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# Multiplicative indices of $TUC_4C_6C_8[m,n]$ nanotube and $C_4C_6C_8[m,n]$ nanotori

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**Abstract**

Chemical graph theory is a branch of graph theory. Topological indices of molecular graph correlate with chemical properties of the chemical molecules. In this article we compute the degree based topological indices like multiplicative first and second Zagreb, multiplicative first and second hyper Zagreb, general first and second multiplicative Zagreb, multiplicative sum connectivity, multiplicative product connectivity, general multiplicative Zagreb, multiplicative geometric arithmetic indices of $TUC_4C_6C_8[m,n]$ and $C_4C_6C_8[m,n]$ nanotori.

**Keywords**

Molecular graph, topological index, multiplicative indices, nanotubes.

**AMS Subject Classification**

05C05, 05C07, 05C35, 05C90.

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## Contents



## 1. Introduction and Preliminaries

Let G be a simple connected graph with its vertex set is denoted by $V(G)$ and edge set is denoted by $E(G)$. Also in chemical graph theory, the points are corresponds to vertices and the lines corresponds to edges respectively. A single number, which indicates the property of the graph of molecular, is said to be topological index of a graph. In theoretical chemistry, topological indices are useful for modeling physical, pharmacological, biological and other properties of chemical compounds [1, 2]. The degree $d_G(v)$ is the number of adjacent vertices of v. The degree of an edge $e = uv$ in G is defined by $d_G(u) + d_G(v) - 2$. The line graph $L(G)$ which having vertex set represents the edges of G also two vertices are $L(G)$ adjacent if corresponding edges of G are adjacent [3–5].

**First and Second multiplicative Zagreb indices [6]:**

$$II_1(G) = \prod_{u\in V(G)} d_G(u)^2,$$

$$II_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v).$$

**New multiplicative version of first Zagreb index [7]:**

$$II_1^*(G) = \prod_{uv\in E(G)} [d_G(u) + d_G(v)]$$

**First and second multiplicative hyper Zagreb indices [8]:**

$$HII_1(G) = \prod_{uv\in E(G)} [d_G(u) + d_G(v)]^2$$

$$HII_2(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^2.$$

**General first and second multiplicative hyper Zagreb indices [9]:**

$$MZ_1^a(G) = \prod_{uv\in E(G)} [d_G(u) + d_G(v)]^a$$

$$MZ_2^a(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^a.$$

**Multiplicative sum and product connectivity index [10]:**

$$XII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$$

$$\chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

**Multiplicative geometric-arithmetic index [10]:**

$$GAII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}.$$

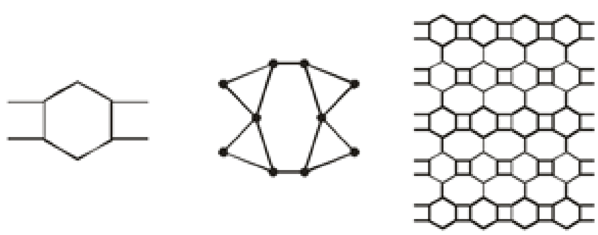
**General multiplicative geometric-arithmetic index [10]:**

$$GA^aII(G) = \prod_{uv\in E(G)} \left(\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}\right)^a$$

The multiplicative connectivity indices of certain nanotubes and nanotori were studied already in [11, 12]. Also various topological indices like eccentric connectivity index, augmented eccentric connectivity Index were studied in [13–15]. Now we compute multiplicative indices for $TUC_4C_6C_8[m,n]$ and $C_4C_6C_8[m,n]$ nanotori.

## 2. Results for $TUC_4C_6C_8[m,n]$ nanotube

We consider the $TUC_4C_6C_8[m,n]$ nanotube. Here figure (a) represents the 2-D lattice of $TUC_4C_6C_8[1,1]$. Figure (b). Represents the line graph of $TUC_4C_6C_8[1,1]$. And figure (c) represents the $TUC_4C_6C_8[4,5]$ nanotube. Generally, we consider the graph $TUC_4C_6C_8[m,n]$ nanotube with m columns and n rows.



(a) (b) (c)

**Lemma 2.1.** *Let G be a $TUC_4C_6C_8[m,n]$ nanotube with m columns and n rows. Consider the line graph of $TUC_4C_6C_8[m,n]$. Then*

(1). *The number of edges are $2m$ having degree (3, 3). It is denoted by $E_1$.*

(2). *The number of edges are $8m$ having degree (3, 4). It is denoted by $E_2$.*

(3). *The number of edges are $18mn-14m$ having degree (4, 4). It is denoted by $E_3$.*

*In addition, total number of edges is $18mn-4m$.*

**Theorem 2.2.** *Let G be a $TUC_4C_6C_8[m,n]$ nanotube with m columns and n rows. Then*

(1). $II_1^*(G) = 3^{2m}\times 7^{8m}\times 2^{m(54n-40)}$

(2). $II_2(G) = 3^{12m}\times 4^{4m(4n-5)}$

(3). $HII_1(G) = 3^{4m}\times 7^{16m}\times 2^{m(78n-80)}$

(4). $HII_2(G) = 3^{24m}\times 4^{8m(4n-5)}E$

(5). $XII(G) = 3^{-m}\times 7^{-4m}\times 2^{20m-27mn}$

(6). $\chi II(G) = 3^{-6m}\times 2^{4m(5-9n)}$

(7). $MZ_1^a(G) = 3^{2am}\times 7^{8am}\times 2^{am(54n-40)}$

(8). $MZ_2^a(T) = 3^{12am}\times 4^{4am(4n-5)}$

(9). $GAII(G) = \left(\frac{2\sqrt{12}}{7}\right)^{8m}$

(10). $GA^aII(G) = \left(\frac{2\sqrt{12}}{7}\right)^{8ma}$

*Proof.* From the definitions of multiplicative indices and partition of edges described in Lemma 2.1, we can see that:

(1).
$$\begin{aligned} II_1^*(G) &= \prod_{uv\in E(G)} [d_G(u)+d_G(v)] \\ &= \prod_{uv\in E_1} 6\times \prod_{uv\in E_2} 7\times \prod_{uv\in E_3} 8 \\ &= 6^{2m}\times 7^{8m}\times 8^{18mn-14m} \\ &= 3^{2m}\times 7^{8m}\times 2^{m(54n-40)}. \end{aligned}$$

(2).
$$\begin{aligned} II_2(G) &= \prod_{uv\in E(G)} d_G(u)d_G(v) \\ &= \prod_{uv\in E_1} 9\times \prod_{uv\in E_2} 12\times \prod_{uv\in E_3} 16 \\ &= 9^{2m}\times 12^{8m}\times 16^{8mn-14m} \\ &= 3^{12m}\times 4^{4m(4n-5)}. \end{aligned}$$

(3).
$$\begin{aligned} HII_1(G) &= \prod_{uv\in E(G)} [d_G(u)+d_G(v)]^2 \\ &= \prod_{uv\in E_1} 6^2\times \prod_{uv\in E_2} 7^2\times \prod_{uv\in E_3} 8^2 \\ &= 6^{4m}\times 7^{16m}\times 8^{36mn-28m} \\ &= 3^{4m}\times 7^{16m}\times 2^{m(78n-80)}. \end{aligned}$$

(4).
$$\begin{aligned} HII_2(G) &= \prod_{uv\in E(G)} [d_G(u)d_G(v)]^2 \\ &= \prod_{uv\in E_1} 9^2\times \prod_{uv\in E_2} 12^2\times \prod_{uv\in E_3} 16^2 \\ &= 9^{4m}\times 12^{16m}\times 16^{36mn-28m} \\ &= 3^{24m}\times 4^{8m(4n-5)}. \end{aligned}$$

$$(5).\ XII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$$
$$= \prod_{uv\in E_1} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{7}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{8}}$$
$$= \left(\frac{1}{\sqrt{6}}\right)^{2m} \times \left(\frac{1}{\sqrt{7}}\right)^{8m} \times \left(\frac{1}{\sqrt{8}}\right)^{18mn-14m}$$
$$= 3^{-m} \times 7^{-4m} \times 2^{20m-27mn}$$

$$(6).\ \chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$
$$= \prod_{uv\in E_1} \frac{1}{\sqrt{9}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{12}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{16}}$$
$$= \left(\frac{1}{\sqrt{9}}\right)^{2m} \times \left(\frac{1}{\sqrt{12}}\right)^{8m} \times \left(\frac{1}{\sqrt{14}}\right)^{18mn-14m}$$
$$= 3^{-6m} \times 2^{4m(5-9n)}$$

$$(7).\ MZ_1^a(G) = \prod_{uv\in E(G)} [d_G(u)+d_G(v)]^a$$
$$= \prod_{uv\in E_1} 6^a \times \prod_{uv\in E_2} 7^a \times \prod_{uv\in E_3} 8^a$$
$$= 6^{2ma} \times 7^{8ma} \times 8^{18mna-14ma}$$
$$= 3^{2am} \times 7^{8am} \times 2^{am(54n-40)}$$

$$(8).\ MZ_2^a(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^a$$
$$= \prod_{uv\in E_1} 9^a \times \prod_{uv\in E_2} 12^a \times \prod_{uv\in E_3} 16^a$$
$$= 9^{2ma} \times 12^{8ma} \times 16^{18mna-14ma}$$
$$= 3^{12am} \times 4^{4am(4n-5)}$$

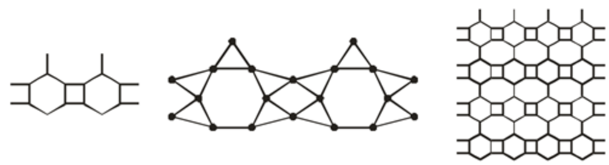
$$(9).\ GAII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$
$$= \prod_{uv\in E_1} \frac{2\sqrt{3\times 3}}{3+3} \times \prod_{uv\in E_2} \frac{2\sqrt{3\times 4}}{3+4}$$
$$\times \prod_{uv\in E_3} \frac{2\sqrt{4\times 4}}{4+4}$$
$$= \left(\frac{2\sqrt{12}}{7}\right)^{8m}$$

$$(10).\ GA^aII(G) = \left[\prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}\right]^a$$
$$= \prod_{uv\in E_1} \left[\frac{2\sqrt{3\times 3}}{3+3}\right]^a \times \prod_{uv\in E_2} \left[\frac{2\sqrt{3\times 4}}{3+4}\right]^a$$
$$\times \prod_{uv\in E_3} \left[\frac{2\sqrt{4\times 4}}{4+4}\right]^a$$
$$= \left(\frac{2\sqrt{12}}{7}\right)^{8ma}$$

□

## 3. Results for $C_4C_6C_8[m,n]$ nanotori

We consider the $C_4C_6C_8[m,n]$ nanotori. Here figure (a) represents the 2-D lattice of $C_4C_6C_8[2,1]$. Figure (b). Represents the line graph of $C_4C_6C_8[2,1]$ nanotori. Also, we consider the graph $C_4C_6C_8[m,n]$ nanotori with m columns and n rows.



(a) (b) (c)

**Lemma 3.1.** *Let G be a $C_4C_6C_8[m,n]$ nanotori with m columns and n rows. Consider the line graph of $C_4C_6C_8[m,n]$. Then*

(1). *The number of edges are 2m having degree (2, 4). It is denoted by $E_1$.*

(2). *The number of edges are m having degree (3, 3). It is denoted by $E_2$.*

(3). *The number of edges are 4m having degree (3, 4). It is denoted by $E_3$.*

(4). *The number of edges are $18mn-9m$ having degree (4, 4). It is denoted by $E_4$.*

*In addition, total number of edges is $18mn-2m$.*

**Theorem 3.2.** *Let G be a $C_4C_6C_8[m,n]$ nanotube with m columns and n rows. Then*

(1). $II_1^*(G) = 3^{3m} \times 7^{4m} \times 2^{m(54n-24)}$

(2). $II_2(G) = 3^{6m} \times 2^{m(72n-22)}$

(3). $HII_1(G) = 3^{6m} \times 7^{8m} \times 2^{m(108n-48)}$

(4). $HII_2(G) = 3^{12m} \times 2^{4m(36n-11)}$

(5). $XII(G) = 3^{-3m/2} \times 7^{-2m} \times 12^{12m-27mn}$

(6). $\chi II(G) = 3^{-3m} \times 2^{m(11-36n)}$

(7). $MZ_1^a(G) = 3^{3am} \times 7^{4am} \times 2^{am(54n-24)}$

(8). $MZ_2^a(T) = 3^{6am} \times 4^{am(36n-11)}$

(9). $GAII(G) = 2^{11m} \times 7^{-4m}$

(10). $GA^aII(G) = 2^{11ma} \times 7^{-4ma}$

*Proof.* From the definitions of multiplicative indices and partition of edges described in Lemma 3.1, we can see that:

$$(1).\ II_1^*(G) = \prod_{uv\in E(G)} [d_G(u)+d_G(v)] = \prod_{uv\in E_1} 6 \times \prod_{uv\in E_2} 6 \times \prod_{uv\in E_3} 7 \times \prod_{uv\in E_4} 8 = 6^{2m} \times 6^{m} \times 7^{4m} \times 8^{18mn-9m} = 3^{3m} \times 7^{4m} \times 2^{m(54n-24)}.$$

$$(2).\ II_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v) = \prod_{uv\in E_1} 8 \times \prod_{uv\in E_2} 9 \times \prod_{uv\in E_3} 12 \times \prod_{uv\in E_4} 16 = 8^{2m} \times 9^{m} \times 12^{4m} \times 16^{18mn-9m} = 3^{6m} \times 2^{m(72n-22)}$$

$$(3).\ HII_1(G) = \prod_{uv\in E(G)} [d_G(u)+d_G(v)]^2 = \prod_{uv\in E_1} 6^2 \times \prod_{uv\in E_2} 6^2 \times \prod_{uv\in E_3} 7^2 \times \prod_{uv\in E_4} 8^2 = 6^{4m} \times 6^{2m} \times 7^{8m} \times 8^{36mn-18m} = 3^{6m} \times 7^{8m} \times 2^{m(108n-48)}$$

$$(4).\ HII_2(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^2 = \prod_{uv\in E_1} 8^2 \times \prod_{uv\in E_2} 9^2 \times \prod_{uv\in E_3} 12^2 \times \prod_{uv\in E_4} 16^2 = 8^{4m} \times 9^{2m} \times 12^{8m} \times 16^{36mn-18m} = 3^{12m} \times 2^{4m(36n-11)}$$

$$(5).\ XII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}} = \prod_{uv\in E_1} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{7}} \times \prod_{uv\in E_4} \frac{1}{\sqrt{8}} = \left(\frac{1}{\sqrt{6}}\right)^{2m} \times \left(\frac{1}{\sqrt{6}}\right)^{m} \times \left(\frac{1}{\sqrt{7}}\right)^{4m} \times \left(\frac{1}{\sqrt{8}}\right)^{18mn-9m} = 3^{-3m/2} \times 7^{-2m} \times 12^{12m-27mn}$$

$$(6).\ \chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} = \prod_{uv\in E_1} \frac{1}{\sqrt{8}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{9}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{12}} \times \prod_{uv\in E_4} \frac{1}{\sqrt{16}} = \left(\frac{1}{\sqrt{8}}\right)^{2m} \times \left(\frac{1}{\sqrt{9}}\right)^{m} \times \left(\frac{1}{\sqrt{12}}\right)^{4m} \times \left(\frac{1}{\sqrt{14}}\right)^{18mn-9m} = 3^{-3m} \times 2^{m(11-36n)}$$

$$(7).\ MZ_1^a(G) = \prod_{uv\in E(G)} [d_G(u)+d_G(v)]^a = \prod_{uv\in E_1} 6^a \times \prod_{uv\in E_2} 6^a \times \prod_{uv\in E_3} 7^a \times \prod_{uv\in E_4} 8^a = 6^{2ma} \times 6^{ma} \times 7^{4ma} \times 8^{18mna-9ma} = 3^{3am} \times 7^{4am} \times 2^{am(54n-24)}$$

$$(8).\ MZ_2^a(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^a = \prod_{uv\in E_1} 8^a \times \prod_{uv\in E_2} 9^a \times \prod_{uv\in E_3} 12^a \times \prod_{uv\in E_4} 16^a = 8^{2ma} \times 9^{2ma} \times 12^{8ma} \times 16^{18mna-14ma} = 3^{6am} \times 2^{2am(36n-11)}$$

$$(9).\ GAII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} = \prod_{uv\in E_1} \frac{2\sqrt{2\times 4}}{2+4} \times \prod_{uv\in E_2} \frac{2\sqrt{3\times 3}}{3+3} \times \prod_{uv\in E_3} \frac{2\sqrt{3\times 4}}{3+4} \times \prod_{uv\in E_4} \frac{2\sqrt{4\times 4}}{4+4} = \left(\frac{2\sqrt{12}}{3}\right)^{2m} \left(\frac{2\sqrt{12}}{7}\right)^{4m} = 2^{11m} \times 7^{-4m}$$

$$(10).\ GA^aII(G) = \left[\prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}\right]^a = \prod_{uv\in E_1} \left[\frac{2\sqrt{2\times 4}}{2+4}\right]^a \times \prod_{uv\in E_2} \left[\frac{2\sqrt{3\times 3}}{3+3}\right]^a \times \prod_{uv\in E_3} \left[\frac{2\sqrt{3\times 4}}{3+4}\right]^a \times \prod_{uv\in E_4} \left[\frac{2\sqrt{4\times 4}}{4+4}\right]^a = 2^{11ma} \times 7^{-4ma}$$

□

## 4. Conclusion

In this paper, we have obtained some multiplicative connectivity indices of $TUC_4C_6C_8[m,n]$ nanotube and $C_4C_6C_8[m,n]$ nanotori. These formulae possible to correlate chemical structure of nanotubes with an information about physical features.

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