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Edge vertex prime labeling of graphs

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Abstract
A bijective labeling $f: V(G)\bigcup E(G) \rightarrow \{1,2,3,...,|V(G)\bigcup E(G)|\}$ is an *edge vertex prime labeling* if for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. A graph G which admits edge vertex prime labeling is called an *edge vertex prime graph*. In this paper, we have obtained some class of graphs such as $p+q$ is odd for $G\hat{O}W_n$, $G\hat{O}f_n$, $G\hat{O}F_n$, $p+q$ is even for $G\hat{O}P_n$, crown graph and union of cycles are edge vertex prime graph.

Keywords
Prime labeling, edge vertex prime labeling, relatively prime.

AMS Subject Classification
05C78.

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Contents

1. Introduction

All our graphs are simple, finite and undirected and we follow Balakrishnan and Ranganathan [1] for standard notations and terminology. $G = (V(G), E(G))$, where $V(G)$ is vertex set and $E(G)$ is edge set of the graph. $|V(G)|$ and $|E(G)|$ are denoted by the number of vertices and edges respectively, which is *order* and *size* of G. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. See the dynamic graph labeling survey [2] by Gallian is regularly updated. Prime labeling is a type of graph labeling developed by Roger Entriger that was first formally introduced by Tout, Dabboucy and Howalla [7]. We define $[n] = 1,2,...,n$, where n is a positive integers. Given a simple graph G of order n, a *prime labeling* consists of labeling the vertices with integers from the set $[n]$ so that the labels of any pair of adjacent vertices are relatively prime.

Edge vertex prime labeling is a variation of prime labeling. A bijective function $f: V(G)\bigcup E(G) \rightarrow \{1,2,3,...,|V(G)\bigcup E(G)|\}$ is said to be an *edge vertex prime labeling* if for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Jagadesh and Baskar Babujee [3] was introduced the concept of an edge vertex prime labeling and proved the existence of the same paths, cycles and star $K_{1,n}$. In [4], if $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ are two connected graphs, then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1\hat{O}G_2$. The resultant graph $G = G_1\hat{O}G_2$ contains $p_1 + p_2 - 1$ vertices $q_1 + q_2$ edges. In general, there are p_1p_2 possibilities of getting $G_1\hat{O}G_2$ from G_1 and G_2. In [4], they also proved that edge vertex prime labeling, for some class of graphs such as generalized star, generalized cycle star, $p+q$ is odd for $G\hat{O}K_{1,n}$, $G\hat{O}P_n$, $G\hat{O}C_n$. Parmer [5] proved that wheel W_n, fan f_n, friendship graph F_n are an edge vertex prime labeling. In [6], they also proved that $K_{2,n}$, for all n and $K_{3,n}$ for $n = \{2,3,...,29\}$ are edge vertex prime labeling. We [8] proved that triangular and rectangular book, butterfly graph, Drums graph D_n, Jahangir graph $J_{n,3}$ and $J_{n,4}$ are edge vertex prime labeling.

A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. Fan graph f_n, $n \geq 2$ obtained by joining all vertices of a path P_n to a further vertex called centre. That is, $f_n = P_n + K_1$. Friendship graph F_n is a graph which consists of n-triangles with a common vertex. The crown graph is obtained from a cycle C_n by attaching a pendant edge at each vertex of the n- cycle. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The graph $G = (V(G), E(G))$, where $V = V_1\bigcup V_2$ and $E = E_1\bigcup E_2$, is called the *union* of G_1 and G_2 is denoted by $G_1\bigcup G_2$.

In this paper, we established that $p+q$ is odd for $G\hat{O}W_n$,

$G\hat{O}f_n$, $G\hat{O}F_n$, $p+q$ is even for $G\hat{O}P_n$, crown graph, union of cycles and some class of several graphs are edge vertex prime.

2. Main Results

Theorem 2.1. *If G ($G \neq P_m$ and W_4) has an edge vertex prime labeling with $p+q$ is odd, then there exists a graph from the class $G\hat{O}W_n$ that admits edge vertex prime labeling.*

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph and $G \neq P_m$ and W_4, when $p+q$ is odd, with bijective function $f: V(G)\bigcup E(G) \to \{1,2,3,...,p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph W_n with vertex set $\{w, w_i : 1 \leq i \leq n\}$ and edge set $\{ww_i : 1 \leq i \leq n\}\bigcup\{w_iw_{i+1} : 1 \leq i \leq n-1\}\bigcup\{w_1w_n\}$. We superimpose one of the vertex say w of W_n on selected vertex v_1 in G. Now, we define new graph $G_1 = G\hat{O}W_n$ with vertex set $V_1 = V\bigcup\{w, w_i : 1 \leq i \leq n\}$ and edge set $E_1 = E\bigcup\{ww_i : 1 \leq i \leq n\}\bigcup\{w_iw_{i+1} : 1 \leq i \leq n-1\}\bigcup\{w_1w_n\}$. Define a bijective function $g: V_1\bigcup E_1 \to \{1,2,3,...,p+q,p+q+1,...,p+q+3n+1\}$ by $g(v) = f(v)$ for all $v \in V(G)$ and $g(uv) = f(uv)$ for all $uv \in E(G)$.
Consider $G\hat{O}W_n$ the following cases.
Case(i). When n is even.

$$g(w) = 1$$

$$g(w_i) = \begin{cases} p+q+3i-1; & i \text{ is odd} \\ p+q+3i-2; & i \text{ is even} \end{cases}$$

$$g(ww_i) = \begin{cases} p+q+3i-2; & i \text{ is odd} \\ p+q+3i-1; & i \text{ is even} \end{cases}$$

$$g(w_iw_{i+1}) = p+q+3i, \forall i$$

We have to prove that G_1 is an edge vertex prime labeling. Already, G is an edge vertex prime labeling, it is enough to prove that for any edge $uv \in E_1$ which is not in G, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime.
(i) For any edge $ww_i \in E_1$ $(1 \leq i \leq n)$,

$$gcd(g(w), g(w_i)) = \begin{cases} gcd(1, p+q+3i-1); & i \text{ is odd} \\ gcd(1, p+q+3i-2); & i \text{ is even} \end{cases} = 1$$

$$gcd(g(w), g(ww_i)) = \begin{cases} gcd(1, p+q+3i-2); & i \text{ is odd} \\ gcd(1, p+q+3i-1); & i \text{ is even} \end{cases} = 1$$

$$gcd(g(w_i), g(ww_i)) = \begin{cases} gcd(p+q+3i-1, p+q+3i-2); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i-1); & i \text{ is even} \end{cases} = 1$$

(ii) For any edge $w_iw_{i+1} \in E_1$ $(1 \leq i \leq n-1)$,

$$gcd(g(w_i), g(w_{i+1})) = \begin{cases} gcd(p+q+3i-1, p+q+3i+1); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i+2); & i \text{ is even} \end{cases} = 1$$

$$gcd(g(w_i), g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i-1, p+q+3i); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i); & i \text{ is even} \end{cases} = 1$$

$$gcd(g(w_{i+1}), g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i+1, p+q+3i); & i \text{ is odd} \\ gcd(p+q+3i+2, p+q+3i); & i \text{ is even} \end{cases} = 1$$

Case(ii). When n is odd.

$$g(w) = 1$$

$$g(w_i) = \begin{cases} p+q+3i-1; & i = 1,3,5,...,n-2 \\ p+q+3i-2; & i = 2,4,6,...,n-1 \end{cases}$$

$$g(ww_i) = \begin{cases} p+q+3i-2; & i = 1,3,5,...,n-2 \\ p+q+3i-1; & i = 2,4,6,...,n-1 \end{cases}$$

$g(w_iw_{i+1}) = p+q+3i$, for $i = 1,2,...,n-2$,
$g(w_n) = p+q+3n-3$,
$g(w_{n-1}w_n) = p+q+3n-1$,
$g(w_1w_n) = p+q+3n+1$.
Now, our claims are (i) $g(w), g(w_i)$ and $g(ww_i)$, (ii) $g(w_i)$, $g(w_{i+1})$ and $g(w_iw_{i+1})$ are pairwise relatively prime.
(i) For any edge $ww_i \in E_1$ $(1 \leq i \leq n)$,

$$gcd(g(w), g(w_i)) = \begin{cases} gcd(1, p+q+3i-1); & i = 1,3,5,...,n-2 \\ gcd(1, p+q+3i-2); & i = 2,4,6,...,n-1 \\ gcd(1, p+q+3i-3); & i = n \end{cases} = 1$$

$$gcd(g(w), g(ww_i)) = \begin{cases} gcd(1, p+q+3i-2); & i \text{ is odd} \\ gcd(1, p+q+3i-1); & i \text{ is even} \\ gcd(1, p+q+3n); & i = n \end{cases} = 1$$

$$gcd(g(w_i),g(ww_i)) = \begin{cases} gcd(p+q+3i-1,p+q+3i-2); & i \text{ is odd} \\ gcd(p+q+3i-2,p+q+3i-1); & i \text{ is even} \\ gcd(p+q+3i-3,p+q+3n); & i=n \end{cases}$$

$$=1$$

(ii) For any edge $w_iw_{i+1} \in E_1$ $(1 \le i \le n-1)$,

$$gcd(g(w_i),g(w_{i+1})) = \begin{cases} gcd(p+q+3i-1,p+q+3i+1); & i \text{ is odd} \\ gcd(p+q+3i-2,p+q+3i+2i); & i \text{ is even} \\ gcd(p+q+3i-2,p+q+3n); & i=n-1 \end{cases}$$

$$gcd(g(w_i),g(w_{i+1})) = \begin{cases} gcd(p+q+3i-1,p+q+3i+1); & i=1,3,5,...,n-2 \\ gcd(p+q+3i-2,p+q+3i+2i); & i=2,4,6,...,n-3 \\ gcd(p+q+3i-2,p+q+3i+3); & i=n-1 \end{cases}$$

$$=1$$

$$gcd(g(w_i),g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i-1,p+q+3i); & i=1,3,5,...,n-2 \\ gcd(p+q+3i-2,p+q+3i); & i=2,4,6,...,n-3 \\ gcd(p+q+3n-2,p+q+3n-1); & i=n-1 \end{cases}$$

$$gcd(g(w_i),g(w_iw_{i+1}) = \begin{cases} gcd(p+q+3i-1,p+q+3i); & i=1,3,5,...,n-2 \\ gcd(p+q+3i-2,p+q+3i); & i=2,4,6,...,n-3 \\ gcd(p+q+3i-2,p+q+3i+2); & i=n-1 \end{cases}$$

$$=1$$

$$gcd(g(w_{i+1}),g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i+1,p+q+3i); & i=1,3,5,...,n-2 \\ gcd(p+q+3i+2,p+q+3i); & i=2,4,6,...,n-3 \\ gcd(p+q+3n-3,p+q+3n-1); & i=n-1 \end{cases}$$

$$gcd(g(w_{i+1}),g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i+1,p+q+3i); & i=1,3,5,...,n-2 \\ gcd(p+q+3i+2,p+q+3i); & i=2,4,6,...,n-3 \\ gcd(p+q+3i,p+q+3i+2); & i=n-1 \end{cases}$$

$$=1$$

Therefore, for any edge $uv \in E_1$ which is not in G, the numbers $g(u)$, $g(v)$ and $g(uv)$ are pairwise relatively prime.
Hence $G\hat{O}W_n$ is an edge vertex prime labeling,
where G ($G \neq P_m$ and W_4). □

The above theorem is not applicable for G ($G \neq P_m$ and W_4). But, first we apply Jagadesh et. al.,[4] proved that if $G = W_n$ is an edge vertex prime labeling, then there exist a graph from the class $W_n\hat{O}P_m$ that admits edge vertex prime labeling.

Next, consider a graph $G = W_4\hat{O}W_5$. Then $V(G) = \{u, u_i, v_j : 1 \le i \le 4, 1 \le j \le 5\}$ and $E(G) = \{uu_i, uv_j : 1 \le i \le 4, 1 \le j \le 5\}$. Also,$|V(G)| = 10$ and $|E(G)| = 18$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,28\}$ by
$f(u) = 1, f(u_1) = 3, f(u_2) = 5, f(u_3) = 9, f(u_4) = 11,$
$f(uu_1) = 2, f(uu_2) = 6, f(uu_3) = 8, f(uu_4) = 12,$
$f(u_1u_2) = 4, f(u_2u_3) = 7, f(u_3u_4) = 10, f(u_1u_4) = 13,$
$f(v_1) = 15, f(v_2) = 17, f(v_3) = 19, f(v_4) = 21, f(v_5) = 23,$
$f(v_1v_2) = 16, f(v_2v_3) = 18, f(v_3v_4) = 20, f(v_4v_5) = 22,$
$f(v_1v_5) = 14, f(uv_1) = 28, f(uv_2) = 24, f(uv_3) = 25,$
$f(uv_4) = 26, f(uv_5) = 27$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = W_4\hat{O}W_5$ is an edge vertex prime labeling.

Theorem 2.2. *If $G(p,q)$ has an edge vertex prime labeling with $p+q$ is odd, then there exists a graph from the class $G\hat{O}f_n$ that admits edge vertex prime labeling.*

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is odd with bijective function $f : V(G) \bigcup E(G) \to \{1,2,...,p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph f_n with vertex set $\{w, w_i : 1 \le i \le n\}$ and edge set $\{ww_i : 1 \le i \le n\} \bigcup \{w_iw_{i+1} : 1 \le i \le n-1\}$. We superimpose one of the vertex say, w of f_n on selected vertex v_1 in G. Now, we define new graph $G_1 = G\hat{O}f_n$ with vertex set $V_1 = V \bigcup \{w, w_i : 1 \le i \le n\}$ and edge set $E_1 = E \bigcup \{ww_i : 1 \le i \le n\} \bigcup \{w_iw_{i+1} : 1 \le i \le n-1\}$. Define a bijective function $g : V_1 \bigcup E_1 \to \{1,2,...,p+q,p+q+1,...,p+q+3n\}$ by $g(v) = f(v)$ for all $v \in V(G)$ and $g(uv) = f(uv)$ for all $uv \in E(G)$,

$$g(w) = 1$$

$$g(w_i) = \begin{cases} p+q+3i-1; & i \text{ is odd} \\ p+q+3i-2; & i \text{ is even} \end{cases}$$

$$g(ww_i) = \begin{cases} p+q+3i-2; & i \text{ is odd} \\ p+q+3i-1; & i \text{ is even} \end{cases}$$

$$g(w_iw_{i+1}) = p+q+3i, \forall i$$

We have to prove that G_1 is an edge vertex prime labeling. Already, G is an edge vertex prime labeling, it is enough to prove that for any edge $uv \in E_1$ which is not in G, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime.
(i) For any edge $ww_i \in E_1$ $(1 \leq i \leq n)$,

$$gcd(g(w), g(w_i)) = \begin{cases} gcd(1, p+q+3i-1); & i \text{ is odd} \\ gcd(1, p+q+3i-2); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w), g(ww_i)) = \begin{cases} gcd(1, p+q+3i-2); & i \text{ is odd} \\ gcd(1, p+q+3i-1); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_i), g(ww_i)) = \begin{cases} gcd(p+q+3i-1, p+q+3i-2); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i-1); & i \text{ is even} \end{cases}$$
$$= 1$$

(ii) For any edge $w_iw_{i+1} \in E_1$ $(1 \leq i \leq n-1)$,

$$gcd(g(w_i), g(w_{i+1})) = \begin{cases} gcd(p+q+3i-1, p+q+3i+1); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i+2); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_i), g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i-1, p+q+3i); & i \text{ is odd} \\ gcd(p+q+3i-2, p+q+3i); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_{i+1}), g(w_iw_{i+1})) = \begin{cases} gcd(p+q+3i+1, p+q+3i); & i \text{ is odd} \\ gcd(p+q+3i+2, p+q+3i); & i \text{ is even} \end{cases}$$
$$= 1$$

Therefore, for any edge $uv \in E_1$ which is not in G, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Hence there exists a graph from the class $G\hat{O}f_n$ admits edge vertex prime labeling. □

The *planter graph* R_n $(n \geq 3)$ can be constructed by joining fan graph f_n $(n \geq 2)$ and cycle $C_n, (n \geq 3)$ with sharing a common vertex, where n is any positive integer, that is $R_n = f_n\hat{O}C_n$

Corollary 2.3. *The planter graph R_n $(n \geq 3)$ admits edge vertex prime labeling graph, where n is any positive integer.*

Theorem 2.4. *If $G(p,q)$ has an edge vertex prime labeling with $p+q$ is odd, then there exists a graph from the class $G\hat{O}F_n$ that admits edge vertex prime labeling.*

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is odd with bijective function $f : V(G) \bigcup E(G) \rightarrow \{1, 2, ..., p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph F_n with vertex set $\{w, w_i : 1 \leq i \leq 2n\}$ and edge set $\{ww_i : 1 \leq i \leq 2n\} \bigcup \{w_{2i-1}w_{2i} : 1 \leq i \leq n\}$. We superimpose one of the vertex say w of F_n on selected vertex v_1 in G. Now, we define new graph $G_1 = G\hat{O}F_n$ with vertex set $V_1 = V \bigcup \{w, w_i : 1 \leq i \leq 2n\}$ and edge set $E_1 = E \bigcup \{ww_i : 1 \leq i \leq 2n\} \bigcup \{w_{2i-1}w_{2i} : 1 \leq i \leq n\}$. Define a bijective function $g : V_1 \bigcup E_1 \rightarrow \{1, 2, 3, ..., p+q, p+q+1, ..., p+q+5n+1\}$ by $g(v) = f(v)$ for all $v \in V(G)$ and $g(uv) = f(uv)$ for all $uv \in E(G)$,

$$g(w) = 1,$$

$$g(w_{2i-1}) = \begin{cases} p+q+5i-3; & i \text{ is odd} \\ p+q+5i-4; & i \text{ is even} \end{cases}$$

$$g(w_{2i}) = \begin{cases} p+q+5i-1; & i \text{ is odd} \\ p+q+5i; & i \text{ is even} \end{cases}$$

$$g(ww_{2i-1}) = \begin{cases} p+q+5i-4; & i \text{ is odd} \\ p+q+5i-3; & i \text{ is even} \end{cases}$$

$$g(ww_{2i}) = \begin{cases} p+q+5i; & i \text{ is odd} \\ p+q+5i-1; & i \text{ is even} \end{cases}$$

$$g(w_{2i-1}w_{2i}) = p+q+5i-2, \forall i$$

We have to prove that G_1 is an edge vertex prime labeling. Already, G is an edge vertex prime labeling, it is enough to prove that for any edge $uv \in E_1$ which is not in G, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Now, our claims are (i) $g(w), g(w_{2i-1})$ and $g(ww_{2i-1})$, (ii) $g(w), g(w_{2i})$ and $g(ww_{2i})$, (iii) $g(w_{2i-1}), g(w_{2i})$ and $g(w_{2i-1}w_{2i})$ are pairwise relatively prime.
(i) For any edge $ww_i \in E_1$ $(1 \leq i \leq 2n)$,

$$gcd(g(w), g(w_{2i-1})) = \begin{cases} gcd(1, p+q+5i-3); & i \text{ is odd} \\ gcd(1, p+q+5i-4); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w), g(ww_{2i-1})) = \begin{cases} gcd(1, p+q+5i-4); & i \text{ is odd} \\ gcd(1, p+q+5i-3); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_{2i-1}), g(ww_{2i-1})) = \begin{cases} gcd(p+q+5i-3, p+q+5i-4); i \text{ is odd} \\ gcd(p+q+5i-4, p+q+5i-3); i \text{ is even} \end{cases}$$
$$= 1$$

(ii)

$$gcd(g(w),g(w_{2i})) = \begin{cases} gcd(1,p+q+5i-1); & i \text{ is odd} \\ gcd(1,p+q+5i); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w),g(ww_{2i})) = \begin{cases} gcd(1,p+q+5i); & i \text{ is odd} \\ gcd(1,p+q+5i-1); & i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_{2i}),g(ww_{2i})) = \begin{cases} gcd(p+q+5i-1,p+q+5i); \\ \quad i \text{ is odd} \\ gcd(p+q+5i,p+q+5i-1); \\ \quad i \text{ is even} \end{cases}$$
$$= 1$$

(iii)

$$gcd(g(w_{2i-1}),g(w_{2i})) = \begin{cases} gcd(p+q+5i-3,p+q+5i-1); \\ \quad i \text{ is odd} \\ gcd(p+q+5i-4,p+q+5i); \\ \quad i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_{2i-1}),g(w_{2i-1}w_{2i})) = \begin{cases} gcd(p+q+5i-3,p+q+ \\ \quad 5i-2); i \text{ is odd} \\ gcd(p+q+5i-4,p+q+ \\ \quad 5i-2); i \text{ is even} \end{cases}$$
$$= 1$$

$$gcd(g(w_{2i}),g(w_{2i-1}w_{2i})) = \begin{cases} gcd(p+q+5i-1,p+q+ \\ \quad 5i-2); i \text{ is odd} \\ gcd(p+q+5i,p+q+5i-2); \\ \quad i \text{ is even} \end{cases}$$
$$= 1$$

Therefore, for any edge $uv \in E_1$, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Hence there exists a graph from the class $G\hat{O}F_n$ admits edge vertex prime labeling. □

Theorem 2.5. *If $G(p,q)$ has an edge vertex prime labeling with $p+q$ is even, then there exists a graph from the class $G\hat{O}P_n$ that admits edge vertex prime labeling.*

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is even with bijective function $f : V(G) \bigcup E(G) \to \{1,2,3,...,p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph P_n with vertex set $\{w_i : 1 \le i \le n\}$ and edge set $\{w_iw_{i+1} : 1 \le i \le n-1\}$. We superimpose one of the vertex say w_1 of P_n on selected vertex v_1 in G. Now, we define a new graph $G_1 = G\hat{O}P_n$ with vertex set $V_1 = V \bigcup \{w_i : 2 \le i \le n\}$ and edge set $E_1 = E \bigcup \{w_iw_{i+1} : 1 \le i \le n-1\}$. Define a bijective function $g : V_1 \bigcup E_1 \to \{1,2,3,...,p+q,p+q+1,...,p+q+2n-2\}$ by $g(v) = f(v)$, for all $v \in V$ and $g(uv) = f(uv)$, for all $uv \in E(G)$, $g(w_1) = 1$, $g(w_i) = p+q+2n+1-2i$ for $2 \le i \le n$, $g(w_iw_{i+1}) = p+q+2n-2i$ for $1 \le i \le n-1$.
For any edge $w_iw_{i+1} \in E_1 (2 \le i \le n-1)$, $gcd(g(w_1), g(w_2)) = gcd(1, p+q+2n-3) = 1$, $gcd(g(w_1), g(w_1w_2)) = gcd(1, p+q+2n-2) = 1$, $gcd(g(w_2), g(w_1w_2)) = gcd(p+q+2n-3, p+q+2n-2) = 1$, $gcd(g(w_i), g(w_{i+1})) = gcd(p+q+2n+1-2i, p+q+2n-2i-1) = 1$, $gcd(g(w_i), g(w_iw_{i+1})) = gcd(p+q+2n+1-2i, p+q+2n-2i) = 1$, $gcd(g(w_{i+1}), g(w_iw_{i+1})) = gcd(p+q+2n-2i-1, p+q+2n-2i) = 1$. Therefore, for any edge $uv \in E_1$, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Hence $G\hat{O}P_n$ admits edge vertex prime labeling. □

Corollary 2.6. *The graph $C_l\hat{O}K_{1,m}\hat{O}P_n$ is an edge vertex prime labeling.*

Proof. Let $G = C_l\hat{O}K_{1,m}\hat{O}P_n$ be a graph. Then $V(G) = \{u_i : 1 \le i \le l\} \bigcup \{v, v_j : 1 \le j \le m\} \bigcup \{w_k : 1 \le k \le n\}$ and $E(G) = \{u_iu_{i+1} : 1 \le i \le l-1\} \bigcup \{u_1u_l\} \bigcup \{u_1v_j : 1 \le j \le m\} \bigcup \{u_1w_{n-1}\} \bigcup \{w_kw_{k+1} : 1 \le k \le n-2\}$. Also, $|V(G)| = l+m+n-1$ and $|E(G)| = l+m+n-1$. We superimpose two of the vertices say, v of $K_{1,m}$ and w_n of P_n on selected vertex u_1 in C_l. Define a bijective function $f : V(G) \bigcup E(G) \to \{1,2,3,...,2(l+m+n-1)\}$ by $f(u_1) = f(v) = f(w_n) = 1$, $f(u_i) = 2i-1$ for $2 \le i \le l$, $f(u_iu_{i+1}) = 2i$ for $1 \le i \le l-1$, $f(u_1u_l) = 2l$, $f(v_j) = 2l+2j-1$ for $1 \le j \le m$, $f(u_1v_j) = 2l+2j$ for $1 \le j \le m$, $f(w_k) = 2(l+m+k)-1$ for $1 \le k \le n-1$, $f(w_kw_{k+1}) = 2(l+m+k)$ for $1 \le k \le n-1$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_l\hat{O}K_{1,m}\hat{O}P_n$ admits edge vertex prime labeling. □

Parmer [5] proved that f_m is an edge vertex prime labeling . Jagadesh, Baskar Babujee [4] proved that if G has an edge vertex prime labeling, then there exist a graph from the class $G\hat{O}P_n$ admits edge vertex prime labeling.

An *Umbrella graph* $U(m,n)$ is the graph obtained by joining a path P_n with the central vertex of a fan f_m.

Corollary 2.7. *The Umbrella graph $U(m,n)$ is an edge vertex prime labeling.*

Theorem 2.8. *If G has an edge vertex prime labeling with $p+q$ is even, then there exists a graph from the class $G\hat{O}C_3$ that admits edge vertex prime labeling.*

Proof. Let $G(p,q)$ be an edge vertex prime labeling graph when $p+q$ is even with bijective function from $f : V(G) \bigcup E(G) \to \{1,2,3,...,p+q\}$ with the property that given any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Consider the graph C_3 with vertex set $\{w_1, w_2, w_3\}$ and edge set $\{w_1w_2, w_2w_3, w_3w_1\}$. We superimpose one of the vertex say w_1 of C_3 on selected vertex v_1 in G. Now, we define new graph $G_1 = G\hat{O}C_3$ with vertex set $V_1 = V \bigcup \{w_1, w_2, w_3\}$ and edge set $E_1 = E \bigcup \{w_1w_2, w_2w_3, w_3w_1\}$. Define a bijective function $g : V_1 \bigcup E_1 \to \{1,2,3,...,p+q,p+q+1,p+q+2,p+q+3\}$ by $g(v) = f(v)$ for all $v \in V(G)$ and $g(uv) = f(uv)$ for all $uv \in E(G)$, $g(w_1) = 1$, $g(w_2) = p+q+1$, $g(w_3) =$

$p+q+3$, $g(w_1w_2) = p+q+2$, $g(w_2w_3) = p+q+5$ and $g(w_1w_3) = p+q+4$. We have to prove that G_1 is an edge vertex prime labeling. Already, G is an edge vertex prime labeling, it is enough to prove that for any edge $uv \in E_1$ which is not in G, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. For any edge $w_iw_{i+1} \leq E_1$, $gcd(g(w_1), g(w_2)) = gcd(1, p+q+1) = 1$, $gcd(g(w_1), g(w_1w_2)) = gcd(1, p+q+2) = 1$, $gcd(g(w_2), g(w_1w_2)) = gcd(p+q+1, p+q+2) = 1$,
$gcd(g(w_2), g(w_3)) = gcd(p+q+1, p+q+3) = 1$,
$gcd(g(w_2), g(w_2w_3)) = gcd(p+q+1, p+q+5) = 1$,
$gcd(g(w_3), g(w_2w_3)) = gcd(p+q+3, p+q+5) = 1$,
$gcd(g(w_1), g(w_3)) = gcd(1, p+q+3) = 1$,
$gcd(g(w_1), g(w_1w_3)) = gcd(1, p+q+4) = 1$,
$gcd(g(w_3), g(w_1w_3)) = gcd(p+q+3, p+q+4) = 1$.
Therefore, for any edge $uv \in E_1$, the numbers $g(u), g(v)$ and $g(uv)$ are pairwise relatively prime. Hence there exists a graph from the class $G\hat{O}C_3$ admits edge vertex prime labeling. □

Theorem 2.9. *The crown graph $C_n.K_1$ is an edge vertex prime labeling, where n is a positive integer.*

Proof. Let $G = C_n.K_1$ be a graph. The degree of the vertices of a crown graph is either 3 or 1. Consider $u_1, u_2, ..., u_n$ be the vertices with degree 3 and $v_1, v_2, ..., v_n$ be the vertices with degree 1. The edges of the crown graph are $\{u_iv_i : 1 \leq i \leq n\} \bigcup \{u_iu_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_1u_n\}$. Here $|V(G)| = 2n$ and $|E(G)| = 2n$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1, 2, ..., 4n\}$. For any edge $1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2$, $f(u_{3i-2}) = 12i - 11$, $f(v_{3i-2}) = 12i - 9$, $f(u_{3i-1}) = 12i - 7$, $f(v_{3i-1}) = 12i - 5$,
$f(u_{3i-2}u_{3i-1}) = 12i - 8$, $f(u_{3i-1}u_{3i}) = 12i - 4$,
$f(u_{3i-2}v_{3i-2}) = 12i - 10$, $f(u_{3i-1}v_{3i-1}) = 12i - 6$,
$f(u_{3i}v_{3i}) = 12i - 2$.
Consider the following cases.
Case (i). When n is even.
For each $1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 3$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$.
Case (ii). When n is odd.
For each $1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2$, $f(u_{3i}) = 12i - 1$, $f(v_{3i}) = 12i - 3$, $f(u_1u_n) = 4n$.
Next, we show that the edge vertex prime labeling.
$gcd(f(u_1), f(u_n)) = gcd(1, 4n-3) = 1$,
$gcd(f(u_1), f(u_1u_n)) = gcd(1, 4n) = 1$,
$gcd(f(u_n), f(u_1u_n)) = gcd(4n-3, 4n) = 1$,
$gcd(f(u_{3i-2}), f(u_{3i-1})) = gcd(12i-11, 12i-7) = 1$,
$gcd(f(u_{3i-2}), f(u_{3i-2}u_{3i-1})) = gcd(12i-11, 12i-8) = 1$,
$gcd(f(u_{3i-1}), f(u_{3i-2}u_{3i-1})) = gcd(12i-7, 12i-8) = 1$,
$gcd(f(u_{3i-1}), f(u_{3i})) = gcd(12i-7, 12i-1) = 1$,
$gcd(f(u_{3i-1}), f(u_{3i-1}u_{3i})) = gcd(12i-7, 12i-4) = 1$,
$gcd(f(u_{3i}), f(u_{3i-1}u_{3i})) = gcd(12i-1, 12i-4) = 1$,
$gcd(f(u_{3i-2}), f(v_{3i-2})) = gcd(12i-11, 12i-9) = 1$,
$gcd(f(u_{3i-2}), f(u_{3i-2}v_{3i-2})) = gcd(12i-11, 12i-10) = 1$,
$gcd(f(v_{3i-2}), f(u_{3i-2}v_{3i-2})) = gcd(12i-9, 12i-10) = 1$,
$gcd(f(u_{3i-1}), f(v_{3i-1})) = gcd(12i-7, 12i-5) = 1$,
$gcd(f(u_{3i-1}), f(u_{3i-1}v_{3i-1})) = gcd(12i-7, 12i-6) = 1$,
$gcd(f(v_{3i-1}), f(u_{3i-1}v_{3i-1})) = gcd(12i-5, 12i-6) = 1$,
$gcd(f(u_{3i}), f(v_{3i}) = gcd(12i-1, 12i-3) = 1$,
$gcd(f(u_{3i}), f(u_{3i}v_{3i})) = gcd(12i-1, 12i-2) = 1$,
$gcd(f(v_{3i}), f(u_{3i}v_{3i})) = gcd(12i-3, 12i-2) = 1$.
Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence the crown graph $C_n.K_1$ is an edge vertex prime labeling. □

Theorem 2.10. *The graph $C_n \bigcup C_n \bigcup C_n$, $n \geq 3$ and $n \equiv 0 (mod\ 3)$ is an edge vertex prime labeling.*

Proof. Let $G = C_n \bigcup C_n \bigcup C_n$, $n \geq 3$ and $n \equiv 0 (\text{mod } 3)$ be a graph. Then $V(G) = \{v_i : 1 \leq i \leq 3n\}$ and $E(G) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_1v_n\} \bigcup \{v_iv_{i+1} : n+1 \leq i \leq 2n-1\} \bigcup \{v_{n+1}v_{2n}\} \bigcup \{v_iv_{i+1} : 2n+1 \leq i \leq 3n-1\} \bigcup \{v_{2n+1}v_{3n}\}$. Also, $|V(G)| = 3n$ and $|E(G)| = 3n$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1, 2, 3, ..., 6n\}$ as follows.
Case 1. $n \equiv 0 (\text{mod } 3)$ and n is not congruent to 6 modulo 15.
$f(v_i) = 2i - 1$ for $1 \leq i \leq 3n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(v_1v_n) = 2n$, $f(v_{n+1}v_{2n}) = 4n$, $f(v_iv_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-1$, $f(v_{2n+1}v_{3n}) = 6n$, $f(v_iv_{i+1}) = 2i$ for $2n+1 \leq i \leq 3n-1$.
Case 2. $n \equiv 6 (\text{mod } 15)$.
$f(v_i) = 2i - 1$ for $1 \leq i \leq 2n$, $f(v_{2n+1}) = 4n+3$, $f(v_{2n+2}) = 4n+1$, $f(v_i) = 2i$ for $2n+3 \leq i \leq 3n$, $f(v_iv_{i+1}) = 2i$ for $1 \leq i \leq n-1$, $f(v_1v_n) = 2n$, $f(v_{n+1}v_{2n}) = 4n$, $f(v_iv_{i+1}) = 2i$ for $n+1 \leq i \leq 2n-1$, $f(v_{2n+1}v_{3n}) = 6n$, $f(v_iv_{i+1}) = 2i$ for $2n+1 \leq i \leq 3n-1$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_n \bigcup C_n \bigcup C_n$, $n \geq 3$ and $n \equiv 0 (\text{mod } 3)$ is an edge vertex prime labeling. □

Theorem 2.11. *The graph $C_n \bigcup C_n \bigcup, ..., \bigcup C_n$, $n \equiv 0 (mod\ 5)$ is an edge vertex prime labeling.*

Proof. Let $G = C_n \bigcup C_n \bigcup, ..., \bigcup C_n$, $n \equiv 0 (\text{mod } 5)$ be a graph. Then $V(G) = \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 5\}$ and $E(G) = \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 4\} \bigcup \{v_{i5}v_{i1} : 1 \leq i \leq m\}$. Also, $|V(G)| = 5m$ and $|E(G)| = 5m$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1, 2, 3, ..., 10m\}$ by $f(v_{ij}v_{ij+1}) = 10(i-1) + 2j$ for $1 \leq i \leq m, 1 \leq j \leq 4$, $f(v_{i5}v_{i1}) = 10(i-1) + 2n$ for $1 \leq i \leq m$, $f(v_{ij}) = 10(i-1) + 2j - 1$ for $1 \leq i \leq m, 1 \leq j \leq 5$. Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence $G = C_n \bigcup C_n \bigcup, ..., \bigcup C_n$, $n \equiv 0 (\text{mod } 5)$ is an edge vertex prime labeling. □

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