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Soft mildly I-normal spaces

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Abstract

In this paper the concept of soft mildly I-normal spaces have been introduced and studied.

Keywords

Soft sets, Soft topology, Soft normal, Soft Ideal, Soft mildly I-normal, Soft I-paracompact and Soft nearly I-paracompact spaces.

AMS Subject Classification

Primary 54A40 ,54C08, 54D10 ; Secondary 06D72

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Contents

1. Introduction

Molodtsov **[18]** introduced the concept of soft set theory as a new mathematical approach to remove problems that contains uncertainty. In 2011, Shabir and Naz **[25]** introduced the topological structure of soft sets and derived their basic properties. Singals **[26]** have introduced the concept of mildly normal spaces and obtained several properties. Recently, Hussain and Ahmad **[11]** introduced the notion of soft normal spaces. Guler and Kale **[9]** introduced the notion of soft I-normal spaces. Kandil et al.**[12]** introduced the concept of soft ideal and soft local function. Hamlett et al.**[10]** defined the concept of paracompactness with respect o ideals. Qahis **[22]** defined the concept of almost Lindelöf with respect to ideals.The main aim of this paper is to introduce a new soft separation axiom called soft mildly I-normality which is a weak form of soft I-normality and investigate some of their properties and characterizations.

2. Preliminaries

Throughout this paper X denotes a nonempty set, E denotes the set of parameters and S(X,E) denotes the family of soft sets over X. For definition and basic properties of soft sets, reader should refer (**[2]**,**[4]**,**[5]**,**[7]**,**[16]**,**[18]**,**[20]**,**[21]**,**[24]**,**[26]**, **[27]**).

Definition 2.1. *[25] A subfamily τ of S(X,E) is called a soft topology on X if :*

(a) $\widetilde{\phi},\widetilde{X}$ *belongs to* τ.

(b) The union of any number of soft sets in τ belongs to τ.

(c) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) *is called a soft topological space. The members of τ are called soft open sets in X and their complements called soft closed sets in X.*

Lemma 2.2. *[25] Let* (X,τ,E) *be a soft topological space. Then the collection* $\tau_\alpha = \{F(\alpha) : (F,E) \in \tau\}$ *for each* $\alpha \in E$, *defines a topology on X.*

Definition 2.3. *[25] In a soft topological space* (X,τ,E) *the intersection of all soft closed super sets of* (F,E) *is called the soft closure of* (F,E). *It is denoted by* $\mathrm{Cl}(F,E)$.

Definition 2.4. *[28] In a soft topological space* (X,τ,E) *the union of all soft open subsets of* (F,E) *is called soft interior of* (F,E). *It is denoted by* $\mathrm{Int}\ (F,E)$.

Definition 2.5 (**[25]**,**[28]**). *Let* (X,τ,E) *be a soft topological space and let* $(F,E),(G,E) \in S(X,E)$. *Then:*

(a) (F,E) *is soft closed if and only if* $(F,E) = \mathrm{Cl}(F,E)$

(b) If $(F,E) \subseteq (G,E)$, *then* $Cl\ (F,E) \subseteq Cl(G,E)$.

(c) (F,E) *is soft open if and only if* $(F,E) = Int\ (F,E)$.

(d) If $(F,E) \subseteq (G,E)$,*then* $Int\ (F,E) \subseteq Int\ (G,E)$.

(e) $(Cl\ (F,E))^C = Int\ ((F,E)^C)$.

(f) $(Int\ (F,E))^C = Cl\ ((F,E)^C)$.

Lemma 2.6. ***[11]*** *Let* (X,τ,E) *be a soft topological space over* X *and* Y *be a nonempty subset of* X. *Then* $\tau_Y = \{(F_Y, E) : (F,E) \in\tau\}$ *is said to be the soft relative topology on* Y *and* (Y,τ_Y,E) *is called a soft subspace of* (X,τ,E).

Lemma 2.7. ***[11]*** *Let* (Y,τ_Y, E) *be a soft subspace of a soft topological space* (X,τ,E) *and* (F,E) *be a soft open set in* Y. *If* $\widetilde{Y}\in\tau$ *then* $(F,E)\in\tau$.

Definition 2.8. ***[25]*** *Let* (Y,τ_Y,E) *be a soft topological subspace of a soft topological space* (X,τ,E) *and* (F,E) *be a soft set over* X *,then :*

(a) (F,E) *is soft open in* Y *if and only if* $(F,E) = \widetilde{Y} \cap (G,E)$ *for some soft open set* (G,E) *in* X.

(b) (F,E) *is soft closed in* Y *if and only if* $(F,E) = \widetilde{Y} \cap(G,E)$ *for some soft closed set* (G,E) *in* X.

Lemma 2.9. ***[23]*** *Let* (X,τ,E) *be a soft topological space and* (Y,τ_Y,E) *be a soft subspace of* (X,τ,E) *, then a soft closed set* (F_Y, E) *of* Y *is soft closed in* X *if and only if* $\widetilde{Y}$ *is soft closed in X.*

Definition 2.10. ***[13]*** *Let* $S(X,E)$ *and* $S(Y,K)$ *be families of soft sets over* X and Y. *Let* $u: X \to Y$ *and* $p: E\to K$ *be mappings. Then a mapping* $f_{pu}: S(X,E)\to S(Y,K)$ *is defined as:*

(a) Let (F,A) *be a soft set in* $S(X,E)$. *The image of* (F,A) *under* f_{pu} *is written as* $f_{pu}\ (F,A) = (f_{pu}(F), p\ (A))$*, is a soft set in* $S(Y,K)$ *such that*

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)\cap A} u(F(e)), & p^{-1}(k)\cap A\neq\phi \\ \phi, & p^{-1}(k)\cap A =\phi. \end{cases}$$

For all $k \in K$.

(b) Let (G,B) *be a soft set in* $S(Y,K)$.*The inverse image of* (G,B) *under* f_{pu} *is written as*

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e) \in B \\ \phi, & \text{otherwise} \end{cases}$$

For all $e \in E$.

Definition 2.11. ***[17]*** *Let* $f_{pu}: S(X,E)\to S(Y,K)$ *be a mapping and* $u: X\to Y$ *and* $p: E\to K$ *be mappings. Then* f_{pu} *is soft injective (respectively surjective, bijective) if* $u: X \to Y$ *and* $p: E\to K$ *are injective (respectively surjective, bijective).*

Definition 2.12. ***[29]*** *Let* (X,τ,E) *and* (Y,η,K) *be a soft topological space. A soft mapping* $f_{pu}: (X,\tau,E)\to (Y,\eta,K)$ *is called soft open if* $f_{pu}(F,E)$ *is soft open in* Y, *for all soft open sets* (F,E) *in* X.

Definition 2.13. ***[28]*** *Let* (X,τ,E) *and* (Y,η,K) *be a soft topological space. A soft mapping* $f_{pu}: (X,\tau,E)\to (Y,\eta,K)$ *is called soft continuous if* $f_{pu}^{-1}(G,K)$ *is soft open in* X *for all soft open sets* (G,K) *in* Y.

Definition 2.14. ***[9]*** *Let* (X,τ,E) and (Y,η,K) *be a soft topological space and* $f_{pu}: S(X,E)\to S(Y,K)$ *be a mapping. If* f_{pu} *is bijection, soft continuous and soft open mapping, then* f_{pu} *is called soft homeomorphism from* X *to* Y.

Definition 2.15. ***([6],[15])*** *The soft set* $(F,E) \in S(X, E)$ *is called a soft point if there exists* $x \in X$ *and* $e \in E$ *such that* $F(e) = \{x\}$ *and* $F(e^C) = \phi$ *for each* $e^C\in E-\{e\}$ *, and the soft point* (F,E) *is denoted by* x_e. *We denote the family of all soft points over* X *by* $SP(X,E)$.

Definition 2.16. ***[28]*** *The soft point* x_e *is said to be in the soft set* (G,E), *denoted by* $x_e\in(G,E)$ *if* $x_e \subseteq (G, E)$.

Definition 2.17. ***([6],[19])*** *Let* $(F,E),(G,E) \in S(X,E)$ *and* $x_e\in SP(X,E)$. *Then we have:*

(a) $x_e\in (F,E)$ *if and only if* $x_e\notin(F,E)^c$.

(b) $x_e\in (F,E) \cup(G, E)$ *if and only if* $x_e\in(F,E)$ *or* $x_e\in(G, E)$.

(c) $x_e\in(F,E)\cap(G, E)$ *if and only if* $x_e\in (F,E)$ *and* $x_e\in(G,E)$.

(d) $(F,E) \subseteq (G,E)$ *if and only if* $x_e\in (F,E)$ *implies* $x_e\in (G,E)$.

Definition 2.18. ***[12]*** *Let* I *be a non-empty collection of soft sets over* X *with the same set of parameters* E.*Then* $I\in S(X,E)$ *is said to be a soft ideal on* X *if,*

(a) $(F,E)\in I$ *and* $(G,E) \in I$ *implies* $(F,E) \cup(G, E)\in I$.

(b) $(F,E)\in I$ *and* $(G,E)\subseteq(F,E)$ *implies* $(G,E)\in I$.

A soft topological space (X,τ,E) *with a soft ideal* I *called soft ideal topological space and is denoted by* (X,τ,E, I).

Definition 2.19. ***[12]*** *Let* (X, τ,E,I) *be a soft ideal topological space. Then,* $(F,E)^*(I, \tau)= \cup\{x_e\in \widetilde{X}:U,E)\cap(F,E)\notin I,\forall\ (U,E) \in \tau$ *containing* $x_e\}$ *is called the soft local function of (F,E) with respect to I and* τ.

Definition 2.20. ***[12]*** *Let* (X,τ,E, I) *be a soft ideal topological space and* $Cl^*: S(X,E)\to S(X,E)$ *be the soft closure operator such that* $Cl^*(F,E) = (F,E)\cup(F,E)^*$. *Then, there exists a unique soft topology over* X *with the same set of parameters* E*, finer than* τ*,called the* $*$ *-soft topology, denoted by* τ^*.

Definition 2.21. ***[12]*** *Let* (X,τ, E, I) *be a soft ideal topological space.Then,* $\beta(I,\tau)= \{(F,E)\text{-}(G,E): (F,E)\in\tau,(G,E)\in I\}$ *is a soft basis for the soft topology* τ^*.

Definition 2.22. *[21] A soft topological space* (X,τ,E) *is said to be soft almost regular if for each soft regular closed sets* (F,E) *and each soft point* x_e *such that* $x_e \notin (F,E)$*, there exists soft open sets* (U,E) *and* (V,E) *such that* $x_e \in (U,E)$, $(F,E) \subseteq (V,E)$ *and* $(U,E) \cap (V,E) = \phi$.

Definition 2.23. *[9] A soft ideal topological space* (X,τ,E,I) *is said to be a soft I-normal if for each pair of soft closed sets* (F,E) *and* (G,E) *such that* $(F,E) \cap (G,E) = \phi$*, there exists soft open sets* (U,E) and (V,E) *such that* $(F,E)-(U,E) \in I$, $(G,E)-(V,E) \in I$ *and* $(U,E) \cap (V,E) = \phi$.

Definition 2.24. *[9] Let* (X,τ,E) *be a soft topological space,* (F,E) *be a soft set and* $x_e \in X$*. Then* (F,E) *is called soft neighborhood of* x_e*, if there exists soft open set* (G,E) *such that* $x_e \in (G,E) \subseteq (F,E)$.

Definition 2.25. *[27] Let (X,τ,E) be a soft topological space and (F,E) and (G,E) be soft subsets of X. Then, (F,E) and (G,E) are said to be soft weakly separated if there exists soft open sets (U,E) and (V,E) of X such that (F,E) ⊆ (U,E), (U,E) ∩ (G,E) = ϕ and (G,E) ⊆ (V,E), (V,E) ∩ (F,E) = ϕ .*

Definition 2.26. *[27] Let (X,τ,E) be a soft topological space and (F,E) and (G,E) be soft subsets of X. Then, (F,E) and (G,E) are said to be soft strongly separated if there exists soft open sets (U,E) and (V,E) of X such that (F,E) ⊆ (U,E), (G,E) ⊆ (V,E) and (U,E) ∩ (V,E) = ϕ .*

Definition 2.27. *[27] A soft topological space (X,τ,E) is said to be soft weakly regular, if every soft weakly separated pair consisting of a soft regular closed set and a soft point can be soft strongly separated.*

Definition 2.28. *[3] A collection* $\{(G_i,E) : i \in I\}$ *of soft open sets is called soft open cover of* (X,τ,E) *if* $\widetilde{X} = \bigcup_{i \in I} (G_i,E)$.

Definition 2.29. *[3] A collection* $\{(G_i,E) : i \in I\}$ *of* (X,τ,E) *is called soft locally finite if for each soft point* x_e *of* $\widetilde{X}$ *, there is a soft open set* (W,E) *satisfies that* $x_e \in (W,E)$ *and a set* $\{m: (W,E) \cap (G_m,E) \neq \phi\}$ *is finite.*

Definition 2.30. *[3] A soft topological space* (X,τ,E) *is soft compact (resp. soft Lindelöf) if every soft open cover of* $\widetilde{X}$ *has a soft finite (resp. countable) sub-collection which covers* $\widetilde{X}$.

Definition 2.31. *[3] A soft topological space* (X,τ,E) *is called soft almost compact (resp. soft almost Lindelöf) if every soft open cover of* $\widetilde{X}$ *has a soft finite (resp. countable) sub-cover,the soft closure of whose members cover* $\widetilde{X}$.

Definition 2.32. *[8] A soft topological space* (X,τ,E) *is called soft paracompact (resp. soft nearly paracompact) if every soft open (resp. soft regular open) covering admits a soft locally finite open refinements. A subset* (U,E) *is called soft nearly paracompact if the relative topology defined on it is soft nearly paracompact.*

3. Main Results

Definition 3.1. *A soft ideal topological space* (X,τ,E,I) *is said to be soft mildly I-normal if for each pair of soft regular closed subsets* (F,E) *and* (G,E) *of* X*, such that* $(F,E) \cap (G,E) = \emptyset$ *,there exists soft open sets* (U,E) *and* (V,E) *such that* $(F,E)-(U,E) \in I$, $(G,E)-(V,E) \in I$ *and* $(U,E) \cap (V,E) = \emptyset$.

Remark 3.2. *Every soft almost I-normal space is soft mildly I-normal but the converse may not be true. For,*

Example 3.3. *Let (X,τ,E) be soft topological spaces where* $X=\{a,b\}$, $E=\{e_1,e_2\}$ $\tau=\{\phi, X, \{(e_1,X),(e_2,\{b\})\}, \{(e_1,\{a\}),(e_2,X)\}, \{(e_1,\{a\}),(e_2,\{b\})\}\}$, $I=\{\emptyset, \{(e_1,\{b\}),(e_2,\phi)\}, \{(e_1,\phi),(e_2,\{a\})\}, \{(e_1,\{b\}),(e_2,\{a\})\}\}$*. Then* (X,τ,E,I) *is soft mildly I-normal but it is not soft almost I-normal.*

Theorem 3.4. *Let* (X,τ,E,I) *be a soft ideal topological space over* X*. Then the following conditions are equivalent:*

(i) (X,τ,E,I) *is soft mildly I-normal.*

(ii) *For each soft regular closed set* (F,E) *and each soft regular open set* (G,E) *containing* (F,E)*, there exists a soft open set* (V,E) *such that* $(F,E)-(V,E) \in I$ and $Cl(V,E)-(G,E) \in I$.

(iii) *For each soft regular open set* (G,E) *containing a soft regular closed set* (F,E) *, there exists a soft regular open set* (U,E) *such that* $(F,E)-(U,E) \in I$ *and* $Cl(U,E)-(G,E) \in I$.

(iv) *For each pair of soft regular closed sets* (F,E) *and* (G,E) *such that* $(F,E) \widetilde{\cap} (G,E) = \phi$*, there exists soft open sets* (U,E) *and* (V,E) *such that* $(F,E)-(U,E) \in I$, $(G,E)-(V,E) \in I$ *and* $Cl(U,E) \widetilde{\cap} Cl(V,E) = \phi$.

Proof. **(i)** ⇒**(ii)** Let (G,E) be a soft regular open set containing a soft regular closed set (F,E), then $(F,E) \widetilde{\cap} (G,E)^C = \phi$, where (F,E) and $(G,E)^C$ are soft regular closed subsets of X.Therefore there exists soft open sets (U,E) and (V,E) such that $(F,E)-(V,E) \in I$, $(G,E)^C-(U,E) \in I$ and $(U,E) \widetilde{\cap} (V,E) = \phi$. It follows that $(U,E)^C-(G,E) \in I$ and thus $(F,E)-(V,E) \in I$ and $Cl(V,E)-(G,E) \in I$.

(ii) ⇒**(iii)** Let (G,E) be a soft regular open set containing a soft regular closed set (F,E), then there exists a soft open set (V,E) such that $(F,E)-(V,E) \in I$ and $Cl(V,E)-(G,E) \in I$. Let $Int(Cl(V,E)) = (U,E)$, then $(F,E)-(U,E) \in I$ and $Cl(U,E)-(G,E) \in I$, where (U,E) is soft regular open.

(iii)⇒**(iv)** Let (F,E) and (G,E) be a soft regular closed sets such that $(F,E) \widetilde{\cap} (G,E) = \phi$, then $(F,E)-(G,E)^C \in I$ .Therefore there exists a soft regular open set (M,E) such that $(F,E)-(M,E) \in I$ and $Cl(M,E)-(G,E)^C \in I$. Again, since (M,E) is a soft regular open set containing the soft regular closed set (F,E) , therefore there exists a soft regular open set (U,E) such that $(F,E)-(U,E) \in I$ and $Cl(U,E)-(M,E) \in I$.

Let $(\mathrm{Cl}(\mathrm{M,E}))^{\mathrm{C}}=(\mathrm{V,E})$. Then, $(\mathrm{F,E})-(\mathrm{U,E})\in\mathrm{I}$, $(\mathrm{G,E})-(\mathrm{V,E})\in\mathrm{I}$ and $\mathrm{Cl}(\mathrm{U,E})\ \widetilde{\cap}\ \mathrm{Cl}(\mathrm{V,E})=\phi$.

(iv)⇒(i) is obvious. □

Lemma 3.5. *Let* $(\mathrm{F,A}),(\mathrm{G,A})\in\mathrm{S}(\mathrm{X,E})$ *and* $\mathrm{f_{pu}}:\mathrm{S}(\mathrm{X,E})\rightarrow\mathrm{S}(\mathrm{Y,K})$ *is a injective mapping. Then* $\mathrm{f_{pu}}((\mathrm{F,A})-(\mathrm{G,A}))=\mathrm{f_{pu}}(\mathrm{F,A})-\mathrm{f_{pu}}(\mathrm{G,A})$.

Theorem 3.6. *Soft mildly I-normality is preserved under soft closed, soft continuous and soft open mappings.*

Proof. Let $\mathrm{f_{pu}}$ be a soft closed, soft continuous and soft open mapping of a soft mildly I-normal space $(\mathrm{X},\tau,\mathrm{E,I})$ onto a space $(\mathrm{Y},\eta,\mathrm{K,J})$. Let $(\mathrm{F,K})$ be a soft closed subset of Y and $(\mathrm{G,K})$ be a soft regular open subset of Y containing $(\mathrm{F,K})$. Put $\mathrm{f_{pu}^{-1}}(\mathrm{F,K})=(\mathrm{M,E})$ and $\mathrm{f_{pu}^{-1}}(\mathrm{G,K})=(\mathrm{N,E})$. Then $(\mathrm{M,E})$ is a soft closed subset of $(\mathrm{X},\tau,\mathrm{E,I})$ contained in the soft open set $(\mathrm{N,E})$ because $\mathrm{f_{pu}}$ is soft continuous. Since $(\mathrm{N,E})$ is soft open, $(\mathrm{N,E})-\mathrm{Int}(\mathrm{Cl}(\mathrm{N,E}))\in\mathrm{I}$.Thus $\mathrm{Int}(\mathrm{Cl}(\mathrm{N,E}))$ is a soft regular open set containing the soft closed set $(\mathrm{M,E})$. Since $(\mathrm{X},\tau,\mathrm{E,I})$ is soft mildly I-normal, there exists a soft open set $(\mathrm{U,E})$ such that $(\mathrm{M,E})-(\mathrm{U,E})\in\mathrm{I}$ and $\mathrm{Cl}(\mathrm{U,E})-\mathrm{Int}(\mathrm{Cl}(\mathrm{N,E}))\in\mathrm{I}$. And So, by **Lemma 3.5**, $\mathrm{f_{pu}}(\mathrm{M,E})-\mathrm{f_{pu}}(\mathrm{U,E})\in\mathrm{J}$ and $\mathrm{f_{pu}}(\mathrm{Cl}(\mathrm{U,E}))-\mathrm{f_{pu}}(\mathrm{Int}(\mathrm{Cl}(\mathrm{N,E})))\in\mathrm{J}$. Since $\mathrm{f_{pu}}$ is soft continuous and soft open therefore $\mathrm{Cl}(\mathrm{N,E})=\mathrm{Cl}(\mathrm{f_{pu}^{-1}}(\mathrm{G,K}))=\mathrm{f_{pu}^{-1}}(\mathrm{Cl}(\mathrm{G,K}))$. Also, since $\mathrm{f_{pu}}$ is soft open and soft continuous, $\mathrm{Int}(\mathrm{f_{pu}^{-1}}(\mathrm{Cl}(\mathrm{G,K}))=\mathrm{f_{pu}^{-1}}(\mathrm{Int}(\mathrm{Cl}(\mathrm{G,K}))$. Thus $\mathrm{f_{pu}}(\mathrm{Int}(\mathrm{Cl}(\mathrm{N,E})))=\mathrm{Int}(\mathrm{Cl}(\mathrm{G,K}))$. Again, since $\mathrm{f_{pu}}$ is soft open, $\mathrm{f_{pu}}(\mathrm{U,E})$ is soft open and $\mathrm{f_{pu}}$ is soft closed and soft continuous, $\mathrm{f_{pu}}(\mathrm{Cl}(\mathrm{U,E}))=(\mathrm{Cl}(\mathrm{f_{pu}}(\mathrm{U,E}))$.Thus $\mathrm{f_{pu}}(\mathrm{U,E})$ is a soft open set of Y such that $(\mathrm{F,K})-\mathrm{f_{pu}}(\mathrm{U,E})\in\mathrm{J}$ and $\mathrm{Cl}(\mathrm{f_{pu}}(\mathrm{U,E}))-(\mathrm{G,K})\in\mathrm{J}$. Hence $(\mathrm{Y},\eta,\mathrm{K,J})$ is soft mildly I- normal. □

Lemma 3.7. *If* Y *is a soft regular closed subset of* X *and* $(\mathrm{F,E})$ *be a soft regular closed subset of* Y *then* $(\mathrm{F,E})$ *is soft regular closed subset of* X.

Theorem 3.8. *Every soft regular closed subspace of a soft mildly I-normal space is soft mildly I-normal.*

Proof. Let $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E,I})$ be a soft regular closed subspace of a soft mildly I-normal space $(\mathrm{X},\tau,\mathrm{E,I})$. Let $(\mathrm{F,E})$ and $(\mathrm{G,E})$ be a soft regular closed subsets of Y such that $(\mathrm{F,E})\cap(\mathrm{G,E})=\phi$. Therefore by **Lemma 3.7** ,$(\mathrm{F,E})$ and $(\mathrm{G,E})$ are soft regular closed subsets of X . Since $(\mathrm{X},\tau,\mathrm{E,I})$ is soft mildly I-normal there exists soft open sets $(\mathrm{U,E})$ and $(\mathrm{V,E})$ of X such that $(\mathrm{F,E})-(\mathrm{U,E})\in\mathrm{I}$, $(\mathrm{G,E})-(\mathrm{V,E})\in\mathrm{I}$ and $(\mathrm{U,E})\cap(\mathrm{V,E})=\phi$. And so, $(\mathrm{U,E})\cap\widetilde{\mathrm{Y}}$ and $(\mathrm{V,E})\cap\widetilde{\mathrm{Y}}$ are soft open sets of Y such that$(\mathrm{F,E})-((\mathrm{U,E})\cap\widetilde{\mathrm{Y}})\in\mathrm{I}$ and $(\mathrm{G,E})-((\mathrm{V,E})\cap\widetilde{\mathrm{Y}})\in\mathrm{I}$ and $\left((\mathrm{U,E})\cap\widetilde{\mathrm{Y}}\right)\cap\left((\mathrm{V,E})\cap\widetilde{\mathrm{Y}}\right)=((\mathrm{U,E})\cap(\mathrm{V,E}))\cap\widetilde{\mathrm{Y}}=\phi$. Hence $(\mathrm{Y},\tau_{\mathrm{Y}},\mathrm{E,I})$ is soft mildly I-normal. □

Definition 3.9. *A soft ideal topological space (X, τ,E,I) is said to be soft almost I- regular, if for every soft regular closed set (G, E) of X such that for each soft point $x_e\notin$ (G,E) there exists disjoint soft open sets (U,E) and (V,E) such that,$x_e\in$(U,E), (G,E)-(V,E) ∈I .*

Definition 3.10. *[1] A soft subset* $(\mathrm{F,E})$ *of a soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft I-compact , if for every soft open cover* $\{(\mathrm{U,E})_\alpha:\alpha\in\bigwedge\}$ *of* $(\mathrm{F,E})$ *, there exists a soft finite sub-collection* $\{(\mathrm{U,E})_{\alpha_i}:i=1,2,\ldots n\}$ *such that* $(\mathrm{F,E})-\bigcup_{i=1}^{n}(\mathrm{U,E})_{\alpha_i}\in\mathrm{I}$.

Definition 3.11. *[1] A soft subset* $(\mathrm{F,E})$ *of a soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft nearly I-compact , if for every soft regular open cover* $\{(\mathrm{U,E})_\alpha:\alpha\in\bigwedge\}$ *of* $(\mathrm{F,E})$*, there exists a soft finite sub-collection* $\{(\mathrm{U,E})_{\alpha_i}:i=1,2,\ldots n\}$ *such that* $(\mathrm{F,E})-\bigcup_{i=1}^{n}(\mathrm{U,E})_{\alpha_i}\in\mathrm{I}$.

Definition 3.12. *[14] A soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft almost I-compact if every soft open cover of the space has a soft finite sub-collection, the soft closures (resp. the soft interiors of the soft closures) of whose members cover the space*

Definition 3.13. *[10] A soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft I- paracompact if and only if every soft open cover* $\mathscr{U}$ *of* $\widetilde{\mathrm{X}}$ *has a soft locally finite open refinement* $\mathscr{V}$ *such that* $\widetilde{\mathrm{X}}-\bigcup\{(\mathrm{V,E}):(\mathrm{V,E})\in\mathscr{V}\}\in\mathrm{I}$.

Definition 3.14. *[10] A soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft nearly I- paracompact if and only if every soft regular open cover* $\mathscr{U}$ *of* $\widetilde{\mathrm{X}}$ *has a soft locally finite open refinement* $\mathscr{V}$ *such that* $\widetilde{\mathrm{X}}-\bigcup\{(\mathrm{V,E}):(\mathrm{V,E})\in\mathscr{V}\}\in\mathrm{I}$.

Definition 3.15. *[22] A soft subset* $(\mathrm{F,E})$ *of a soft ideal topological space* $(\mathrm{X},\tau,\mathrm{E,I})$ *is said to be soft almost I-Lindelöf if for every soft open cover* $\{(\mathrm{V,E})_\alpha:\alpha\in\Lambda\}$ *of* $(\mathrm{F,E})$*, there exists a soft countable subset* Λ_0 *of* Λ *such that* $(\mathrm{A,E})$ - $\bigcup\{\mathrm{Cl}(\mathrm{V,E})_\alpha:\alpha\in\Lambda_0\}\in I$.

Lemma 3.16. *Every soft regular closed subset of a soft almost I-compact space is soft almost I-compact*

Theorem 3.17. *Every soft almost I-regular , soft almost I-compact space is soft mildly I- normal.*

Proof. Let (A,E) and (B,E) be a soft regular closed subsets of a soft almost I-regular , soft almost I-compact space $(\mathrm{X},\tau,\mathrm{E,I})$ such that (A,E)⋂(B,E) =ϕ .Since $(\mathrm{X},\tau,\mathrm{E,I})$ being soft almost I-regular , for each $x_e\in$ (A,E) there exists soft open set $(\mathrm{G,E})_{\mathrm{x_e}}$ and $(\mathrm{H,E})_{\mathrm{x_e}}$ such that $x_e\in(\mathrm{G,E})_{\mathrm{x_e}}$, (B,E) - $(\mathrm{H,E})_{\mathrm{x_e}}\in\mathrm{I}$ and $\mathrm{Cl}(\mathrm{G,E})_{\mathrm{x_e}}\cap\mathrm{Cl}(\mathrm{H,E})_{\mathrm{x_e}}=\phi$. Then $\{(\mathrm{G,E})_{\mathrm{x_e}}\cap(\mathrm{A,E}):\mathrm{x_e}\in(\mathrm{A,E})\}$ is a relatively soft open covering of $(\mathrm{A,E})$. By **Lemma 3.16** ,(A,E) is soft almost I-compact. It follows that there exists a soft finite subfamily $\{(\mathrm{G,E})_{\mathrm{x_{e_i}}}\cap(\mathrm{A,E}):\ \mathrm{i}=1,\ldots..\mathrm{n}\}$ whose soft closure covers $(\mathrm{A,E})$ .Obviously, then $(\mathrm{A,E})-\bigcup_{i=1}^{n}\mathrm{Cl}((\mathrm{G,E})_{\mathrm{x_{e_i}}})\in\mathrm{I}$. Let $(\mathrm{H,E})=\bigcap_{i=1}^{n}(\mathrm{H,E})_{\mathrm{x_{e_i}}}$and $(\mathrm{G,E})=\{\bigcap_{i=1}^{n}\mathrm{Cl}(\mathrm{H,E})_{\mathrm{x_{e_i}}}\}^{C}$. Also $(\mathrm{A,E})-(\mathrm{G,E})\in\mathrm{I}$, $(\mathrm{B,E})-(\mathrm{H,E})\in\mathrm{I}$, and $(\mathrm{G,E})\cap(\mathrm{H,E})=\phi$. Hence $(\mathrm{X},\tau,\mathrm{E,I})$ is soft mildly I-normal. □

Theorem 3.18. *Every soft closed subset of a soft almost I-Lindelöf space is soft almost I-Lindelöf.*

Proof. Obvious. □

Theorem 3.19. *Every soft almost I-regular, soft almost I-Lindelof space is soft mildly I-normal.*

Proof. Since (X,τ,E,I) be soft almost I- regular, soft almost I-Lindelof space and let (A,E) , (B,E) be soft regular closed subsets of X such that $(A,E)\cap(B,E)=\phi$. For each $x_e\in(A,E)$, there exists soft open set $(U,E)_{x_e}$ such that $x_e\in(U,E)_{x_e}$ and $Cl(U,E)_{x_e}-(B,E)^C\in I$. It follows that for each soft point $x_e\in(A,E)$, there is a soft open set $(U,E)_{x_e}$ such that $x_e\in(U,E)_{x_e}$ and $(Cl(U,E)_{x_e})\cap(B,E)=\phi$. Then $\mathscr{U}=\{(U,E)_{x_e}:x_e\in(A,E)\}$ is a soft open covering of (A,E). By **Theorem 3.18** , $\mathscr{U}$ admits of a soft countable subcovering $\{(U,E)_n : n=1,2\ldots.\}$. Similarly, for each soft point $y_e\in(B,E)$, there exists soft open set $(V,E)_{y_e}$ such that $y_e\in(V,E)_{y_e}$ and $Cl\,(V,E)_{y_e}\cap(A,E)=\phi$. Again $\mathscr{V}=\{(V,E)_{y_e}:y_e\in(B,E)\}$ is a soft open covering of the soft almost I-Lindelöf set (B,E) and therefore $\mathscr{V}$ has a soft countable subcovering $\{(V,E)_n : n=1,2\ldots.\}$. Let $(A,E)_n=(U,E)_n\sim\bigcup\{Cl(V,E)_k : k\leq n\}$ and let $(B,E)_n=(V,E)_n\sim\bigcup\{Cl(U,E)_k : k\leq n\}$ for each n=1,2...,where $\sim$ denotes the relative soft compliment . Since $(A,E)_n\cap(V,E)_m=\phi$ for all $n\geq m$,therefore $(A,E)_n\cap(B,E)_m=\phi$ for all $n\geq m$. Similarly $(A,E)_n\cap(B,E)_m=\phi$ for all $n\leq m$. Hence $(A,E)_n\cap(B,E)_m=\phi$ for all m,n. If $(G,E)=\bigcup\{(A,E)_n : n=1,2\ldots\}$ and $(H,E)=\bigcup\{(B,E)_n : n=1,2\ldots\}$, then (G,E) and (H,E) are soft open sets such that $(A,E)-(G,E)\in I$, $(B,E)-(H,E)\in I$ and $(G,E)\cap(H,E)=\phi$. Hence (X,τ,E,I) is soft mildly I-normal. □

Lemma 3.20. *A soft ideal topological space* $(X,\tau,E,I$ *) is soft weakly I-regular if for each soft point* x_e *and each soft regular open set* (U,E) *containing* x_e *,there is a soft open set* (V,E) *such that* $x_e\in(V,E)$ *and* $Cl(V,E)-(U,E)\in I$.

Theorem 3.21. *Every soft weakly I-regular, soft nearly I-paracompact space is soft mildly I-normal.*

Proof. Since (X,τ,E,I) is soft weakly I- regular, soft nearly I-paracompact space and (A,E) , (B,E) be a soft regular closed subsets of X , such that $(A,E)\cap(B,E)=\phi$. Let $x_e\in(A,E)$. Then $Cl\{x_e\}-(A,E)\in I$ and $(A,E)-(B,E)^C\in I$. Since(X,τ,E,I) is soft weakly I-regular , there exists a soft open sets $(V,E)_{x_e}$ such that $Cl(V,E)_{x_e}\cap(B,E)=\phi$ and $x_e\in(V,E)_{x_e}$. Then, $\{Int(Cl(V,E)_{x_e}) : x_e\in(A,E)\}\cup(A,E)^C\}$ is a soft regular open covering of X. Since (X,τ,E,I) is soft nearly I-paracompact , therefore this soft covering has a soft locally finite open refinement .Let $\mathscr{U}=\{(U,E)_\alpha : \alpha\in\bigwedge\}$ be the family of those members of this refinement which intersects (A,E) . Let $(U,E)=\bigcup\{(U,E)_\alpha : \alpha\in\bigwedge\}$. Then (U,E) is a soft open set containing (A,E). Let $(U,E)^*=(\bigcup\{Cl(U,E)_\alpha : \alpha\in\bigwedge\})^C$. Then $(U,E)^*$ is a soft open set such that $(U,E)\cap(U,E)^*=\phi$. For each $\alpha\in\bigwedge$, there exists $x_e\in(A,E)$ such that $(U,E)_\alpha-((Int(Cl(V,E))_{x_e})\in I$. Since $Cl(U,E)_\alpha-Cl(Int(Cl(V,E)_{x_e}))=Cl(V,E)_{x_e}\in I$and $Cl(V,E)_{x_e}-(B,E)^C\in I$, Thus, $(B,E)\cap Cl(U,E)_\alpha=\phi$ for all α. Thus $(B,E)-(\bigcup\{Cl(U,E)_\alpha : \alpha\in\bigwedge\})^C=(W,E)\in I$. Then $(A,E)-(U,E)\in I$, $(B,E)-(W,E)\in I$ and $(U,E)\cap(W,E)=\phi$. □

Lemma 3.22. *Every soft almost I- regular space is soft weakly I- regular.*

Corollary 3.23. *Every soft almost I-regular , soft nearly I-paracompact space is soft mildly I- normal.*

Proof. It follows from **Lemma 3.22 .** □

4. Conclusion

In the present paper, we extended the concept of soft mildly I-normality to soft sets and presented its studies in soft topological spaces.

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