On continuous selections of weakly $(1,2)^*$-$\ddot{g}$-closed maps

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Abstract

In this paper, the concepts of weakly $(1,2)^*$-$\ddot{g}$-continuous maps, weakly $(1,2)^*$-$\ddot{g}$-compact spaces and weakly $(1,2)^*$-$\ddot{g}$-connected spaces are introduced and some of their properties are investigated.

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Contents

1. Introduction and Preliminaries

Quite recently, several authors investigated some new maps and notions (see for example [14], [10]. It is the objective of this paper, the concepts of weakly $(1,2)^*$-$\ddot{g}$-continuous maps, weakly $(1,2)^*$-$\ddot{g}$-compact spaces and weakly $(1,2)^*$-$\ddot{g}$-connected spaces are introduced and some of their properties are investigated with addition Examples.
Throughout this paper, (X,τ_1,τ_2), (Y,τ_1,τ_2) and (Z,η_1,η_2) (briefly, X, Y and Z) will denote bitopological spaces.

Definition 1.1. *Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [8] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.*

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 1.2. *[8] Let S be a subset of a bitopological space X. Then*

1. *the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-$cl(S)$, is defined as $\cap\{F : S \subseteq F$ and F is $\tau_{1,2}$-closed $\}$.*

2. *the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-$int(S)$, is defined as $\cup\{F : F \subseteq S$ and F is $\tau_{1,2}$-open $\}$.*

Definition 1.3. *A subset S of a bitopological space X is called*

1. *$(1,2)^*$-generalized closed (briefly, $(1,2)^*$-g-closed) set [11] if $\tau_{1,2}$-$cl(S) \subseteq U$ whenever $S \subseteq U$ and U is $\tau_{1,2}$-open in X.*

2. *$(1,2)^*$-semi-generalized closed (briefly, $(1,2)^*$-sg-closed) set [9] if $(1,2)^*$-$scl(S) \subseteq U$ whenever $S \subseteq U$ and U is $(1,2)^*$-semi-open in X.*

3. *$(1,2)^*$-α-generalized closed (briefly, $(1,2)^*$-αg-closed) set [12] if $(1,2)^*$-$\alpha cl(S) \subseteq U$ whenever $S \subseteq U$ and U is $\tau_{1,2}$-open in X.*

4. *$(1,2)^*$-$\ddot{g}$-closed set [3] if $\tau_{1,2}$-$cl(S) \subseteq U$ whenever $S \subseteq U$ and U is $(1,2)^*$-sg-open in X.*

5. *$(1,2)^*$-πg-closed set [10] if $\tau_{1,2}$-$cl(S) \subseteq U$ whenever $S \subseteq U$ and U is $(1,2)^*$-π-open in X.*

The complements of the above mentioned open sets are called their respective closed sets.

The family of all $(1,2)^*$-$\ddot{g}$-open (resp. $(1,2)^*$-$\ddot{g}$-closed) sets in X is denoted by $(1,2)^*$-$\ddot{G}O(X)$ (resp. $(1,2)^*$-$\ddot{G}C(X)$).

Definition 1.4. *[4] For every set $S \subseteq X$, we define the $(1,2)^*$-$\ddot{g}$-closure of S to be the intersection of all $(1,2)^*$-$\ddot{g}$-closed sets containing S. That is $(1,2)^*$-$\ddot{g}$-$cl(S) = \cap\{F : S \subseteq F \in (1,2)^*$-$\ddot{G}C(X)\}$.*

Definition 1.5. *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces. A function* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called*

1. *completely* $(1,2)^*$*-continuous [10] (resp.* $(1,2)^*$*-R-map [10]) if* $f^{-1}(V)$ *is regular* $(1,2)^*$*-open in X for each* $\sigma_{1,2}$*-open (resp. regular* $(1,2)^*$*-open) set V of Y.*

2. *perfectly* $(1,2)^*$*-continuous [10] if* $f^{-1}(V)$ *is both* $\tau_{1,2}$*-open and* $\tau_{1,2}$*-closed in X for each* $\sigma_{1,2}$*-open set V of Y.*

3. $(1,2)^*$*-$\ddot{g}$-continuous [4] if* $f^{-1}(V)$ *is* $(1,2)^*$*-$\ddot{g}$-closed in X for every* $\sigma_{1,2}$*-closed set V of Y.*

4. $(1,2)^*$*-$\ddot{g}$-irresolute [4] if* $f^{-1}(V)$ *is* $(1,2)^*$*-$\ddot{g}$-closed in X for every* $(1,2)^*$*-$\ddot{g}$-closed set V of Y.*

5. $(1,2)^*$*-sg-irresolute [13] if* $f^{-1}(V)$ *is* $(1,2)^*$*-sg-open in X for every* $(1,2)^*$*-sg-open set V of Y.*

6. $(1,2)^*$*-$\ddot{g}$-closed [4] if the image of every* $\tau_{1,2}$*-closed set in X is* $(1,2)^*$*-$\ddot{g}$-closed in Y.*

Definition 1.6. *A subset S of a bitopological space X is called*

1. *weakly* $(1,2)^*$*-g-closed (briefly,* $(1,2)^*$*-wg-closed) set [14] if* $\tau_{1,2}$*-cl$(\tau_{1,2}$-int$(S)) \subseteq U$ whenever* $S \subseteq U$ *and U is* $\tau_{1,2}$*-open in X.*

2. *weakly* $(1,2)^*$*-πg-closed (briefly,* $(1,2)^*$*-wπg-closed) set [14] if* $\tau_{1,2}$*-cl$(\tau_{1,2}$-int$(S)) \subseteq U$ whenever* $S \subseteq U$ *and U is* $(1,2)^*$*-πg-open in X.*

3. *regular weakly* $(1,2)^*$*-generalized closed (briefly,* $(1,2)^*$*-rwg-closed) set [14] if* $\tau_{1,2}$*-cl$(\tau_{1,2}$-int$(S)) \subseteq U$ whenever* $S \subseteq U$ *and U is regular* $(1,2)^*$*-open in X.*

Definition 1.7. *[4] A function* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is said to be an* $(1,2)^*$*-$\ddot{g}$-open map if the image* $f(S)$ *is* $(1,2)^*$*-$\ddot{g}$-open in Y for each* $\tau_{1,2}$*-open set S of X.*

Remark 1.8. *[4] Every* $\tau_{1,2}$*-open set is* $(1,2)^*$*-sg-open but not conversely.*

Remark 1.9. *[14] For a subset of a bitopological space, we have following implications:*

regular $(1,2)^*$*-open* $\rightarrow (1,2)^*$*-π-open* $\rightarrow \tau_{1,2}$*-open*

Definition 1.10. *A subset S of a bitopological space X is said to be nowhere dense if* $\tau_{1,2}$*-int$(\tau_{1,2}$-cl$(S)) = \phi$.*

Definition 1.11. *Let* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *be a function. Then f is said to be*

1. *contra-*$(1,2)^*$*-$\ddot{g}$-continuous [14] if* $f^{-1}(V)$ *is* $(1,2)^*$*-$\ddot{g}$-closed in X for every* $\sigma_{1,2}$*-open set of Y.*

2. $(1,2)^*$*-continuous [14] if* $f^{-1}(V)$ *is* $\tau_{1,2}$*-closed in X for every* $\sigma_{1,2}$*-closed set of Y.*

Remark 1.12. *[4] Every* $(1,2)^*$*-continuous function is* $(1,2)^*$*-$\ddot{g}$-continuous but not conversely.*

Definition 1.13. *[5] A subset S of a bitopological space X is called a weakly* $(1,2)^*$*-$\ddot{g}$-closed (briefly,* $(1,2)^*$*-w$\ddot{g}$-closed) set if* $\tau_{1,2}$*-cl$(\tau_{1,2}$-int$(S)) \subseteq U$ whenever* $S \subseteq U$ *and U is* $(1,2)^*$*-sg-open in X.*

Corollary 1.14. *[5] If a subset S of a bitopological space X is both* $\tau_{1,2}$*-open and* $(1,2)^*$*- w$\ddot{g}$-closed, then it is both regular* $(1,2)^*$*-open and regular* $(1,2)^*$*-closed in X.*

Proposition 1.15. *[5] Every* $(1,2)^*$ *-$\ddot{g}$-open set is* $(1,2)^*$ *-w$\ddot{g}$-open but not conversely.*

2. Weakly $(1,2)^*$-$\ddot{g}$-Continuous maps

Definition 2.1. *A map* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called* $((1,2)^*$*-$\ddot{g}, s)$*-continuous if the inverse image of each regular* $(1,2)^*$*-open set of Y is* $(1,2)^*$*-$\ddot{g}$-closed in X.*

Definition 2.2. *A space X is called* $(1,2)^*$*-$\ddot{g}$-connected if X is not the union of two disjoint nonempty* $(1,2)^*$*-$\ddot{g}$-open sets.*

Definition 2.3. *A subset of a bitopological space X is said to be*

1. *almost* $(1,2)^*$*-connected if X cannot be written as a disjoint union of two non-empty regular* $(1,2)^*$*-open sets.*

2. $(1,2)^*$*-connected if X cannot be written as a disjoint union of two non-empty* $\tau_{1,2}$*-open sets.*

Definition 2.4. *Let X and Y be two bitopological spaces. A map* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \tau_1, \tau_2)$ *is called weakly* $(1,2)^*$*-$\ddot{g}$-continuous (briefly* $(1,2)^*$*-w$\ddot{g}$-continuous) if* $f^{-1}(U)$ *is a* $(1,2)^*$*-w$\ddot{g}$-open set in X for each* $\sigma_{1,2}$*-open set U of Y.*

Example 2.5. *Let* $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$ *and* $\tau_2 = \{\phi, X, \{b, c\}\}$*. Then the sets in* $\{\phi, \{a\}, \{b, c\}, X\}$ *is called* $\tau_{1,2}$*-open. Let* $\sigma_1 = \{\phi, Y, \{a\}\}$ *and* $\sigma_2 = \{\phi, Y\}$*. Then the sets in* $\{\phi, \{a\}, Y\}$ *is called* $\sigma_{1,2}$*-open.*

The mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *defined by* $f(a) = b$, $f(b) = c$ *and* $f(c) = a$ *is* $(1,2)^*$*-w$\ddot{g}$-continuous, because every subset of Y is* $(1,2)^*$*-w$\ddot{g}$-closed in X.*

Theorem 2.6. *Every* $(1,2)^*$*-$\ddot{g}$-continuous function is* $(1,2)^*$*-w$\ddot{g}$-continuous.*

Proof. It follows from Proposition 1.15.

The converse of Theorem 2.6 need not be true as seen in the following example.

Example 2.7. *Let* $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$ *and* $\tau_2 = \{\phi, X, \{b, c\}\}$*. Then the sets in* $\{\phi, X, \{a\}, \{b, c\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, X, \{a\}, \{b, c\}\}$ *are called* $\tau_{1,2}$*-closed. Let* $\sigma_1 = \{\phi, Y, \{a, b\}\}$ *and* $\sigma_2 = \{\phi, Y\}$*. Then the sets in* $\{\phi, Y, \{a, b\}\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, Y, \{c\}\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *be the identity function. Then f is* $(1,2)^*$*-w$\ddot{g}$-continuous but not* $(1,2)^*$*-$\ddot{g}$-continuous.*

Theorem 2.8. *A map* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* $(1,2)^*$-
wg̈-continuous if and only if $f^{-1}(U)$ *is a* $(1,2)^*$-*wg̈-closed
set in* X *for each* $\sigma_{1,2}$-*closed set* U *of* Y.

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. According to the
assumption $f^{-1}(U^c) = X \backslash f^{-1}(U)$ is $(1,2)^*$-*wg̈*-open in X, so
$f^{-1}(U)$ is $(1,2)^*$-*wg̈*-closed in X.

The converse can be proved in a similar manner.

Theorem 2.9. *Suppose that* X *and* Y *are bitopological spaces
and* $(1,2)^*$-*G̈C(X) is closed under arbitrary intersection. If a
map* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is contra* $(1,2)^*$-*g̈-continuous
and* Y *is* $(1,2)^*$-*regular, then* f *is* $(1,2)^*$-*g̈-continuous.*

Proof. Let x be an arbitrary point of X and V be an $\sigma_{1,2}$-
open set of Y containing $f(x)$. Since Y is $(1,2)^*$-regular,
there exists an $\sigma_{1,2}$-open set G in Y containing $f(x)$ such that
$\sigma_{1,2}$-$cl(G) \subseteq V$. Since f is contra $(1,2)^*$-*g̈*-continuous, there
exists $U \in (1,2)^*$-*G̈O(X)* containing x such that $f(U) \subseteq \sigma_{1,2}$-
$cl(G)$. Then $f(U) \subseteq \sigma_{1,2}$-$cl(G) \subseteq V$. Hence, f is $(1,2)^*$-*g̈*-
continuous.

Theorem 2.10. *Suppose that* X *and* Y *are bitopological spaces
and the family of* $(1,2)^*$-*g̈-closed sets in* X *is closed under
arbitrary intersection. If a map* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is
contra* $(1,2)^*$-*g̈-continuous and* Y *is* $(1,2)^*$-*regular, then* f *is*
$(1,2)^*$-*wg̈-continuous.*

Proof. The proof is obvious from Theorem .

Definition 2.11. *A bitopological space* X *is said to be locally
$(1,2)^*$-g̈-indiscrete if every* $(1,2)^*$-*g̈-open set of* X *is* $\tau_{1,2}$-
closed in X.

Theorem 2.12. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a func-
tion. If* f *is* $(1,2)^*$-*g̈-continuous and* X *is locally* $(1,2)^*$-*g̈-
indiscrete, then* f *is* $(1,2)^*$-*continuous.*

Proof. Let V be an $\sigma_{1,2}$-open in Y. Since f is $(1,2)^*$-*g̈*-
continuous, $f^{-1}(V)$ is $(1,2)^*$-*g̈*-open in X. Since X is locally
$(1,2)^*$-*g̈*-indiscrete, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Hence f is
$(1,2)^*$-continuous.

Theorem 2.13. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function.
If* f *is contra* $(1,2)^*$-*g̈-continuous and* X *is locally* $(1,2)^*$-*g̈-
indiscrete, then* f *is* $(1,2)^*$-*wg̈-continuous.*

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be contra $(1,2)^*$-*g̈*-
continuous and X is locally *g̈*-indiscrete. By Theorem 2.12,
f is $(1,2)^*$-continuous, then f is $(1,2)^*$-*wg̈*-continuous by
Remark 1.12 and Theorem 2.6.

Corollary 2.14. *Let* Y *be a* $(1,2)^*$-*regular space and* f :
$(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function. Suppose that the collec-
tion of* $(1,2)^*$-*g̈-closed sets in* X *is closed under arbitrary in-
tersections. Then if* f *is* $((1,2)^*$-*g̈,s)-continuous,* f *is* $(1,2)^*$-
wg̈-continuous.

Proof. Let f be $((1,2)^*$-*g̈,s)*-continuous. By Theorem 2.13,
f is $(1,2)^*$-*g̈*-continuous. Thus, f is $(1,2)^*$-*wg̈*-continuous
by Theorem 2.6.

Proposition 2.15. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is perfectly
$(1,2)^*$-continuous and* $(1,2)^*$-*wg̈-continuous, then it is* $(1,2)^*$-
R-map.

Proof. Let V be any regular $(1,2)^*$-open subset of Y. Ac-
cording to the assumption, $f^{-1}(V)$ is both $\tau_{1,2}$-open and
$\tau_{1,2}$-closed in X. Since $f^{-1}(V)$ is $\tau_{1,2}$-closed, it is $(1,2)^*$-
wg̈-closed. We have $f^{-1}(V)$ is both $\tau_{1,2}$-open and $(1,2)^*$-*wg̈*-
closed. Hence, by Corollary 1.14, it is regular $(1,2)^*$-open in
X, so f is $(1,2)^*$-*R*-map.

Definition 2.16. *A bitopological space* X *is called* $(1,2)^*$-
g̈-compact (resp. $(1,2)^*$-*compact) if every cover of* X *by
$(1,2)^*$-g̈-open (resp.* $\tau_{1,2}$-*open) sets has finite subcover.*

Definition 2.17. *A bitopological space* X *is weakly* $(1,2)^*$-
g̈-compact (briefly, $(1,2)^*$-*wg̈-compact) if every* $(1,2)^*$-*wg̈-
open cover of* X *has a finite subcover.*

Remark 2.18. *Every* $(1,2)^*$-*wg̈-compact space is* $(1,2)^*$-*g̈-
compact.*

Theorem 2.19. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be surjective
$(1,2)^*$-wg̈-continuous map. If* X *is* $(1,2)^*$-*wg̈-compact, then
Y *is* $(1,2)^*$-*compact.*

Proof. Let $\{A_i : i \in I\}$ be an $\sigma_{1,2}$-open cover of Y. Then
$\{f^{-1}(A_i) : i \in I\}$ is a $(1,2)^*$-*wg̈*-open cover in X. Since X is
$(1,2)^*$-*wg̈*-compact, it has a finite subcover, say $\{f^{-1}(A_1),$
$f^{-1}(A_2), ..., f^{-1}(A_n)\}$. Since f is surjective $\{A_1, A_2, ..., A_n\}$
is a finite subcover of Y and hence Y is $(1,2)^*$-compact.

Definition 2.20. *A bitopological space* X *is weakly* $(1,2)^*$-
g̈-connected (briefly, $(1,2)^*$-*wg̈-connected) if* X *cannot be
written as the disjoint union of two non-empty* $(1,2)^*$-*wg̈-
open sets.*

Theorem 2.21. *If a bitopological space* X *is* $(1,2)^*$-*wg̈-connected,
then* X *is almost* $(1,2)^*$-*connected (resp.* $(1,2)^*$-*g̈-connected).*

Proof. It follows from the fact that each regular $(1,2)^*$-open
set (resp. $(1,2)^*$-*g̈*-open set) is $(1,2)^*$-*wg̈*-open.

Theorem 2.22. *For a bitopological space* X, *the following
statements are equivalent:*

1. *X is* $(1,2)^*$-*wg̈-connected.*

2. *The empty set* ϕ *and* X *are only subsets which are both
$(1,2)^*$-wg̈-open and* $(1,2)^*$-*wg̈-closed.*

3. *Each* $(1,2)^*$-*wg̈-continuous function from* X *into a dis-
crete bitopological space* Y *which has at least two
points is a constant function.*

Proof. $(1) \Rightarrow (ii)$. Let $S \subseteq X$ be any proper subset, which is both $(1,2)^*$-$w\ddot{g}$-open and $(1,2)^*$-$w\ddot{g}$-closed. Its complement $X \backslash S$ is also $(1,2)^*$-$w\ddot{g}$-open and $(1,2)^*$-$w\ddot{g}$-closed. Then $X = S \cup (X \backslash S)$ is a disjoint union of two non-empty $(1,2)^*$-$w\ddot{g}$-open sets which is a contradiction with the fact that X is $(1,2)^*$-$w\ddot{g}$-connected. Hence, $S = \phi$ or X.

$(2) \Rightarrow (1)$. Let $X = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are $(1,2)^*$-$w\ddot{g}$-open. Since $A = X \backslash B$, A is $(1,2)^*$-$w\ddot{g}$-closed. According to the assumption $A = \phi$, which is a contradiction.

$(2) \Rightarrow (3)$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$-$w\ddot{g}$-continuous function where Y is a discrete bitopological space with at least two points. Then $f^{-1}(y)$ is $(1,2)^*$-$w\ddot{g}$-closed and $(1,2)^*$-$w\ddot{g}$-open for each $y \in Y$ and $X = \cup \{f^{-1}(y)|y \in Y\}$. According to the assumption, $f^{-1}(y) = \phi$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, f will not be a function. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is a constant function.

$(3) \Rightarrow (2)$. Let $S \neq \phi$ be both $(1,2)^*$-$w\ddot{g}$-open and $(1,2)^*$-$w\ddot{g}$-closed in X. Let $f : X \rightarrow Y$ be a $(1,2)^*$-$w\ddot{g}$-continuous function defined by $f(S) = \{a\}$ and $f(X \backslash S) = \{b\}$ where $a \neq b$. Since f is constant function we get $S = X$.

Theorem 2.23. *Let* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *be a* $(1,2)^*$-$w\ddot{g}$-*continuous surjective function. If* X *is* $(1,2)^*$-$w\ddot{g}$-*connected, then* Y *is* $(1,2)^*$-*connected.*

Proof. We suppose that Y is not $(1,2)^*$-connected. Then $Y = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are $\sigma_{1,2}$-open sets in Y. Since f is $(1,2)^*$-$w\ddot{g}$-continuous surjective function, $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint union of two non-empty $(1,2)^*$-$w\ddot{g}$-open subsets. This is contradiction with the fact that X is $(1,2)^*$-$w\ddot{g}$-connected.

3. Conclusion

The notions of sets and functions in bitopological spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences. By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all bitopological sets and functions defined will have many possibilities of applications in digital topology and computer graphics.

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