-s_i)^2$$

Proof. Any soft tournament T with outdegree sequence $(s_1,s_2,\dots,s_n)$ have indegree sequence $(n-1-s_1,n-1-s_2,\dots,n-1-s_n)$ since any vertex e_i has in T associated with $(n-1)$ arcs. Hence the result follows. □

6. Applications of soft digraph and soft tournament

6.1 Decision Making Problem

Molodstov showed various applications of soft set theory in real life situations in his paper[15]. In [17] applications of soft digraph in decission making, medical diagnosis, soft entropy calculation have been shown. In this subsection we will use soft tournament in solving decision making problem.

Algorithm– Now we give an algorithm for selection of flats for a prospective buyer using soft tournament. For this, the following steps are to be followed:

(1) Input the soft set (F,E).

(2) Draw the soft digraph T_2 corresponding to the soft set (F,E).

(3) Find out the score sequence of the soft tournament T_2.

(4) Find out the indegree sequence of T_2 using score sequence.

(5) Choose k, for which $x_k = max\ id(e_i), i\ne a$.

Then x_k is the optimal selection. If there exists more than one optimal solution, then any solution can be taken.

Example 6.1. Consider the following problem:

The author wish to buy a flat. Suppose a soft set (F,E) describes choice of parameters.

U = the set of all flats under consideration = $\{x_1,x_2,x_3\}$. E = {roadside, riverside, parkside} be a set of parameters = $\{e_1,e_2,e_3\}$. Suppose $F(e_1)=\{x_2,x_3,x_4\}$, $F(e_2)=\{x_4\}$,

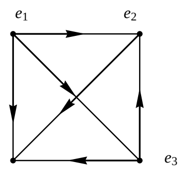

Figure 3. Soft Tournament T_2

$F(e_3) = \{x_2, x_4\}$. The author is interested to buy a flat on the basis of his choice of parameters. Please note that the choice of parameters is dependent on the author i.e. the choice of parameters vary from person to person. So the solution will vary according to the choice of the person. Now we consider the score sequence i.e. outdegree sequence of the vertices e_i's of T_2. The score sequence of the vertices of T_2 i.e. $od(e_1, e_2, e_3, e_a)$ is $(3,1,2,0)$. Then the indegree sequence of the vertices of T_2 is $(0,2,1,3)$, follows from Corollary 5.9. Now we choose the vertex of maximum indegree among e_i's, $i \neq a$ of T_2. e_2 is such a vertex whose indegree is 2.
Decision: The author can buy the flat x_2.

6.2 Traffic Flow Problem

Example 6.2. In this subsection we will consider traffic flow problem for a traveler. In this problem he/she wants to visit all the cities which are well connected once.
Algorithm– Now we give an algorithm for traffic flow problem for a traveler using soft digraph. For this, the following steps are to be followed:

(1) Input the soft set (F, E).

(2) Draw the soft digraph D corresponding to the soft set (F, E).

(3) Find out the matrix representation of the soft digraph D.

(4) To visit n cities once at a time, compute M^n.

(5) If the starting and ending point is the city x_i, calculate $(x_i, x_i) - th$ entry of M^n.

(6) If (x_i, x_i)-th entry is k, find out the k number of paths from x_i to x_i in D.

(7) Finally choose desired path in which all cities can be visited once providing the starting and ending point is the city x_i.

Suppose the author wants to visit 5 cities x_1, x_2, x_3, x_4, x_5 starting from x_1. Suppose a soft set (G, E) describes choice of parameters.

U = the set of cities under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$. E = {train, road, metro-railway, flight, ship} be the set of transport parameters by which the cities x_i, x_j are connected $= \{e_1, e_2, e_3, e_4, e_5\}$. Now $x_j \in F(e_i)$ denotes that author can visit the city x_i to the city x_j by the transport e_i but not conversely.

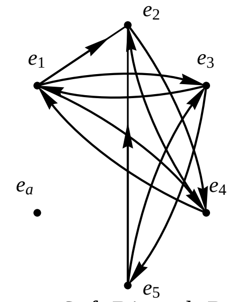

Figure 4. Soft Digraph D_2

Suppose $F(e_1) = \{x_2, x_3, x_4\}, F(e_2) = \{x_4\}, F(e_3) = \{x_1, x_5\}, F(e_4) = \{x_2, x_1\}, F(e_5) = \{x_3\}$. The author is interested to visit all the cities at most once from x_1 and wants to return back into x_1. He wish to find the number of 5-step connections from x_1 to x_1. Now we draw the soft digraph D_2 corresponding to the soft set (G, E) and calculate the matrix representation M of the digraph D_2 as follows:

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now we compute the matrix

$$M^5 = \begin{bmatrix} 6 & 14 & 9 & 10 & 1 & 0 \\ 1 & 5 & 4 & 5 & 0 & 0 \\ 9 & 6 & 2 & 9 & 5 & 0 \\ 9 & 6 & 1 & 6 & 4 & 0 \\ 5 & 9 & 5 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now by Theorem 4.2, we can see that number of 5-step connections from x_1 to x_1 is 6. The connections are (x_1, x_4, x_1), $(x_1, x_3, x_1), (x_1, x_2, x_4, x_1), (x_1, x_3, x_5, x_2, x_4, x_1), (x_1, x_3, x_5, x_3, x_1), (x_1, x_3, x_1, x_4, x_1)$.
Decision: The only way that the author can visit all cities at most 1 from x_1 and return to x_1 is

$$(x_1, x_3, x_5, x_2, x_4, x_1).$$

6.3 Determination of strongest team in cricket championship by using Soft Tournament.

In this subsection we will determine the strongest team in a upcoming tournament by soft tournament. Now we give an algorithm for this purpose using soft digraph. For this, the following steps are to be followed: **Algorithm**–

(1) Input the soft set (F, E).

(2) Draw the soft tournament T corresponding to the soft set (F, E).

(3) Find out the matrix representation of the soft digraph D.

(4) Compute M^2 and calculate $B = M + M^2$.

(5) Find out the row sum of the matrix B and write down the row sum of the matrix B in ascending order.

(6) Finally the strongest team has highest row sum.

Suppose the author wants to determine the strongest team in a cricket championship. He knew the previous results of some tournaments. Suppose a soft set (H,E) describes choice of parameters.

U = the set of registered teams in the championship = $\{e_1,e_2,e_3,e_4,e_5,e_6\}$. E =the teams which are originally playing in the tournament= $\{e_1,e_2,e_3,e_4,e_5\}$. Suppose $F(e_1) = \{e_2,e_3,e_5,e_6\}$, $F(e_2) = \{e_4,e_5,e_6\}$, $F(e_3) = \{e_2,e_4,e_5,e_6\}$, $F(e_4) = \{e_1,e_6\}$, $F(e_5) = \{e_4,e_6\}$ where $e_j \in F(e_i)$ denotes that according to the schedule (e_i,e_j) will play between them and e_i can defeat e_j. Here we have assumed that e_i can defeat e_j if e_i has defeated e_j more number of times than e_j defeated e_i in previously played matches. Now we draw the soft tournament T_3 in Figure 5 where we have,

$$V_{T_3} = E \cup \{e_a\}$$
$$A_{T_3} = \{(e_i,e_j) : \text{if } e_j \in F(e_i); i,j = 1,2,\dots 5\} \bigcup \{(e_i,e_a) \text{ if } e_6 \in F(e_i)\}.$$

Now by Theorem 5.6, in any soft tournament there is at least one vertex from which there is a 1-step or a 2-step connection to any other vertex in the soft tournament. We calculate the matrix representation M of T_3 as follows:

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Finally we calculate

$$M + M^2 = \begin{bmatrix} 0 & 2 & 1 & 3 & 3 & 4 \\ 1 & 0 & 0 & 2 & 1 & 3 \\ 1 & 1 & 0 & 3 & 2 & 4 \\ 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now R_i denotes the sum of the elements of row i. Here, $R_1 = 13$, $R_2 = 7$, $R_3 = 11$, $R_4 = 6$, $R_5 = 4$. Since the first row has the largest sum, the vertex e_1 must have a 1-step or 2-step connection to any other vertex. The ranking of the strong teams according to the powers of the corresponding vertices is: Team e_1 (first), Team e_3 (second), Teams e_2 (third) and e_4 (fourth) and Team e_5 (last).

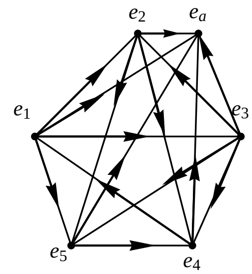

Figure 5. Soft tournament T_3

7. Conclusion

Molodtstov introduced the soft set theory in his paper [15] to deal with the uncertainties in real life problems. Currently research in SST is going on at a high phase. Many authors have studied SST in various way and have applied this theory in solving many practical problems ([8], [12] ,[14]). We have introduced the soft digraph theory in our previous paper [17]. In that paper we have developed the soft digraph theory in combination of soft set theory and digraph theory. In this paper we have studied the properties of soft digraphs and soft tournaments and showed many possible application of this theory. The novelty of soft digraphs is that they can graphically represent any problem that is originally represented by a soft set. One can further study soft tournament and use the soft tournament in many other real life problems.

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