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The analysis of the M/M/1 queue with two vacation policies using pentagonal fuzzy numbers

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Abstract

This study investigates the analysis of the M/M/1 queue with two vacation policies and Fuzzy parameters (FM/FM/1/SWV+MV). For this fuzzy queuing model, the researcher obtains some performance measure of interest such as the server is in the working vacation period, server is in the vacation period, the server is in the regular service period. Finally, numerical results are presented using pentagonal fuzzy numbers to show the effects of system parameters.

Keywords

FM/FM/S model, membership values, pentagonal fuzzy numbers.

AMS Subject Classification

68M20, 90B22.

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Contents

1. Introduction

In 2002, Servi and Finn [15] introduced a class of semi-vacation, called working vacation, during which the customers are served with a lower rate rather than completely stopping serving. And they first analyzed an M/M/1 queue with this class of semi-vacation policy, denoted by M/M/1/WV, and obtained the PGF of the queue length and the LST of the sojourn time of a customer in steady state and applied their results to analyses a WDM optical access network using multiple wavelengths which can be reconfigured. Subsequently, Liu et al. [11] studied the M/M/1 queue with working vacations by matrix-geometric solution and established the stochastic decomposition structures of the queue length and sojourn time of a customer in steady state. Later, by the same method, Tian and Zhao [16] analyzed the M/M/1 queue with single working vacation, denoted by M/M/1/SWV, and various indicators in steady state were derived.

On the M/G/1-type queue with working vacations, Wu and Takagi [17] studied the M/G/1 queue with working vacations based on the Laplace–Stieltjes transform method. Using the results in systems with disasters, Kim et al. [10] gave an analysis on the M/G/1 queue with exponential working vacations. By the method of matrix analysis, Baba [1] first investigated a GI/M/1-type queue with multiple working vacations utilizing the method of matrix analysis. Subsequently, Liu and Tian [11] analyzed the GI/M/1 queue with single working vacation. For a finite-buffer GI/M/1-type queue with multiple working vacations, we can see the survey of Banik et al. [2]. In contrast to the queuing model in above references, in which the exhaustive discipline has been applied.

R.Kalayanaraman et.al [3] introduced a single server vacation queue with fuzzy service time and vacation time distributions with some performance measures. R.Kalyanaraman, et.al [4, 5] gave a single server fuzzy queue with group arrivals and server vacation. R.Kalyanaraman, et.al [6] investigated a fuzzy bulk queue with modified Bernoulli vacation and restricted admissible customers. They obtained some performance measures in fuzzy parameters.

G. Kannadasan and N. Sathiyamoorthi [8] investigate the $FM/FM/1$ queue with single working vacation. We obtain some system characteristic such as the number of customer in the system in study-state, the virtual time of a customer in the system, the server is in idle period, the server is in regular busy period. G. Kannadasan, et.al [8] also gave analysis for the $FM/FM/1$ queue with multiple working vacation with N-Policy, using non-linear programming method, with mean queue length, mean waiting time, at N=2. G.Kannadasan and N. Sathiyamoorthi [7] established the $FM^X/FM/1$ queue with multiple working vacation and some performance measure of interest. G.Kannadasan and N.Sathiyamoorthi [9] worked in fuzzy analysis technique in the FM/FM/1 queue with single working vacation and set-up times.

In this paper, we analysis of the FM/FM/1 queue with two vacation policies (FM/FM/1/SWV+MV). In section 2, we describe the queue model. In section 3 and 4, we discuss the fuzzy model the server is in the working vacation period, server is in the vacation period, the server is in the regular service period are studied in fuzzy environment respectively. In section 5 includes numerical study about the performance measures.

2. The crisp model

The queuing model we consider here is defined explicitly as follows:

(1) Customers arrive to the system according to the Poisson process with rate λ, and service times in a regular service period are exponential distribution with mean μ_b^{-1}.

(2) The working vacation is a class of semi-vacation policy during which the customers arriving are served at a lower service rate $\mu_v(\mu_v < \mu_b)$, rather than completely stopping service as that the server is in the working vacation period.

(3) The durations of the working vacations are exponential distributions with mean θ_w^{-1} and the durations of the vacations are exponential distributions with mean θ_w^{-1}

(4) The two vacation policies are described as follows:After a regular service period, the server starts to take a working vacation. At the working vacation completion epoch, if there are customers left in the system, the server will change the service rate from $\mu_v to \mu_b$, and another regular service period will start. Otherwise, the system chooses to enter into a vacation. If there are customers staying the queue when a vacation completes, the server is resumed to a regular service period. Otherwise, the server continues the vacations until there are arrivals in the system at the vacation completion epoch, and a regular service period will start.

(5) The inter-arrival times, service times in regular service period, service times in working vacation period, working vacation times and vacation times are all assumed to be mutually independent. In addition, the service discipline is First Come First Served.

3. The model in fuzzy environment

In this section the arrival rate, regular service rate,lower service rate, vacation rate and working vacation rate are assumed to be fuzzy numbers $\bar{\lambda}, \bar{\beta}_1, \bar{\beta}_2, \bar{\theta}_1, \bar{\theta}_2$ respectively. Now

$\bar{\lambda} = \{x, \mu_{\bar{\lambda}}(x); x \in S(\bar{\lambda})\}$,

$\bar{\beta}_1 = \left\{y_1, \mu_{\bar{\beta}_1}(y_1); y_1 \in S(\bar{\beta}_1)\right\}$,

$\bar{\beta}_2 = \left\{y_2, \mu_{\bar{\beta}_2}(y_2); y_2 \in S(\bar{\beta}_2)\right\}$,

$\bar{\theta}_1 = \left\{z_1, \mu_{\bar{\theta}_1}(z_1); z_1 \in S(\bar{\theta})\right\}$ and

$\bar{\theta}_2 = \left\{z_2, \mu_{\bar{\theta}_2}(z_2); z_2 \in S(\bar{\theta})\right\}$.

Where $S(\bar{\lambda})$, $S(\bar{\beta}_1)$, $S(\bar{\beta}_2)$, $S(\bar{\theta}_1)$ and $S(\bar{\theta}_2)$ are the universal set's of arrival rate, regular service rate, lower service rate, vacation rate and working vacation rate respectively. Define $f(x, y_1, y_2, z_1, z_2)$ as the system performance measure related to the above defined fuzzy queuing model which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\beta}_1, \bar{\beta}_2, \bar{\theta}_1, \bar{\theta}_2)$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $f(\bar{\lambda}, \bar{\beta}_1, \bar{\beta}_2, \bar{\theta}_1, \bar{\theta}_2)$ can be defined as,

$$\mu_{\bar{f}(\bar{\lambda},\bar{\beta}_1,\bar{\beta}_2,\bar{\theta}_1,\bar{\theta}_2)}(D) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\beta}_1)\\ y_2\in S(\bar{\beta}_2)\\ z_1\in S(\bar{\theta}_1)\\ z_2\in S(\bar{\theta}_2)}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}_1}(y_1), \mu_{\bar{\beta}_2}(y_2), \mu_{\bar{\theta}_1}(z_1), \mu_{\bar{\theta}_2}(z_2)\right\}$$

$$/D = f(x, y_1, y_2, z_1, z_2)$$

If the α- cuts of $f(\bar{\lambda}, \bar{\beta}_1, \bar{\beta}_2, \bar{\theta}_1, \bar{\theta}_2)$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The server is in the working vacation period

$P_0 = \sum_{k=0}^{\infty} \pi_{k0} = K\dfrac{1}{1-r}$,

$$P_0 = K\left(\frac{2\mu_v - (A)}{2\mu_v}\right).$$

Where,

$A = \lambda + \theta_w + \mu_v - \sqrt{(\lambda + \theta_w + \mu_v)^2 - 4\lambda\mu_v}$.

and,

$$K = \frac{2\mu_v - A}{2\mu_v}\left(\frac{\theta_v\mu_b - \lambda\theta_v}{\lambda\mu_b + \theta_v\mu_b}\right)\left[\left(\frac{\theta_v\mu_b - \theta_v\lambda}{\lambda\mu_b + \theta_v\mu_b}\right) + \left(\frac{2\mu_v - A}{2\mu_v}\right)\left(\frac{\theta_w\mu_b - \theta_w\lambda}{\mu_b\lambda}\right) + \frac{2\theta_w\mu_v - A\theta_w}{2\mu_v\mu_b} + \left(\frac{\theta_w\theta_v A}{2\lambda\mu_v + 2\mu_v\theta_v}\right)\left(\frac{2\mu_v}{2\mu_v\mu_b - A\mu_b}\right)\right]^{-1}, 0 < r < 1$$

The server is in the vacation period

$P_1 = \sum_{n=1}^{\infty} \pi_{k1} = K\frac{\theta_w}{\lambda}\frac{1}{1-\beta}$,

$$P_1 = \sum_{n=1}^{\infty} \pi_{k1} = K\frac{\theta_w}{\lambda}\left(\frac{\lambda + \theta_v}{\theta_v}\right).$$

The server is in the regular service period

$$P_2 = \sum_{k=1}^{\infty} \pi_{k2} = K\left[\frac{\theta_w r}{\mu_b(1-r)^2(1-\rho)} + \frac{\theta_w}{\mu_b(1-\beta)(1-\rho)}\right],$$

$$P_2 = K\left[\frac{2A\theta_w\mu\nu}{(2\mu\nu-A)^2(\mu_b-\lambda)} + \frac{\theta_w(\lambda\mu_b+\theta_\nu\mu_b)}{\mu_b(\theta_\nu\mu_b-\lambda\theta_\nu)}\right].$$

We obtain the membership function of some performance measures namely the server is in the working vacation period, the server is in the vacation period, the server is in the regular service period for the system in terms of this membership function are, as follows:

$$\mu_{\overline{P_0}}(B) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\beta_1})\\ y_2\in S(\bar{\beta_2})\\ z_1\in S(\bar{\theta_1})\\ z_2\in S(\bar{\theta_2})}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta_1}}(y_1), \mu_{\bar{\beta_2}}(y_2), \mu_{\bar{\theta_1}}(z_1), \mu_{\bar{\theta_2}}(z_2)/B\right\}, \tag{3.1}$$

Where, $B = K\left(\frac{2y_2-A}{2y_2}\right)$, and,

$$K = \frac{2y_2-A}{2y_2}\left(\frac{z_1y_1-xz_1}{xy_1+z_1y_1}\right)\left[\left(\frac{z_1y_1-xz_1}{xy_1+z_1y_1}\right)+\left(\frac{2y_2-A}{2y_2}\right)\right.$$
$$\left(\frac{z_2y_1-xz_1}{xy_1}\right)+\left(\frac{2y_2z_2-Az_2}{2y_1y_2}\right)+\left(\frac{y_2z_2A}{2(xy_2+y_2z_1)}\right)$$
$$\left.\left(\frac{2y_2}{2y_1y_2-Ay_1}\right)\right]^{-1}.$$

Where,
$A = x+z_2+y_2-\sqrt{(x+z_2+y_2)^2-4xy_2}$

$$\mu_{\overline{P_1}}(C) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\beta}_1)\\ y_2\in S(\bar{\beta}_2)\\ z_1\in S(\bar{\theta}_1)\\ z_2\in S(\bar{\theta}_2)}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}_1}(y_1), \mu_{\bar{\beta}_2}(y_2), \mu_{\bar{\theta}_1}(z_1), \mu_{\bar{\theta}_2}(z_2)/C\right\}, \tag{3.2}$$

Where,
$$C = K\,\frac{z_2}{x}\left(\frac{x+z_1}{z_1}\right).$$

$$\mu_{\overline{P_2}}(D) = \sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\beta}_1)\\ y_2\in S(\bar{\beta}_2)\\ z_1\in S(\bar{\theta}_1)\\ z_2\in S(\bar{\theta}_2)}} \left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}_1}(y_1), \mu_{\bar{\beta}_2}(y_2), \mu_{\bar{\theta}_1}(z_1), \mu_{\bar{\theta}_2}(z_2)/D\right\}, \tag{3.3}$$

Where,
$$D = K\left[\frac{2Ay_2z_2}{(2y_2-A)^2(y_1-x)} + \frac{z_2(xy_1+y_1z_1)}{y_1(y_1z_1-xz_1)}\right].$$

Using the fuzzy analysis technique explain, we can find the membership of $\overline{P_0}$, $\overline{P_1}$ and $\overline{P_2}$ as a function of the parameter α. Thus the α-cut approach can be used to develop the membership function of $\overline{P_0}$, $\overline{P_1}$ and $\overline{P_2}$.

4. Performance measure of interest

The following performance measure are studied for this model in fuzzy environment.

The server is in the working vacation period

Based on Zadeh's extension principle, $\mu_{P_0}(B)$ is the superimum of minimum over
$\left\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}_1}(y_1),\ \mu_{\bar{\beta}_2}(y_2),\ \mu_{\bar{\theta}_1}(z_1),\ \mu_{\bar{\theta}_2}(z_2)\right\}$
$: B = f(x, y_1, y_2, z_1, z_2)$ to satisfying
$\mu_{\overline{P_0}}(B) = \alpha,\ \ 0 < \alpha \leq 1.$

The following five case arise:

Case(i) : $\mu_{\bar{\lambda}}(x) = \alpha,\ \mu_{\bar{\beta}_1}(y_1) \geq \alpha,\ \mu_{\bar{\beta}_2}(y_2) \geq \alpha,$
$\mu_{\bar{\theta}_1}(z_1) \geq \alpha,\ \mu_{\bar{\theta}_2}(z_2) \geq \alpha,$
Case(ii) : $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\beta}_1}(y_1) = \alpha,\ \mu_{\bar{\beta}_2}(y_2) \geq \alpha,$
$\mu_{\bar{\theta}_1}(z_1) \geq \alpha,\ \mu_{\bar{\theta}_2}(z_2) \geq \alpha,$
Case(iii) : $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\beta}_1}(y_1) \geq \alpha,\ \mu_{\bar{\beta}_2}(y_2) = \alpha,$
$\mu_{\bar{\theta}_1}(z_1) \geq \alpha,\ \mu_{\bar{\theta}_2}(z_2) \geq \alpha,$
Case(iv) : $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\beta}_1}(y_1) \geq \alpha,\ \mu_{\bar{\beta}_2}(y_2) \geq \alpha,$
$\mu_{\bar{\theta}_1}(z_1) = \alpha,\ \mu_{\bar{\theta}_2}(z_2) \geq \alpha,$
Case(v) : $\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\beta}_1}(y_1) \geq \alpha,\ \mu_{\bar{\beta}_2}(y_2) \geq \alpha,$
$\mu_{\bar{\theta}_1}(z_1) \geq \alpha,\ \mu_{\bar{\theta}_2}(z_2) = \alpha.$

For case (i), the lower and upper bound of α- cuts of $\overline{P_0}$ can be obtained through the corresponding parametric nonlinear programs,

$$[\overline{P_0}]_\alpha^{L_1} = \min_{\Omega}\left\{\left[K\left(\frac{2y_2-A}{2y_2}\right)\right]\right\}$$

and

$$[\overline{P_0}]_\alpha^{U_1} = \max_{\Omega}\left\{\left[K\left(\frac{2y_2-A}{2y_2}\right)\right]\right\}.$$

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{P_0}$ for the case $(ii), (iii), (iv)$ and (v).

By considering the cases, simulatuosly the lower and upper bounds of the α-cuts of $\overline{P_0}$ can be written as,

$$[\overline{P_0}]_\alpha^{L} = \min_{\Omega}\left\{\left[K\left(\frac{2y_2-A}{2y_2}\right)\right]\right\}$$

and

$$[\overline{P_0}]_\alpha^{U} = \max_{\Omega}\left\{\left[K\left(\frac{2y_2-A}{2y_2}\right)\right]\right\}.$$

such that

$$x_\alpha^L \leq x \leq x_\alpha^U,\ {y_1}_\alpha^L \leq y_1 \leq {y_1}_\alpha^U,\ {y_2}_\alpha^L \leq y_2 \leq {y_2}_\alpha^U,$$
$${z_1}_\alpha^L \leq z_1 \leq {z_1}_\alpha^U,\ {z_2}_\alpha^L \leq z_2 \leq {z_2}_\alpha^U.$$

If both $(\overline{P_0})_\alpha^L$ and $(\overline{P_0})_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(B) = [(P_0)_\alpha^L]^{-1}$ and $R(B) = [(P_0)_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{P_0}}(B)$ can be constructed as,

$$\mu_{\overline{P_0}}(B) = \begin{cases} L(B), & (P_0)_{\alpha=0}^L \leq B \leq (P_0)_{\alpha=0}^U \\ 1, & (P_0)_{\alpha=1}^L \leq B \leq (P_0)_{\alpha=1}^U \\ R(B), & (P_0)_{\alpha=1}^L \leq B \leq (P_0)_{\alpha=0}^U \end{cases} \tag{4.1}$$

In the same way as we said before we get the following results.

The server is in the vacation period

$$\mu_{\overline{P_1}}(C) = \begin{cases} L(C), & (P_1)^L_{\alpha=0} \leq C \leq (P_1)^U_{\alpha=0} \\ 1, & (P_1)^L_{\alpha=1} \leq C \leq (P_1)^U_{\alpha=1} \\ R(C), & (P_1)^L_{\alpha=1} \leq C \leq (P_1)^U_{\alpha=0} \end{cases} \tag{4.2}$$

The server is in the regular service period

$$\mu_{\overline{P_2}}(D) = \begin{cases} L(D), & (P_2)^L_{\alpha=0} \leq D \leq (P_2)^U_{\alpha=0} \\ 1, & (P_2)^L_{\alpha=1} \leq D \leq (P_2)^U_{\alpha=1} \\ R(D), & (P_2)^L_{\alpha=1} \leq D \leq (P_2)^U_{\alpha=0} \end{cases} \tag{4.3}$$

5. Numerical study

The server is in the working vacation period

Suppose the arrival rate $\overline{\lambda}$, regular service rate $\overline{\beta_1}$, lower service rate $\overline{\beta_2}$, vacation time $\overline{\theta_1}$ and working vacation rate $\overline{\theta_2}$ are assumed to be Pentagonal fuzzy numbers described by: $\overline{\lambda} = [1,2,3,4,5]$, $\overline{\beta_1} = [6,7,8,9,10]$, $\overline{\beta_2} = [11,12,13,14,15]$, $\overline{\theta_1} = [16,17,18,19,20]$ & $\overline{\theta_2} = [21,22,23,24,25]$ per hours respectively.
Then,

$$\lambda(\alpha) = \min_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\}, \max_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\}.$$

where

$$G(x) = \begin{cases} 0 & ,if \ \ x \leq a_1 \\ 1-(1-r)\dfrac{x-a_2}{a_3-a_2} & ,if \ a_2 \leq \ x \leq a_3 \\ 1 \ if \ x = a_3 & \\ 1-(1-r)\dfrac{a_4-x}{a_4-a_3} & ,if \ a_3 \leq \ x \leq a_4 \\ r\dfrac{a_5-x}{a_5-a_4} & ,if \ a_4 \leq \ x \leq a_5 \\ 0 & ,if \ x \geq a_5 \end{cases}$$

(i,e).,
$\lambda(\alpha) = [1+\alpha, 5-\alpha]$, $\beta_1(\alpha) = [6+\alpha, 10-\alpha]$,
$\beta_2(\alpha) = [11+\alpha, 15-\alpha]$, $\theta_1(\alpha) = [16+\alpha, 20-\alpha]$
& $\theta_2(\alpha) = [21+\alpha, 25-\alpha]$.
It is clear that, when $x = x^U_\alpha$, $y_1 = y_1{}^U_\alpha$, $y_2 = y_2{}^U_\alpha$, $z_1 = z_1{}^U_\alpha$, & $z_2 = z_2{}^U_\alpha$ B attains its maximum value and when $x = x^L_\alpha$, $y_1 = y_1{}^L_\alpha$, $y_2 = y_2{}^L_\alpha$, $z_1 = z_1{}^L_\alpha$ & $z_2 = z_2{}^L_\alpha$ B attains its minimum value.
From the generated for the given input value of $\overline{\lambda}$, $\overline{\beta}_1$, $\overline{\beta}_2$, $\overline{\theta}_1$ & $\overline{\theta}_2$.

i) For fixed values of x, y_1, y_2 & z_1, B decreases as z_2 increase.
ii) For fixed values of y_1, y_2, z_1 & z_2, B decreases as x increase.
iii) For fixed values of y_2, z_1, z_2 & x, B decreases as y_1 increase.
iv) For fixed values of z_1, z_2, x & y_1, B decreases as y_2 increase.
v) For fixed values of z_2, x, y_1 & y_2, B decreases as z_1 increase.

The smallest value of occurs when x-takes its lower bound. i.e)., $x = 1+\alpha$ and y_1, y_2, z_1 and z_2 take their upper bounds given by $y_1 = 10-\alpha$, $y_2 = 15-\alpha$, $z_1 = 20-\alpha$ and $z_2 = 25-\alpha$ respectively. And maximum value of P_0 occurs when $x = 5-\alpha, y_1 = 6+\alpha$, $y_2 = 11+\alpha$, $z_1 = 16+\alpha$ and $z_2 = 21+\alpha$. If both $[P_0]^L_\alpha$ & $[P_0]^\alpha_U$ are invertible with respect to $'\alpha'$ then, the left shape function $L(B) = [(P_0)^L_\alpha]^{-1}$ and right shape function $R(B) = [(P_0)^U_\alpha]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{P_0}}(B)$ can be constructed as:

$$\mu_{\overline{P_0}}(B) = \begin{cases} 0, & if \ \ B \leq B_1 \\ 0.4(x-2), & if \ B_1 \leq B \leq B_2, \\ 0.4(4-x), & if \ B_2 \leq B \leq B_3, \\ 0.4(5-x), & if \ B_3 \leq B \leq B_4, \\ 0, & if \ B \leq B_5 \end{cases} \tag{5.1}$$

The values of B_1, B_2, B_3, B_4 and B_5 as obtained from (5.1) are:

$$\mu_{\overline{P_1}}(B) = \begin{cases} 0, & if \ B \leq 0.0000 \\ 0.4(4-x), & if \ 0.0000 \leq B \leq 0.3980, \\ 1, & if \ x = 1 \\ 0.4(4-x), & if \ 0.3980 \leq B \leq 0.7321, \\ 0.4(5-x), & if \ 0.7321 \leq B \leq 0.4381, \\ 0, & if \ B \geq 0.0000 \end{cases}$$

In the same way we get the following results.

The server is in the vacation period

$$\mu_{\overline{P_1}}(C) = \begin{cases} 0, & if \ \ C \leq C_1 \\ 0.6(x-2), & if \ C_1 \leq C \leq C_2, \\ 0.6(4-x), & if \ C_2 \leq C \leq C_3, \\ 0.6(5-x), & if \ C_3 \leq C \leq C_4, \\ 0, & if \ C \geq C_5 \end{cases} \tag{5.2}$$

The values of C_1, C_2, C_3, C_4 and C_5 as obtained from (5.2) are:

$$\mu_{\overline{P_1}}(C) = \begin{cases} 0, & if \ C \leq 0.0000 \\ 0.6(2-x), & if \ 0.0000 \leq C \leq 0.6001, \\ 0.6(4-x), & if \ 0.6001 \leq C \leq 0.9965, \\ 0.6(5-x), & if \ 0.9965 \leq C \leq 0.5951, \\ 0, & if \ C \geq 0.0000 \end{cases}$$

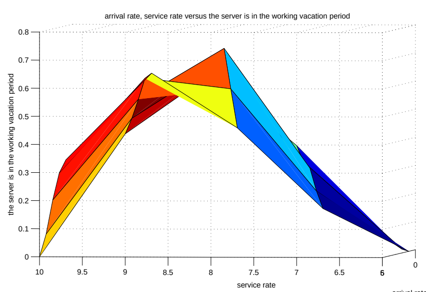
The server is in the regular service period

$$\mu_{\overline{P_1}}(D) = \begin{cases} 0, & if \quad D \leq D_1 \\ -58(x-2), & if\ D_1 \leq D \leq D_2, \\ -58(4-x), & if\ D_2 \leq D \leq D_3, \\ 60(5-x), & if\ D_3 \leq D \leq D_4, \\ 0, & if\ D \geq D_5 \end{cases} \qquad (5.3$$

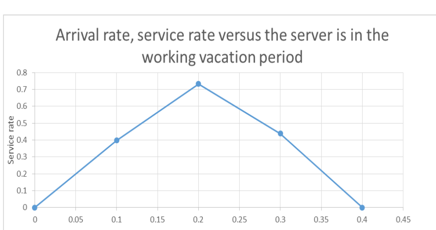
The values of D_1, D_2, D_3, D_4 and D_5 as obtained from (5.3) are:

$$\mu_{\overline{P_1}}(D) = \begin{cases} 0, & if\ D \leq 0.0000 \\ -58(2-x), & if\ 0.0000 \leq D \leq 59.0721, \\ -58(4-x), & if\ 59.0721 \leq D \leq 98.2739, \\ 60(5-x), & if\ 98.2739 \leq D \leq 61.0028, \\ 0, & if\ D \geq 0.0000 \end{cases}$$

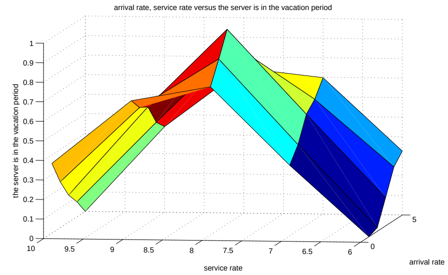
The following three graphs are represent the performanc measures.



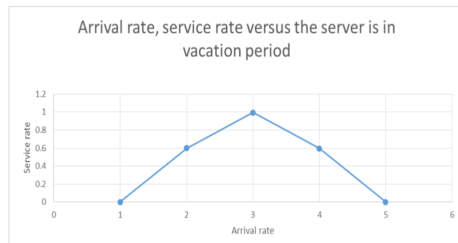
arrival rate, service rate versus the server is in the working vacation period



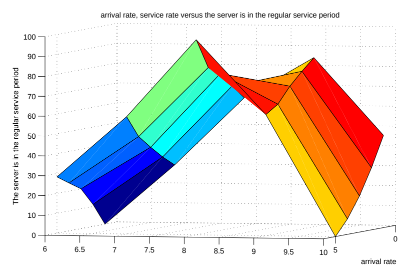
arrival rate, service rate versus the server is in the working vacation period



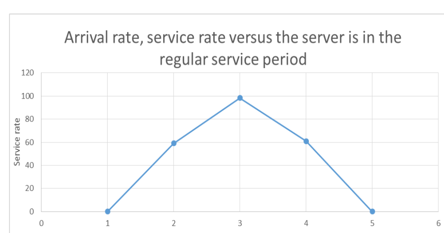
arrival rate, service rate versus the server is in the vacation period



arrival rate, service rate versus the server is in the vacation period



arrival rate, service rate versus the server is in the regular service period



arrival rate, service rate versus the server is in the regular service period

6. Conclusion

In this paper we have studied the analysis of the M/M/1 queue with two vacation policies using Pentagonal fuzzy numbers. We have obtained the server is in the working vacation period, server is in the vacation period, the server is in the regular service period. Consider the examples for these fuzzy queues models. The software company working more than 2000 employees, there are taking vacation for traditional holidays like Christmas, Diwali etc and medical leaves. Both polices are a far cry from the days when two weeks vacation and eight fixed holidays were the norm. Each employee is given Rs.15000 a year to spend on airfare, hotels, meals, petrol and other vacation related expenses. We have obtained numerical results to all the performance measures for this fuzzy queues.

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