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Initial coefficient estimates for a new subclasses of analytic and m-fold symmetric bi-univalent functions

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Abstract
In the present investigation, we define two new subclasses of the function class Σ_m of analytic and m-fold symmetric bi-univalent functions defined in the open unit disk U. Furthermore, for functions in each of the subclasses introduced here, we determine the estimates on the initial coefficients $|a_{m+1}|$ and $|a_{2m+1}|$. Also, we indicate certain special cases for our results.

Keywords
Analytic functions, univalent functions, bi-univalent functions, m-fold symmetric bi-univalent functions, coefficient estimates.

AMS Subject Classification
30C45, 30C50, 30C80.

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Contents

1. Introduction

Let $\mathscr{A}$ stand for the class of functions f that are analytic in the open unit disk $U=\{z\in\mathbb{C}:|z|<1\}$ and normalized by the conditions $f(0)=f'(0)-1=0$ and having the form:

$$f(z)=z+\sum_{k=2}^{\infty}a_kz^k. \tag{1.1}$$

Let S be the subclass of $\mathscr{A}$ consisting of the form (1.1) which are also univalent in U. The Koebe one-quarter theorem (see [4]) states that the image of U under every function $f\in S$ contains a disk of radius $\frac{1}{4}$. Therefore, every function $f\in S$ has an inverse f^{-1} which satisfies $f^{-1}(f(z))=z$, $(z\in U)$ and $f(f^{-1}(w))=w$, $(|w|<r_0(f),\, r_0(f)\geq\frac{1}{4})$, where

$$g(w)=f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3 - \left(5a_2^3-5a_2a_3+a_4\right)w^4+\cdots. \tag{1.2}$$

A function $f\in\mathscr{A}$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U. We denote by Σ the class of bi-univalent functions in U given by (1.1). For a brief history and interesting examples in the class Σ see [14], (see also [6, 7, 10–12]).

For each function $f\in S$, the function $h(z)=\left(f(z^m)\right)^{\frac{1}{m}}$, $(z\in U, m\in\mathbb{N})$ is univalent and maps the unit disk U into a region with m-fold symmetry. A function is said to be m-fold symmetric (see [8]) if it has the following normalized form:

$$f(z)=z+\sum_{k=1}^{\infty}a_{mk+1}z^{mk+1},\ (z\in U, m\in\mathbb{N}). \tag{1.3}$$

We denote by S_m the class of m-fold symmetric univalent functions in U, which are normalized by the series expansion (1.3). In fact, the functions in the class S are one-fold symmetric.

In [15] Srivastava et al. defined m-fold symmetric bi-univalent functions analogues to the concept of m-fold symmetric univalent functions. They gave some important results, such as each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each $m \in \mathbb{N}$. Furthermore, for the normalized form of f given by (1.3), they obtained the series expansion for f^{-1} as follows:

$$\begin{aligned} g(w) = w - a_{m+1}w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 \right. \\ \left. - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots, \end{aligned} \tag{1.4}$$

where $f^{-1} = g$. We denote by Σ_m the class of m-fold symmetric bi-univalent functions in U. It is easily seen that for $m = 1$, the formula (1.4) coincides with the formula (1.2) of the class Σ. Some examples of m-fold symmetric bi-univalent functions are given as follows:

$$\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}, \left[\frac{1}{2}\log\left(\frac{1+z^m}{1-z^m}\right)\right]^{\frac{1}{m}} \text{ and } \left[-\log\left(1-z^m\right)\right]^{\frac{1}{m}}$$

with the corresponding inverse functions

$$\left(\frac{w^m}{1+w^m}\right)^{\frac{1}{m}}, \left(\frac{e^{2w^m}-1}{e^{2w^m}+1}\right)^{\frac{1}{m}} \text{ and } \left(\frac{e^{w^m}-1}{e^{w^m}}\right)^{\frac{1}{m}},$$

respectively.

Recently, many authors investigated bounds for various subclasses of m-fold bi-univalent functions (see [1, 2, 5, 13, 15–17]).

The aim of the present paper is to introduce the new subclasses $E_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$ and $E^*_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$ of Σ_m and find estimates on the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in each of these new subclasses.

In order to prove our main results, we require the following lemma.

Lemma 1.1. *[3] If $h \in \mathscr{P}$, then $|c_k| \le 2$ for each $k \in \mathbb{N}$, where $\mathscr{P}$ is the family of all functions h analytic in U for which*

$$Re\,(h(z)) > 0, \quad (z \in U),$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots, \quad (z \in U).$$

2. Coefficient estimates for the function class $\mathscr{E}_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$

Definition 2.1. *A function $f \in \Sigma_m$ given by (1.3) is said to be in the class $E_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$ if it satisfies the following conditions:*

$$\left|arg\left(\frac{zf'(z)}{f(z)}\right)^{\delta}\left[(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma}\right| < \frac{\alpha\pi}{2} \tag{2.1}$$

and

$$\left|arg\left(\frac{wg'(w)}{g(w)}\right)^{\delta}\left[(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma}\right| < \frac{\alpha\pi}{2}, \tag{2.2}$$

$(z, w \in U, 0 < \alpha \le 1, 0 \le \delta \le 1, 0 \le \gamma \le 1, 0 \le \lambda \le 1, m \in \mathbb{N})$, *where the function $g = f^{-1}$ is given by (1.4).*

In particular, for one-fold symmetric bi-univalent functions, we denote the class $E_{\Sigma_1}(\delta,\gamma,\lambda;\alpha) = E_{\Sigma}(\delta,\gamma,\lambda;\alpha)$.

Remark 2.2. *It should be remarked that the classes $E_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$ and $E_{\Sigma}(\delta,\gamma,\lambda;\alpha)$ are a generalization of well-known classes consider earlier. These classes are:*
(1) For $\delta = \lambda = 0$ and $\gamma = 1$, the class $E_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$ reduce to the class $S^{\alpha}_{\Sigma_m}$ which was considered by Altınkaya and Yalçın [1].
(2) For $\delta = 0$ and $\gamma = 1$, the class $E_{\Sigma}(\delta,\gamma,\lambda;\alpha)$ reduce to the class $M_{\Sigma}(\alpha,\lambda)$ which was introduced by Liu and Wang [9].
(3) For $\delta = \lambda = 0$ and $\gamma = 1$, the class $E_{\Sigma}(\delta,\gamma,\lambda;\alpha)$ reduce to the class $S^*_{\Sigma}(\alpha)$ which was given by Brannan and Taha [3].

Theorem 2.3. *Let $f \in E_{\Sigma_m}(\delta,\gamma,\lambda;\alpha)$,*

$(z, w \in U, 0 < \alpha \le 1, 0 \le \delta \le 1, 0 \le \gamma \le 1, 0 \le \lambda \le 1, m \in \mathbb{N})$,

be given by (1.3). Then

$$|a_{m+1}| \le \frac{2\alpha}{m\sqrt{(\alpha+\delta)(\delta+2\gamma(1+\lambda m)) + \gamma(\gamma-\alpha)(1+\lambda m)^2}} \tag{2.3}$$

and

$$|a_{2m+1}| \le \frac{2\alpha^2(m+1)}{m^2(\delta+\gamma(1+\lambda m))^2} + \frac{\alpha}{m(\delta+\gamma(1+2\lambda m))}. \tag{2.4}$$

Proof. It follows from conditions (2.1) and (2.2) that

$$\left(\frac{zf'(z)}{f(z)}\right)^{\delta}\left[(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma} = [p(z)]^{\alpha} \tag{2.5}$$

and

$$\left(\frac{wg'(w)}{g(w)}\right)^{\delta}\left[(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma} = [q(w)]^{\alpha} \tag{2.6}$$

where $g = f^{-1}$ and p, q in $\mathscr{P}$ have the following series representations:

$$p(z) = 1 + p_m z^m + p_{2m} z^{2m} + p_{3m} z^{3m} + \cdots \tag{2.7}$$

and

$$q(w) = 1 + q_m w^m + q_{2m} w^{2m} + q_{3m} w^{3m} + \cdots . \tag{2.8}$$

Comparing the corresponding coefficients of (2.5) and (2.6) yields

$$m\left(\delta + \gamma(1+\lambda m)\right) a_{m+1} = \alpha p_m, \tag{2.9}$$

$$\begin{aligned} & m\left[2\left(\delta + \gamma(1+2\lambda m)\right) a_{2m+1} \right. \\ & -\left(\delta + \gamma(1+2\lambda m + \lambda m^2)\right)\Big] a_{m+1}^2 + \tfrac{m^2}{2}\left[\delta(\delta-1)\right. \\ & \left. +\gamma(1+\lambda m)\left(2\delta + (\gamma-1)(1+\lambda m)\right)\right] a_{m+1}^2 \\ & = \alpha p_{2m} + \tfrac{\alpha(\alpha-1)}{2} p_m^2 \end{aligned} \tag{2.10}$$

$$-m\left(\delta + \gamma(1+\lambda m)\right) a_{m+1} = \alpha q_m \tag{2.11}$$

and

$$\begin{aligned} & m\Big[\left(\delta(2m+1) + \gamma\left(3\lambda m^2 + 2(\lambda+1)m + 1\right)\right) a_{m+1}^2 \\ & \quad - 2\left(\delta + \gamma(1+2\lambda m)\right) a_{2m+1}\Big] + \frac{m^2}{2}\Big[\delta(\delta-1) \\ & \quad + \gamma(1+\lambda m)\left(2\delta + (\gamma-1)(1+\lambda m)\right)\Big] a_{m+1}^2 \\ & = \alpha q_{2m} + \frac{\alpha(\alpha-1)}{2} q_m^2 . \end{aligned} \tag{2.12}$$

Making use of (2.9) and (2.11), we obtain

$$p_m = -q_m \tag{2.13}$$

and

$$2m^2\left(\delta + \gamma(1+\lambda m)\right)^2 a_{m+1}^2 = \alpha^2(p_m^2 + q_m^2). \tag{2.14}$$

Also, from (2.10), (2.12) and (2.14), we find that

$$\begin{aligned} & m^2\left[(1+\delta)\left(\delta + 2\gamma(1+\lambda m)\right) + \gamma(\gamma-1)(1+\lambda m)^2\right] a_{m+1}^2 \\ & = \alpha(p_{2m} + q_{2m}) + \frac{\alpha(\alpha-1)}{2}\left(p_m^2 + q_m^2\right) \\ & = \alpha(p_{2m} + q_{2m}) + \frac{m^2(\alpha-1)\left(\delta + \gamma(1+\lambda m)\right)^2}{\alpha} a_{m+1}^2 . \end{aligned}$$

Therefore, we have

$$a_{m+1}^2 = \frac{\alpha^2(p_{2m} + q_{2m})}{m^2\left[(\alpha+\delta)\left(\delta + 2\gamma(1+\lambda m)\right) + \gamma(\gamma-\alpha)(1+\lambda m)^2\right]} . \tag{2.15}$$

Now, taking the absolute value of (2.15) and applying Lemma 1.1 for the coefficients p_{2m} and q_{2m}, we deduce that

$$|a_{m+1}| \le \frac{2\alpha}{m\sqrt{(\alpha+\delta)\left(\delta + 2\gamma(1+\lambda m)\right) + \gamma(\gamma-\alpha)(1+\lambda m)^2}} .$$

This gives the desired estimate for $|a_{m+1}|$ as asserted in (2.3).

In order to find the bound on $|a_{2m+1}|$, by subtracting (2.12) from (2.10), we get

$$\begin{aligned} & 2m\left(\delta + \gamma(1+2\lambda m)\right)\left[2a_{2m+1} - (m+1)a_{m+1}^2\right] \\ & = \alpha(p_{2m} - q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2 - q_m^2). \end{aligned} \tag{2.16}$$

It follows from (2.13), (2.14) and (2.16) that

$$a_{2m+1} = \frac{\alpha^2(m+1)\left(p_m^2 + q_m^2\right)}{4m^2\left(\delta + \gamma(1+\lambda m)\right)^2} + \frac{\alpha\left(p_{2m} - q_{2m}\right)}{4m\left(\delta + \gamma(1+2\lambda m)\right)} . \tag{2.17}$$

Taking the absolute value of (2.17) and applying Lemma 1.1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we obtain

$$|a_{2m+1}| \le \frac{2\alpha^2(m+1)}{m^2\left(\delta + \gamma(1+\lambda m)\right)^2} + \frac{\alpha}{m\left(\delta + \gamma(1+2\lambda m)\right)},$$

which completes the proof of Theorem 2.3. □

For one-fold symmetric bi-univalent functions, Theorem 2.3 reduce to the following corollary:

Corollary 2.4. *Let $f \in E_\Sigma(\delta,\gamma,\lambda;\alpha)$ $(z,w \in U, 0 < \alpha \le 1, 0 \le \delta \le 1, 0 \le \gamma \le 1, 0 \le \lambda \le 1)$, be given by (1.1). Then*

$$|a_2| \le \frac{2\alpha}{\sqrt{(\alpha+\delta)\left(\delta + 2\gamma(1+\lambda)\right) + \gamma(\gamma-\alpha)(1+\lambda)^2}}$$

and

$$|a_3| \le \frac{4\alpha^2}{\left(\delta + \gamma(1+\lambda)\right)^2} + \frac{\alpha}{\delta + \gamma(1+2\lambda)} .$$

3. Coefficient estimates for the function class $E^*_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$

Definition 3.1. *A function $f \in \Sigma_m$ given by (1.3) is said to be in the class $E^*_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$ if it satisfies the following conditions:*

$$Re\left\{\left(\frac{zf'(z)}{f(z)}\right)^{\delta}\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma}\right\}>\beta \tag{3.1}$$

and

$$Re\left\{\left(\frac{wg'(w)}{g(w)}\right)^{\delta}\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma}\right\}>\beta, \tag{3.2}$$

$(z,w\in U, 0<\alpha\leq 1,\ 0\leq\delta\leq 1,\ 0\leq\gamma\leq 1,\ 0\leq\lambda\leq 1,\ m\in\mathbb{N})$,

where the function $g=f^{-1}$ is given by (1.4).

In particular, for one-fold symmetric bi-univalent functions, we denote the class $E^{*}_{\Sigma_1}(\delta,\gamma,\lambda;\beta)=E^{*}_{\Sigma}(\delta,\gamma,\lambda;\beta)$.

Remark 3.2. *It should be remarked that the classes $E^{*}_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$ and $E^{*}_{\Sigma}(\delta,\gamma,\lambda;\beta)$ are a generalization of well-known classes consider earlier. These classes are:*
(1) For $\delta=\lambda=0$ and $\gamma=1$, the class $E^{*}_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$ reduce to the class $S^{\beta}_{\Sigma_m}$ which was considered by Altınkaya and Yalçın [1].
(2) For $\delta=0$ and $\gamma=1$, the class $E^{*}_{\Sigma}(\delta,\gamma,\lambda;\beta)$ reduce to the class $B_{\Sigma}(\beta,\tau)$ which was introduced by Liu and Wang [9].
(3) For $\delta=\lambda=0$ and $\gamma=1$, the class $E^{*}_{\Sigma}(\delta,\gamma,\lambda;\beta)$ reduce to the class $S^{*}_{\Sigma}(\beta)$ which was given by Brannan and Taha [3].

Theorem 3.3. *Let $f\in E^{*}_{\Sigma_m}(\delta,\gamma,\lambda;\beta)$ $(0\leq\beta<1,\ 0\leq\delta\leq 1,\ 0\leq\gamma\leq 1,\ 0\leq\lambda\leq 1,\ m\in\mathbb{N})$ be given by (1.3). Then*

$$|a_{m+1}|\leq\frac{2}{m}\sqrt{\frac{1-\beta}{(1+\delta)(\delta+2\gamma(1+\lambda m))+\gamma(\gamma-1)(1+\lambda m)^2}} \tag{3.3}$$

and

$$|a_{2m+1}|\leq\frac{2(m+1)(1-\beta)^2}{m^2(\delta+\gamma(1+\lambda m))^2}+\frac{1-\beta}{m(\delta+\gamma(1+2\lambda m))}. \tag{3.4}$$

Proof. It follows from conditions (3.1) and (3.2) that there exist $p,q\in\mathscr{P}$ such that

$$\left(\frac{zf'(z)}{f(z)}\right)^{\delta}\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma}$$
$$=\beta+(1-\beta)p(z) \tag{3.5}$$

and

$$\left(\frac{wg'(w)}{g(w)}\right)^{\delta}\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma}$$
$$=\beta+(1-\beta)q(w), \tag{3.6}$$

where $p(z)$ and $q(w)$ have the forms (2.7) and (2.8), respectively. Equating coefficients (3.5) and (3.6) yields

$$m(\delta+\gamma(1+\lambda m))a_{m+1}=(1-\beta)p_m, \tag{3.7}$$

$$\begin{aligned}&m\Big[2(\delta+\gamma(1+2\lambda m))a_{2m+1}\\&\quad-\left(\delta+\gamma\left(\lambda m^2+2\lambda m+1\right)\right)a^2_{m+1}\Big]+\frac{m^2}{2}\Big[\delta(\delta-1)\\&\quad+\gamma(1+\lambda m)(2\delta+(\gamma-1)(1+\lambda m))\Big]a^2_{m+1}\\&=(1-\beta)p_{2m},\end{aligned} \tag{3.8}$$

$$-m(\delta+\gamma(1+\lambda m))a_{m+1}=(1-\beta)q_m \tag{3.9}$$

and

$$\begin{aligned}&m\Big[\left(\delta(2m+1)+\gamma\left(3\lambda m^2+2(\lambda+1)m+1\right)\right)a^2_{m+1}\\&\quad-2(\delta+\gamma(1+2\lambda m))a_{2m+1}\Big]+\frac{m^2}{2}\Big[\delta(\delta-1)\\&\quad+\gamma(1+\lambda m)(2\delta+(\gamma-1)(1+\lambda m))\Big]a^2_{m+1}\\&=(1-\beta)q_{2m}.\end{aligned} \tag{3.10}$$

From (3.7) and (3.9), we get

$$p_m=-q_m \tag{3.11}$$

and

$$2m^2(\delta+\gamma(1+\lambda m))^2a^2_{m+1}=(1-\beta)^2\left(p^2_m+q^2_m\right). \tag{3.12}$$

Adding (3.8) and (3.10), we obtain

$$\begin{aligned}&m^2\left[(1+\delta)(\delta+2\gamma(1+\lambda m))+\gamma(\gamma-1)(1+\lambda m)^2\right]a^2_{m+1}\\&=(1-\beta)(p_{2m}+q_{2m}).\end{aligned} \tag{3.13}$$

Therefore, we have

$$a^2_{m+1}=\frac{(1-\beta)(p_{2m}+q_{2m})}{m^2\left[(1+\delta)(\delta+2\gamma(1+\lambda m))+\gamma(\gamma-1)(1+\lambda m)^2\right]}.$$

Applying Lemma 1.1 for the coefficients p_{2m} and q_{2m}, we obtain

$$|a_{m+1}|\leq\frac{2}{m}\sqrt{\frac{1-\beta}{(1+\delta)(\delta+2\gamma(1+\lambda m))+\gamma(\gamma-1)(1+\lambda m)^2}}.$$

This gives the desired estimate for $|a_{m+1}|$ as asserted in (3.3). In order to find the bound on $|a_{2m+1}|$, by subtracting (3.10) from (3.8), we get

$$2m(\delta+\gamma(1+2\lambda m))\left[2a_{2m+1}-(m+1)a_{m+1}^2\right] = (1-\beta)(p_{2m}-q_{2m}).$$

or equivalently

$$a_{2m+1}=\frac{m+1}{2}a_{m+1}^2+\frac{(1-\beta)(p_{2m}-q_{2m})}{4m(\delta+\gamma(1+2\lambda m))}.$$

Upon substituting the value of a_{m+1}^2 from (3.12), it follows that

$$a_{2m+1}=\frac{(m+1)(1-\beta)^2(p_m^2+q_m^2)}{4m^2(\delta+\gamma(1+\lambda m))^2}+\frac{(1-\beta)(p_{2m}-q_{2m})}{4m(\delta+\gamma(1+2\lambda m))}.$$

Applying Lemma 1.1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we obtain

$$|a_{2m+1}|\leq\frac{2(m+1)(1-\beta)^2}{m^2(\delta+\gamma(1+\lambda m))^2}+\frac{1-\beta}{m(\delta+\gamma(1+2\lambda m))}.$$

which completes the proof of Theorem 3.3. □

For one-fold symmetric bi-univalent functions, Theorem 3.3 reduce to the following corollary:

Corollary 3.4. *Let* $f\in E_{\Sigma}^{*}(\delta,\gamma,\lambda;\beta)$, $(0\leq\beta<1,\ 0\leq\delta\leq1,\ 0\leq\gamma\leq1,\ 0\leq\lambda\leq1)$ *be given by (1.1). Then*

$$|a_2|\leq 2\sqrt{\frac{1-\beta}{(1+\delta)(\delta+2\gamma(1+\lambda))+\gamma(\gamma-1)(1+\lambda)^2}}$$

and

$$|a_3|\leq\frac{4(1-\beta)^2}{(\delta+\gamma(1+\lambda))^2}+\frac{1-\beta}{\delta+\gamma(1+2\lambda)}.$$

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