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# Economic order quantity in fuzzy sense with allowable shortage: A Karush Kuhn-tucker conditions approach

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**Abstract**

In this article, a fuzzy inventory model with allowable shortage is formulated and solved. Fuzziness is introduced by allowing the cost components (holding cost, ordering cost, shortages cost and demand). In fuzzy environment, all related inventory parameters are represented to be octagonal fuzzy numbers .These fuzzy numbers have been used in order to determine the optimal order quantity and optimal total cost for the inventory model. The calculation of EOQ is carried out through defuzzification by using ranking function method. The model is solved using Kuhn- tucker conditions method. The results of the models are illustrated with numerical example.

**Keywords**

Economic order quantity (EOQ), Fuzzy EOQ model, Octagonal fuzzy numbers, Fuzzy optimal order quantity, Fuzzy optimal total cost, Ranking function, Karush Kuhn- tucker condition.

**AMS Subject Classification**

65K10.

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## Contents



## 1. Introduction

The classical EOQ model developed in 1915 had the specific requirements of deterministic costs and demand and lack of deterioration of the items in stock. Ghare and Schrader [3] were the first researches to develop an EOQ model for an item with exponential decay and constant demand. Park and Wang [1] studied shortages and partial backlogging of items. Yao and Lee [8] considered an economic production quantity model in the fuzzy sense. Sujit Kumar De, Kundu, Goswami [5] presented an economic production quantity inventory model involving fuzzy demand rate. Dutta and Pavankumar published several papers in the area of fuzzy inventory with or without shortages. In 2013, the same authors Dutta and Pavan Kumar [2] proposed an optimal policy for an inventory model without shortage considering fuzziness in demand, holding cost, ordering cost. Yao and Chiang [9] developed an inventory without backorder with fuzzy total cost and fuzzy storage cost defuzzified by centroid and signed distance method. Bellman and Zadeh [10] first introduced the notation of fuzziness. Menaka [6] formulated ranking of octagonal intuitionistic fuzzy numbers. Kuhn- tucker conditions was first developed by W.Karush in 1939. The same conditions were developed independent in 1951 by W. Kuhn and A.Tuker. A new fuzzy arithmetic operations and on fuzzy inventory model with allowable shortage are discussed by Stephen Dinagar and Rajesh kannan [7] Kasthuri and Seshaiah [4] solved multi-item EOQ model with demand dependent on unit price.

In our present study, in section 2, introduces basic definitions and fuzzy arithmetic operations. In section 3, presents fuzzy notations, assumptions, mathematics for the crisp case of an EOQ model and mathematics of the full-fuzzy version of the EOQ model. Section 4 is for numerical examples and Section 5 is for conclusion.

## 2. Definitions and Preliminaries

### 2.1 Fuzzy Set

A fuzzy set $\tilde{A}$ on the given universal set $X$ is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : x \to [0,1]$ is called membership function or grade membership. The membership function is also a degree of compatibility or a degree of truth of $x$ in $\tilde{A}$.

### 2.2 Fuzzy Number [FN]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the member-ship function.

A fuzzy number is a convex normalized fuzzy set on the real line $R$ such that:

(1) There exist at least one with $x \in R$ with $\mu_{\tilde{A}}(x) = 1$

(2) $\mu_{\tilde{A}}(x)$ is piecewise continuous

### 2.3 Generalized Octagonal Fuzzy number

A fuzzy number is said to be a generalized octagonal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function $\mu_A$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \quad \text{for} \quad x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) \quad \text{for} \quad a_1 \leq x \leq a_2 \\ k \quad \text{for} \quad a_2 \leq x \leq a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right) \quad \text{for} \quad a_3 \leq x \leq a_4 \\ 1 \quad \text{for} \quad a_4 \leq x \leq a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right) \quad \text{for} \quad a_5 \leq x \leq a_6 \\ k \quad \text{for} \quad a_6 \leq x \leq a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) \quad \text{for} \quad a_7 \leq x \leq a_8 \\ 0 \quad \text{for} \quad x > a_8 \end{cases}$$

### 2.4 Ranking function method

We define a ranking function $R : f(R) \to R$ which maps each fuzzy numbers to the real line : $f(R)$ represents the set of all trapezoidal fuzzy number. If R be any linear ranking function, then

$$R(\tilde{A}) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8}{8}\right)$$

### 2.5 The Kuhn-Tucker conditions

**Table 1: The objective function and the solution space**

| Sense of optimization | Objective function | Solution space |
|---|---|---|
| Maximization | Concave | Convex set |
| Minimization | Convex | Convex set |

**Table 2: The sufficiency of the Kuhn-Tucker conditions**

| Problem | Kuhn-Tucker conditions |
|---|---|
| 1) $\max z = f(x)$ subject to $h^i(x) \leq 0, x \geq 0, i = 1,2,\ldots,m$ | $\frac{\partial}{\partial x_j} f(x) - \sum_{i=1}^{n} \lambda_i \frac{\partial}{\partial x_j} h^i(x) = 0; \lambda_i h^i(x) = 0,\ h^i(x) \leq 0,\ i = 1,2,\ldots,m, \lambda_i \geq 0,\ i = 1,2,\ldots,m,$ |
| 2) $\min z = f(x)$ subject to $h^i(x) \geq 0, x \geq 0, i = 1,2,\ldots,m$ | $\frac{\partial}{\partial x_j} f(x) - \sum_{i=1}^{n} \lambda_i \frac{\partial}{\partial x_j} h^i(x) = 0; \lambda_i h^i(x) = 0,\ h^i(x) \geq 0,\ i = 1,2,\ldots,m, \lambda_i \geq 0,\ i = 1,2,\ldots,m,$ |

### 2.6 New Arithmetic Operations

The new arithmetic operationsbetween hexagonal fuzzy numbers proposed are given below. Let us consider $\tilde{A_1} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{A_2} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two octagonal fuzzy numbers. Then,

(1) The addition of $\tilde{A_1}$ and $\tilde{A_2}$ is

$$\tilde{A_1}(+)\tilde{A_2} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$$

(2) The subtraction of $\tilde{A_1}$ and $\tilde{A_2}$ is

$$\tilde{A_1}(-)\tilde{A_2} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8)$$

(3) The multiplication of $\tilde{A_1}$ and $\tilde{A_2}$ is

$$\tilde{A_1}(\times)\tilde{A_2} = \left(\frac{a_1}{8}\sigma_b, \frac{a_2}{8}\sigma_b, \frac{a_3}{8}\sigma_b, \frac{a_4}{8}\sigma_b, \frac{a_5}{8}\sigma_b, \frac{a_6}{8}\sigma_b, \frac{a_7}{8}\sigma_b, \frac{a_8}{8}\sigma_b,\right)$$

where

$$\sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)$$

(4) The division of $\tilde{A_1}$ and $\tilde{A_2}$ is

$$\tilde{A_1}(\div)\tilde{A_2} = \left(\frac{8a_1}{\sigma_b}, \frac{8a_2}{\sigma_b}, \frac{8a_3}{\sigma_b}, \frac{8a_4}{\sigma_b}, \frac{8a_5}{\sigma_b}, \frac{8a_6}{\sigma_b}, \frac{8a_7}{\sigma_b}, \frac{8a_8}{\sigma_b}\right)$$

if

$$\sigma_b \neq 0, \sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)$$

(5) If $k \neq 0$ is a scalar $k$ is defined as

$$k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8) & \text{if} \quad k > 0 \\ (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8) & \text{if} \quad k < 0 \end{cases}$$

(6)

$$\sqrt{\tilde{A}} = \sqrt{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8} = \sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}, \sqrt{a_5}, \sqrt{a_6}, \sqrt{a_7}, \sqrt{a_8}$$

where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are non zero positive real numbers.

## 3. Fuzzy Inventory Model

### 3.1 Notations

We define the following symbols:
$\tilde{C}_h$: Fuzzy holding cost per unit quantity per unit time
$\tilde{C}_a$:Fuzzy Setup cost (or) ordering cost per order
$T$: Length of the plan
$\tilde{C}_s$:Fuzzy Shortage cost per unit quantity
$\tilde{D}_t$: Fuzzy Total demand over the planning time period $[0,T]$
$\tilde{Q}^*$: Fuzzy Order quantity per cycle
$\tilde{T}_c$: Fuzzy total cost for the period $[0,T]$
$\tilde{F}(Q^*)$: Minimum Fuzzy total cost for $[0,T]$
$\tilde{Q}^*_d$ : Fuzzy Optimal order quantity

### 3.2 Assumptions

In the present paper, the assumptions considered are as follows:

(i) Shortage cost is fuzzy in nature

(ii) Total demand is fuzzy in nature

(iii) Time plan is constant

(iv) Holding cost, Ordering cost are fuzzy in nature

### 3.3 Inventory model in crisp sense:

The total cost for the period $[0,T]$ is

$$TC = \frac{HTS^2}{2Q} + \frac{S(Q-S)^2}{2Q} + \frac{AD}{Q}$$

Where $S = \frac{Qs}{HT+s}$ is the maximum order level. Substituting $S$ in $TC$ and simplifying we get,

$$TC = \frac{HTQs^2}{2(HT+s)^2} + \frac{sQ\left(1-\frac{s}{HT+s}\right)^2}{2} + \frac{AD}{Q}$$

Optimal order quantity

$$Q = \sqrt{\frac{2AD(HT+s)}{HTs}}$$

### 3.4 Inventory model in fuzzy sense:

The ordering cost, holding cost, shortage cost are fuzzy in nature. The total demand and time of plan are considered as constants. Now we fuzzifying total cost is given by

$$\tilde{T}C = \frac{\tilde{H}T\tilde{Q}\tilde{s}^2}{2(\tilde{H}T+\tilde{s})^2} + \frac{\tilde{s}\tilde{Q}\left(1-\frac{\tilde{s}}{\tilde{H}T+\tilde{s}}\right)^2}{2} + \frac{\tilde{A}D}{\tilde{Q}}$$

Our goal is to obtain fuzzy total cost and the optimal order quantity in terms of octagonal fuzzy numbers by using simple calculus techniques.Suppose

$$\begin{aligned}
\tilde{C}_h &= (C_{h1},C_{h2},C_{h3},C_{h4},C_{h5},C_{h6},C_{h7},C_{h8})\\
\tilde{D}_t &= (D_{t1},D_{t2},D_{t3},D_{t4},D_{t5},D_{t6},D_{t7},D_{t8})\\
\tilde{C}_a &= (C_{a1},C_{a2},C_{a3},C_{a4},C_{a5},C_{a6},C_{a7},C_{a8})\\
\tilde{C}_s &= (C_{s1},C_{s2},C_{s3},C_{s4},C_{s5},C_{s6},C_{s7},C_{s8})
\end{aligned}$$

are octagonal fuzzy numbers.

$$\tilde{T}C = \frac{\tilde{C}_h T\tilde{Q}\tilde{C_s}^2}{2(\tilde{C}_h T+\tilde{s})^2} + \frac{\tilde{C}_s\tilde{Q}\left(1-\frac{\tilde{C}_s}{\tilde{C}_h T+\tilde{C}_s}\right)^2}{2} + \frac{\tilde{C}_a\tilde{D}_t}{\tilde{Q}}$$

By applying arithmetic function principles and simplifying we get,

$$\tilde{T}C = \begin{cases}
\frac{TQC_{s1}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h1}+C_{s1}-C_{s8}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t1}\partial f,\\
\frac{TQC_{s2}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h2}+C_{s2}-C_{s7}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t2}\partial f,\\
\frac{TQC_{s3}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h3}+C_{s3}-C_{s6}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t3}\partial f,\\
\frac{TQC_{s4}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h4}+C_{s4}-C_{s5}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t4}\partial f,\\
\frac{TQC_{s5}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h5}+C_{s5}-C_{s4}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t5}\partial f,\\
\frac{TQC_{s6}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h6}+C_{s6}-C_{s3}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t6}\partial f,\\
\frac{TQC_{s7}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h7}+C_{s7}-C_{s2}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t7}\partial f,\\
\frac{TQC_{s8}}{8}\frac{\partial a\partial b}{\partial c} + \frac{Q}{16}\left(\frac{TC_{h8}+C_{s8}-C_{s1}}{\partial d}\right)\partial a\partial e + \frac{1}{8Q}D_{t8}\partial f,\\
(a1,a2,a3,a4,a5,a6,a7,a8) = F(\tilde{Q}),
\end{cases}$$

where

$$\begin{aligned}
\partial a &= (C_{s1}+C_{s2}+C_{s3}+C_{s4}+C_{s5}+C_{s6}+C_{s7}+C_{s8})\\
\partial b &= (C_{h1}+C_{h2}+C_{h3}+C_{h4}+C_{h5}+C_{h6}+C_{h7}+C_{h8})\\
\partial c &= \frac{TC_{h1}+C_{s1}}{4}(TC_{h1}+C_{s1}+TC_{h2}+C_{s2}+TC_{h3}+C_{s3}+TC_{h4}+C_{s4}+TC_{h5}+C_{s5}+TC_{h6}\\
&\quad +C_{s6}+TC_{h7}+C_{s7}+TC_h8+C_{s8})+\cdots+\frac{TC_{h8}+C_{s8}}{4}(TC_{h1}+C_{s1}+TC_{h2}+C_{s2}+TC_{h3}+C_{s3}\\
&\quad +TC_{h4}+C_{s4}+TC_{h5}+C_{s5}+TC_{h6}+C_{s6}+TC_{h7}+C_{s7}+TC_h8+C_{s8})\\
\partial d &= (TC_{h1}+C_{s1}+TC_{h2}+C_{s2}+TC_{h3}+C_{s3}+TC_{h4}+C_{s4}+TC_{h5}+C_{s5}+TC_{h6}+C_{s6}\\
&\quad +TC_{h7}+C_{s7}+TC_h8+C_{s8})\\
\partial e &= \frac{8(TC_{h1}+C_{s1}-C_{s8})}{(TC_{h1}+C_{s1}+TC_{h2}+C_{s2}+TC_{h3}+C_{s3}+TC_{h4}+C_{s4}+TC_{h5}+C_{s5}+TC_{h6}+C_{s6}+TC_{h7}+C_{s7}+TC_h8+C_{s8})}+\cdots\\
&\quad +\frac{8(TC_{h8}+C_{s8}-C_{s1})}{(TC_{h1}+C_{s1}+TC_{h2}+C_{s2}+TC_{h3}+C_{s3}+TC_{h4}+C_{s4}+TC_{h5}+C_{s5}+TC_{h6}+C_{s6}+TC_{h7}+C_{s7}+TC_h8+C_{s8})}\\
\partial f &= (C_{a1}+C_{a2}+C_{a3}+C_{a4}+C_5+C_{a6}+C_{a7}+C_{a8})
\end{aligned}$$

Fuzzy order quantity model using ranking method:

Using ranking function method for defuzzifying the fuzzy total cost, then we get,

$$\tilde{R}\tilde{T}C_1 = \frac{1}{8}\left\{\begin{array}{l}
\frac{TQC_{s1}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h1}+C_{s1}-C_{s8}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t1}\partial f+\\
\frac{TQC_{s2}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h2}+C_{s2}-C_{s7}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t2}\partial f+\\
\frac{TQC_{s3}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h3}+C_{s3}-C_{s6}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t3}\partial f+\\
\frac{TQC_{s4}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h4}+C_{s4}-C_{s5}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t4}\partial f+\\
\frac{TQC_{s5}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h5}+C_{s5}-C_{s4}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t5}\partial f+\\
\frac{TQC_{s6}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h6}+C_{s6}-C_{s3}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t6}\partial f+\\
\frac{TQC_{s7}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h7}+C_{s7}-C_{s2}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t7}\partial f+\\
\frac{TQC_{s8}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q}{16}\left(\frac{TC_{h8}+C_{s8}-C_{s1}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q}D_{t8}\partial f+
\end{array}\right.$$

$$\tilde{R}\tilde{T}C_1 = \tilde{F}(Q)^*$$

$$R(\tilde{T}C_1) = \frac{1}{8}\left\{\begin{array}{l}
\frac{TQ}{8}\frac{\partial a\partial b}{\partial c}(C_{s1}+C_{s2}+C_{s3}+C_{s4}+C_{s5}+C_{s6}+C_{s7}+C_{s8})\\
+\frac{Q}{16}\frac{\partial a\partial b}{\partial d}(TC_{h1}+TC_{h2}+TC_{h3}+TC_{h4}+TC_{h5}\\
+TC_{h6}+TC_{h7}+TC_{h8})\\
+\frac{1}{8Q}\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})
\end{array}\right.$$

$$R(\tilde{T}C_1) = \frac{1}{8}\left\{\begin{array}{l}
\frac{TQ}{8}\frac{(\partial a)^2\partial b}{\frac{(T\partial b+\partial a)^2}{4}}+\frac{Q}{16}\partial a\left(\frac{8T\partial b}{T\partial a+\partial b}\right)\frac{T\partial a}{T\partial a+\partial b}\\
+\frac{1}{8Q}\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})
\end{array}\right.$$

To find the fuzzy order quantity $\tilde{Q}^*$ which can be obtained by the solution of the first order fuzzy differential equation $\frac{d}{d\tilde{Q}^*}(R(\tilde{T}C_1)) = 0$. Then the result is,

$$\frac{d}{d\tilde{Q}^*}(R(\tilde{T}C_1)) = 0$$

$$\frac{d}{dQ}\left[\frac{1}{8}\left[\frac{TQ}{8}\frac{(\partial a)^2\partial b}{\frac{(T\partial b+\partial a)^2}{4}}+\frac{Q}{16}\partial a\left(\frac{8T\partial b}{T\partial a+\partial b}\right)\frac{T\partial a}{T\partial a+\partial b}\right.\right.$$

$$\left.\left.+\frac{1}{8Q}\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})\right]\right]=0$$

$$\frac{1}{8}\left[\frac{T}{8}\frac{(\partial a)^2\partial b}{\frac{(T\partial b+\partial a)^2}{4}}+\frac{1}{16}\partial a\left(\frac{8T\partial b}{T\partial a+\partial b}\right)\frac{T\partial a}{T\partial a+\partial b}\right.$$

$$\left.+\frac{1}{8Q^2}\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})\right]=0$$

$$\tilde{Q}^* = \sqrt{\frac{\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})}{4\partial a}+\frac{\partial f(D_{t1}+D_{t2}+D_{t3}+D_{t4}+D_{t5}+D_{t6}+D_{t7}+D_{t8})}{4T\partial b}}$$

Fuzzy optimal order quantity using Kuhn-tucker method. Suppose the fuzzy order quantity $\tilde{Q}$ be a octagonal fuzzy number $\tilde{Q} = (Q_1,Q_2,Q_3,Q_4,Q_5,Q_6,Q_7,Q_8)$ with $Q < Q_1 \le Q_2 \le Q_3 \le Q_4 \le Q_5 \le Q_6 \le Q_7 \le Q_8$. The fuzzy total cost is,

$$\tilde{T}C_2 = \left\{\begin{array}{l}
\frac{TQ_1C_{s1}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_1}{16}\left(\frac{TC_{h1}+C_{s1}-C_{s8}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_8}D_{t1}\partial f,\\
\frac{TQ_2C_{s2}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_2}{16}\left(\frac{TC_{h2}+C_{s2}-C_{s7}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_7}D_{t2}\partial f,\\
\frac{TQ_3C_{s3}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_3}{16}\left(\frac{TC_{h3}+C_{s3}-C_{s6}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_6}D_{t3}\partial f,\\
\frac{TQ_4C_{s4}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_4}{16}\left(\frac{TC_{h4}+C_{s4}-C_{s5}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_5}D_{t4}\partial f,\\
\frac{TQ_5C_{s5}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_5}{16}\left(\frac{TC_{h5}+C_{s5}-C_{s4}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_4}D_{t5}\partial f,\\
\frac{TQ_6C_{s6}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_6}{16}\left(\frac{TC_{h6}+C_{s6}-C_{s3}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_3}D_{t6}\partial f,\\
\frac{TQ_7C_{s7}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_7}{16}\left(\frac{TC_{h7}+C_{s7}-C_{s2}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_2}D_{t7}\partial f,\\
\frac{TQ_8C_{s8}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_8}{16}\left(\frac{TC_{h8}+C_{s8}-C_{s1}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_1}D_{t8}\partial f,
\end{array}\right.$$

We defuzzify the fuzzy total cost using ranking function method. The result is,

$$R\tilde{T}C_2 = \frac{1}{8}\left\{\begin{array}{l}
\frac{TQ_1C_{s1}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_1}{16}\left(\frac{TC_{h1}+C_{s1}-C_{s8}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_8}D_{t1}\partial f+\\
\frac{TQ_2C_{s2}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_2}{16}\left(\frac{TC_{h2}+C_{s2}-C_{s7}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_7}D_{t2}\partial f+\\
\frac{TQ_3C_{s3}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_3}{16}\left(\frac{TC_{h3}+C_{s3}-C_{s6}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_6}D_{t3}\partial f+\\
\frac{TQ_4C_{s4}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_4}{16}\left(\frac{TC_{h4}+C_{s4}-C_{s5}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_5}D_{t4}\partial f+\\
\frac{TQ_5C_{s5}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_5}{16}\left(\frac{TC_{h5}+C_{s5}-C_{s4}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_4}D_{t5}\partial f+\\
\frac{TQ_6C_{s6}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_6}{16}\left(\frac{TC_{h6}+C_{s6}-C_{s3}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_3}D_{t6}\partial f+\\
\frac{TQ_7C_{s7}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_7}{16}\left(\frac{TC_{h7}+C_{s7}-C_{s2}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_2}D_{t7}\partial f+\\
\frac{TQ_8C_{s8}}{8}\frac{\partial a\partial b}{\partial c}+\frac{Q_8}{16}\left(\frac{TC_{h8}+C_{s8}-C_{s1}}{\partial d}\right)\partial a\partial e+\frac{1}{8Q_1}D_{t8}\partial f.
\end{array}\right. \quad (1.1)$$

With

$$Q < Q_1 \le Q_2 \le Q_3 \le Q_4 \le Q_5 \le Q_6 \le Q_7 \le Q_8$$

It will not change the meaning of formula (1.1), if we replace inequality constrains

$$Q < Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5 \leq Q_6 \leq Q_7 \leq Q_8$$

Into the following inequality constrains.

$$Q_2 - Q_1 \geq 0,\ Q_3 - Q_2 \geq 0,\ Q_4 - Q_3 \geq 0,\ Q_5 - Q_4 \geq 0,\ Q_6 - Q_5 \geq 0,\ Q_7 - Q_6 \geq 0,\ Q_8 - Q_7 \geq 0,\ Q_1 > 0$$

Next, the Kuhn-tucker condition is used to find the solution of $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$ and $Q_8$. To minimize $R(\tilde{T}C_2)$in formula (1.1), subject to

$$Q_2 - Q_1 \geq 0,\ Q_3 - Q_2 \geq 0,\ Q_4 - Q_3 \geq 0,\ Q_5 - Q_4 \geq 0,\ Q_6 - Q_5 \geq 0,\ Q_7 - Q_6 \geq 0,\ Q_8 - Q_7 \geq 0,\ Q_1 > 0.$$

The Kuhn-tucker conditions are,

$$\begin{aligned} \lambda &\leq 0 \\ \nabla f(R(\tilde{T}C_2)) - \lambda_i \nabla g(Q) &= 0 \\ \lambda_i \nabla_{g_i}(Q) &= 0 \\ g_i(Q) &\geq 0 \end{aligned}$$

These conditions simplify to the following,
$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8 \leq 0$

$$\begin{cases} \frac{1}{8}\left[\frac{TC_{s1}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h1}+C_{s1}-C_{s8}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_1^2}D_{t8}\partial f\right] + \lambda_1 - \lambda_8 = 0 \\ \frac{1}{8}\left[\frac{TC_{s2}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h2}+C_{s2}-C_{s7}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_2^2}D_{t7}\partial f\right] - \lambda_1 + \lambda_2 = 0 \\ \frac{1}{8}\left[\frac{TC_{s3}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h3}+C_{s3}-C_{s6}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_3^2}D_{t6}\partial f\right] - \lambda_2 + \lambda_3 = 0 \\ \frac{1}{8}\left[\frac{TC_{s4}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h4}+C_{s4}-C_{s5}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_4^2}D_{t5}\partial f\right] - \lambda_3 + \lambda_4 = 0 \\ \frac{1}{8}\left[\frac{TC_{s5}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h5}+C_{s5}-C_{s4}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_5^2}D_{t4}\partial f\right] - \lambda_4 + \lambda_5 = 0 \\ \frac{1}{8}\left[\frac{TC_{s6}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h6}+C_{s6}-C_{s3}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_6^2}D_{t3}\partial f\right] - \lambda_5 + \lambda_6 = 0 \\ \frac{1}{8}\left[\frac{TC_{s7}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h7}+C_{s7}-C_{s2}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_7^2}D_{t2}\partial f\right] - \lambda_6 + \lambda_7 = 0 \\ \frac{1}{8}\left[\frac{TC_{s8}}{8}\frac{\partial a \partial b}{\partial c} + \frac{1}{16}\left(\frac{TC_{h8}+C_{s8}-C_{s1}}{\partial d}\right)\partial a \partial e - \frac{1}{8Q_8^2}D_{t1}\partial f\right] - \lambda_7 = 0 \end{cases} \tag{1.2}$$

If $Q_1 > 0$ and $\lambda_8 Q_1 = 0$ then $\lambda_8 = 0$. If $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$. Then

$$Q_8 < Q_7 < Q_6 < Q_5 < Q_4 < Q_3 < Q_2 < Q_1.$$

It does not satisfy the constrains

$$0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5 \leq Q_6 \leq Q_7 \leq Q_8.$$

Therefore

$$Q_2 = Q_1,\ Q_3 = Q_2,\ Q_4 = Q_3,\ Q_5 = Q_4,\ Q_6 = Q_5,\ Q_7 = Q_6,\ Q_8 = Q_7.$$

i.e,

$$Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q_6 = Q_7 = Q_8 = \tilde{Q}_d^*$$

Hence from (1.2), we find the optimal order quantity $\tilde{Q}_d^*$ by the above equation as,

$$\left[\frac{T}{8}\frac{(\partial a)^2 \partial b}{\frac{(T\partial b + \partial a)^2}{4}} + \frac{1}{16}\partial a\left(\frac{8T\partial b}{T\partial a + \partial b}\right)\frac{T\partial a}{T\partial a + \partial b} - \frac{1}{8Q^2}\partial f(D_{t1} \quad + D_{t2} + D_{t3} + D_{t4} + D_{t5} + D_{t6} + D_{t7} + D_{t8})\right] = 0$$

$$\tilde{Q}_d^* = \sqrt{\frac{\partial f(D_{t1} + D_{t2} + D_{t3} + D_{t4} + D_{t5} + D_{t6} + D_{t7} + D_{t8})}{4\partial a} + \frac{\partial f(D_{t1} + D_{t2} + D_{t3} + D_{t4} + D_{t5} + D_{t6} + D_{t7} + D_{t8})}{4T\partial b}}$$

## 4. Conclusion

This paper is dedicated to solve a economic order quantity in fuzzy sense problem for determining the fuzzy total cost using Kuhn-tucker condition method. To find various fuzzy optimal quantities, the demand, holding cost, ordering cost and shortage cost using octagonal fuzzy numbers have been used and solved by ranking function method. New arithmetic operations of a particular octagonal fuzzy number are also addressed. We conclude that fuzzy values are all closer to crisp values of the real system.

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