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Extension of fuzzy Johnson algorithm to three machines fuzzy flow-shop scheduling problem

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Abstract

This paper deals with the famous two machines Johnson algorithm in the fuzzy environment and extension of the same into three machines case under certain conditions.Here the fuzzification and defuzzification are handled with octagonal fuzzy numbers.

Keywords

Flow-shop scheduling, Fuzzy flowshop,Octagonal fuzzy numbers,Fuzzy Johnson algorithm, Makespan, Ranking methods.

AMS Subject Classification

90C70,97M40.

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Contents

1. Introduction

A 2-machine flow shop problem with the objective to minimize the makespan is called as Johnson's problem. In 1954, Johnson [4] developed a algorithm to solve a 2- machine flow shop problem which was a first leap towards the algorithmic approach to flow shop scheduling problems.In the literature,one could find the elaborate use of triangular and trapezoidal fuzzy numbers ([7],[2],[3],[5]).Octagonal fuzzy numbers are also involved into active research in fuzzy flowshop scheduling by Selvamalar et al.,([8],[9],[10],[11],[12]). This paper deals with the Johnson's 2-machine flow shop problem in fuzzy environment and it has been extended to 3-machine case also under certain conditions. Section 2 deals with the regular Johnson algorithm for 2 machines and section 3 discusses with its extension to three or more machines under the condition of dominance.Section3 gives the conclusion.

2. N-jobs 2-machines Flowshop

2.1 Assumptions

- The number of machines is restricted to two.
- The two machines are continuously available and they execute only one job at a time
- The number of jobs is not restricted
- Every job consists of two operations to be completed in series of two machines
- A job started on one machine must be completed before another job is started

2.2 Algorithm

- Consider a N-jobs 2-machines flow shop problem with processing times in the form of octagonal fuzzy numbers.
- Convert the fuzzy execution times into crisp numbers using the ranking method
Ranking Method(Malini et al.,[6]):
If $\tilde{O} = (o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8; k, 1)$ is a normal octagonal fuzzy number ,then the rank of $\tilde{O}$ is evaluated as follows: If $k = 0.5$ then
$$\Re(\tilde{O}) = \frac{1}{8}[o_1 + o_2 + o_3 + o_4 + o_5 + o_6 + o_7 + o_8] \quad (2.1)$$
In this case the rank is called the general mean value(GMV).
- Sorting the jobs into two sets U and V where
$U = \{J_i/GMV(\tilde{E}(i,1)) < GMV(\tilde{E}(i,2))\}$
$V = \{J_i/GMV(\tilde{E}(i,1)) \geq GMV(\tilde{E}(i,2))\}$
- Arranging the jobs in the set U in an increasing pattern of their GMV's in machine 1
- Arranging the jobs in the set V in a decreasing pattern of their GMV's in machine 2
- An optimal schedule is obtained from the order of jobs in U and V.
- Find the makespan

2.3 Illustrative Example

Consider the flowshop problem with N-jobs,2-Workstations whose execution times are octagonal fuzzy numbers satisfying all the assumption of a flow shop given in table1.

To convert the octagonal fuzzy numbers into crisp numbers, the GMVs of the execution times are calculated using the equation2.1 and are listed in table2.

Now,sorting the jobs according to their GMVs into two sets U and V as:
$U = \{J_i/GMV(\tilde{E}(i,1)) < GMV(\tilde{E}(i,2))\} = \{1,3,4\}$
$V = \{J_i/GMV(\tilde{E}(i,1)) \geq GMV(\tilde{E}(i,2))\} = \{2,5,6\}$
Arranging jobs in U as:$4-1-3$ and arranging jobs in V as $6-5-2$.
$\therefore$ the optimal scheduling of jobs is $4-1-3-6-5-2$.
The waiting time of the machines, the execution and finish times for all the jobs of the illustrative problem are calculated and are listed in the Table3.

2.4 Advantage of Fuzzy Maximization over Fuzzy Subtraction

When waiting time of machines 1 and 2 is evaluated using fuzzy subtraction, one could get a fuzzy number with negative parts.Since the negative idle time is not natural, the negative part of the fuzzy number is replaced with zeroes.This results into a fuzzy number which is no longer octagonal.This will cause the membership functions of the completion times to break into many pieces and make subsequent calculations to fall in a impossible zone..This may have an effect on the mean flow time and the makespan which in turn affects the optimality of the solution. When the finish times are calculated using the equations and the fuzzy maximum operator

$$\tilde{F}(i,j) = \max\{\tilde{F}(i-1,j), \tilde{F}(i,j-1)\}(+)\tilde{E}(i,j) \quad (2.2)$$

with the assumption that *i-1* th job precedes i^{th} job in the sequence, the fuzzy finish times are yielded. (See table4) : The performance criteria is evaluated for both cases and results are compared in the table 5.

The difference found in the makespan due to fuzzy subtraction and fuzzy maximization is clearly depicted in the following figure.

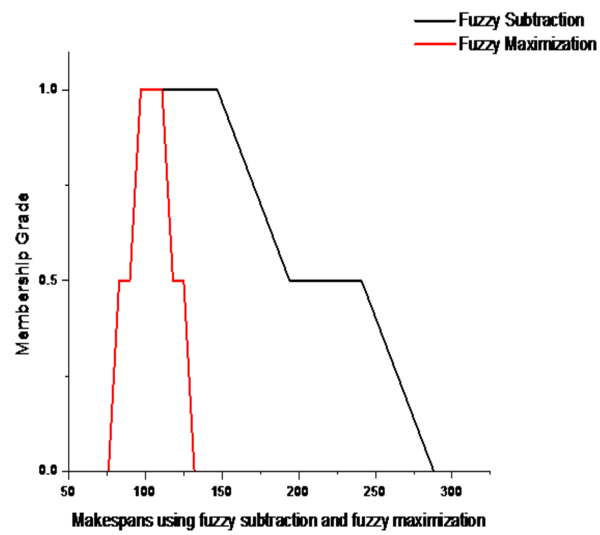

Figure 1. Makespans:Fuzzy Subtraction Vs Fuzzy Maximization

3. Extension of fuzzy Johnson algorithm for 3-machine flow shop

Johnson's algorithm can provide optimal solution to a two machines problem. But it cannot be generalized into a m-machine problem directly. Johnson had given a possibility of generalization only under certain conditions. The necessary condition for extension of Johnson's algorithm is that the 2nd machine should be dominated by either or both the machines 1 and 3.That is

1. Minimum execution time on Machine 1 $\geq$ maximum execution time Machine 2
2. Minimum execution time on Machine 3 $\geq$ maximum execution time Machine 2

Table 1. Illustrative Problem

Jobs	Machine1	Machine2
1	(12,13,14,15,17,18,19,20)	(14,15,16,17,19,20,21,22)
2	(16,17,18,19,21,22,23,24)	(6,7,8,9,11,12,13,14)
3	(15,16,17,18,20,21,22,23)	(16,17,18,19,21,22,23,24)
4	(7,8,9,10,12,13,14,15)	(11,12,13,14,16,17,18,19)
5	(9,10,11,12,14,15,16,17)	(8,9,10,11,13,14,15,16)
6	(11,12,13,14,16,17,18,19)	(10,11,12,13,15,16,17,18)

Table 2. GMVs of fuzzy Execution Times

Jobs	Machine1	Machine2
1	16	18
2	20	10
3	19	20
4	11	15
5	13	12
6	15	14

Table 3. Fuzzy Waiting,Execution and Finish times of the Optimum Schedule

Jobs	$\tilde{w}(i,1)$	$\tilde{E}(i,1)$	$\tilde{F}(i,1)$
4	0	(7,8,9,10,12,13,14,15)	(7,8,9,10,12,13,14,15)
1	(7,8,9,10,12,13,14,15)	(12,13,14,15,17,18,19,20)	(19,21,23,25,29,31,33,35)
3	(19,21,23,25,29,31,33,35)	(15,16,17,18,20,21,22,23)	(34,37,40,43,49,52,55,58)
6	(34,37,40,43,49,52,55,58)	(11,12,13,14,16,17,18,19)	(45,49,53,57,65,69,73,77)
5	(45,49,53,57,65,69,73,77)	(9,10,11,12,14,15,16,17)	(54,59,64,69,79,84,89,94)
2	(54,59,64,69,79,84,89,94)	(16,17,18,19,21,22,23,24)	(70,76,82,88,100,106,112,118)

Jobs	$\tilde{w}(i,2)$	$\tilde{E}(i,2)$	$\tilde{F}(i,2)$
4	0	(11,12,13,14,16,17,18,19)	(18,20,22,24,28,30,32,34)
1	(0,0,0,0,3,7,11,15)	(14,15,16,17,19,20,21,22)	(33,36,39,42,51,58,65,72)
3	(0,0,0,0,8,18,28,38)	(16,17,18,19,21,22,23,24)	(50,54,58,62,78,92,106,120)
6	(0,0,0,0,21,39,57,75)	(10,11,12,13,15,16,17,18)	(55,60,65,70,101,124,147,170)
5	(0,0,0,0,32,60,88,116)	(8,9,10,11,13,14,15,16)	(62,68,74,80,124,158,192,226)
2	(0,0,0,0,36,76,116,156)	(6,7,8,9,11,12,13,14)	(76,83,90,97,147,194,241,288)

Table 4. Fuzzy Finish times of the Optimum Schedule Using Maximization Operator

Job	$\tilde{E}(i,1)$	$\tilde{F}(i,1)$
4	(7,8,9,10,12,13,14,15)	(7,8,9,10,12,13,14,15)
1	(12,13,14,15,17,18,19,20)	(19,21,23,25,29,31,33,35)
3	(15,16,17,18,20,21,22,23)	(34,37,40,43,49,52,55,58)
6	(11,12,13,14,16,17,18,19)	(45,49,53,57,65,69,73,77)
5	(9,10,11,12,14,15,16,17)	(54,59,64,69,79,84,89,94)
2	(16,17,18,19,21,22,23,24)	(70,76,82,88,100,106,112,118)

Job	$\tilde{E}(i,2)$	$\tilde{F}(i,2)$
4	(11,12,13,14,16,17,18,19)	(18,20,22,24,28,30,32,34)
1	(14,15,16,17,19,20,21,22)	(33,36,39,42,48,51,54,57)
3	(16,17,18,19,21,22,23,24)	(50,54,58,62,70,74,78,82)
6	(10,11,12,13,15,16,17,18)	(60,65,70,75,85,90,95,100)
5	(8,9,10,11,13,14,15,16)	(68,74,80,86,98,104,110,116)
2	(6,7,8,9,11,12,13,14)	(76,83,90,97,111,118,125,132)

Table 5. Comparison of Results:Fuzzy Subtraction Vs Fuzzy Maximization

	Makespan $\tilde{M} = \underset{i}{max}\tilde{F}(i,2)$	Mean Flow Time $M\tilde{F}T = \left[(+)_{i=1}^{6}\tilde{F}(i,2)\right]/6$
Fuzzy Subtraction	(76,83,,90,97,147,194,241,288)	(49,53.5,58,62.5,88.17,109.33,130.5,151.67)
Fuzzy Maximization	(76,83,90,97,111,118,125,132)	(50.8,55.3,59.8,64.3,73.3,77.8,82.3,86.8)

When either of these two conditions or both the conditions are satisfied, one could say that the 2^{nd} machine is dominated by either 1^{st} or 3^{rd} or both the machines.
Let us now fuzzify these conditions to extend the two machine fuzzy Johnson algorithm into a three machines fuzzy Johnson's algorithm.
Let $\tilde{E}(i,j)$ be the execution time of a job i on machine $j = 1,2,3$ then the condition for dominance for machine2 becomes:

1. $\underset{i}{min}\tilde{E}(i,1) \succeq \underset{i}{max}\tilde{E}(i,2)$ where $i = 1,2,3,...,n$
2. $\underset{i}{min}\tilde{E}(i,3) \succeq \underset{i}{max}\tilde{E}(i,2)$ where $i = 1,2,3,...,n$

3.1 Assumptions

- The number of machines is restricted to three
- The three machines are continuously available and they execute only one job at a time
- The number of jobs is not restricted
- Every job consists of three operations to be completed in series of three machines
- A process began on one machine must be completed

3.2 Algorithm

1. Consider a n jobs 3 machines flowshop problem with execution times in the form of octagonal fuzzy numbers.
2. Convert the fuzzy execution times into crisp numbers using the ranking method
3. Check the condition for dominance with the execution times on all the three machines
4. Sort the jobs into two sets U and V where
   $U = \{J_i/GMV(\tilde{E}(i,1)) < GMV(\tilde{E}(i,2))\}$
   $V = \{J_i/GMV(\tilde{E}(i,1)) \geq GMV(\tilde{E}(i,2))\}$
5. Arrange the jobs in the set U in an increasing pattern of their GMV's in machine1
6. Arrange the jobs in the set V in a decreasing pattern of their GMV's in machine2
7. An optimal schedule is obtained from the order of jobs in A and B.
8. Find the makespan and minimum flow time

3.3 Illustration

A company has six jobs which go through three machines in the order 123.The execution times are found to be octagonal and are given by the table 6. Find the schedule of jobs that minimizes the total finish time for the process.
The problem has been solved by the step by step procedure:
Step1: Converting the fuzzy numbers into crisp numbers,we get the values tabulated in table 7. *Step2*:Finding the minimum of the execution times in M1 and maximum execution times in M2 Minimum of execution times in M1=12
Maximum of execution times in M2 =12
Since there is a tie between the values , it is necessary to compare their fuzzy relation using the ranking algorithm of Dhanalakshmi et al.,[1]
For a octagonal fuzzy number $\tilde{A} = (a,b,c,d,e,f,g,h;k,w)$ the rank is found using the following equations and the results are tabulated in the table 8 and table 9.

$$\Re_{k,w}(\tilde{A}) = \frac{w(c+d+e+f)}{4}$$
$$w-Divergence(\tilde{A}) = w(f-c)$$
$$w-Mode(\tilde{A}) = \frac{w(d+e)}{2}$$

since $w = ht(\tilde{A}) = 1$,

$$\Re_{k,w}(\tilde{A}) = \frac{(c+d+e+f)}{4}$$
$$w-Divergence(\tilde{A}) = (f-c)$$
$$w-Mode(\tilde{A}) = \frac{(d+e)}{2}$$

$\implies \tilde{A} \succ \tilde{B}.$
$\therefore$ Machine 2 is dominated by machine 1.

Similarly if we check the dominance between M3 and M2,

$\implies \tilde{A} \succ \tilde{B}.$
$\therefore$ Machine 2 is dominated by machine 3.
Step 3: Converting the three machine problem into a 2-machine pseudo problem(Refer table 10)
Step4: Converting into crisp numbers by finding the GMVs,wt get the values in table 11. *Step 5*:Sorting the jobs into two sets:
$U = \{1,4\}$
$V = \{2,3,5,6\}$
Step6: Arranging the jobs in U and V in increasing and decreasing pattern of their GMVs respectively then the optimum schedule $1-4-6-5-3-2$ is obtained.
The fuzzy finish times are evaluated using fuzzy maximum operator and tabulated in table 12.

Table 6. Illustrative Problem

Job	Machine1	Machine2	Machine3
1	(11,13,15,17,19,21,23,25)	(0,2,4,6,8,10,12,14)	(12,14,16,18,20,22,24,26)
2	(4,6,8,10,14,16,18,20)	(6,7,10,11,12,15,16,19)	(5,7,9,11,13,15,17,19)
3	(22,24,26,28,30,32,34,36)	(3,5,7,9,13,15,17,19)	(16,18,20,22,24,26,28,30)
4	(29,31,33,35,37,39,41,43)	(0,2,4,6,10,12,14,16)	(40,42,44,46,48,50,52,54)
5	(36,38,40,42,44,46,48,50)	(0,1,3,5,7,9,11,12)	(21,23,25,27,29,31,33,35)
6	(30,32,34,36,38,40,42,44)	(5,7,9,10,13,15,18,19)	(28,30,32,34,38,40,42,44)

Table 7. GMVs of Fuzzy Execution Times

Job	Machine1	Machine2	Machine3
1	18	7	19
2	**12**	**12**	**12**
3	29	11	23
4	36	8	47
5	43	6	28
6	37	**12**	36

Table 8. Comparison of Various Parameters

	$\tilde{A} = (4,6,8,10,14,16,18,20)$	$\tilde{B} = (6,7,10,11,12,15,16,19)$
$\mathfrak{R}_{k,w}(\tilde{A}) = \frac{(c+d+e+f)}{4}$	12	12
$w - Divergence(\tilde{A}) = (f-c)$	8	5

Table 9. Comparison of Various Parameters

	$\tilde{A} = (5,7,9,11,13,15,17,19)$	$\tilde{B} = (6,7,10,11,12,15,16,19)$
$\mathfrak{R}_{k,w}(\tilde{A}) = \frac{c+d+e+f}{4}$	12	12
$w - Divergence(\tilde{A}) = (f-c)$	6	5

Table 10. Conversion into Two Machine Problem

Job	G1=M1+M2	G2=M2+M3
1	(11,15,19,23,27,31,35,39)	(12,16,20,24,28,32,36,40)
2	(10,13,18,21,26,31,34,39)	(11,14,19,22,25,30,33,38)
3	(25,29,33,37,43,47,51,55)	(19,23,27,31,37,41,45,49)
4	(29,33,37,41,47,51,55,59)	(40,44,48,52,58,62,66,70)
5	(36,39,43,47,51,55,59,62)	(21,24,28,32,36,40,44,47)
6	(35,39,43,46,51,55,60,63)	(33,37,41,44,51,55,60,63)

Table 11. GMVs of Fuzzy Execution Times

Job	G1=M1+M2	G2=M2+M3
1	25	26
2	24	24
3	40	34
4	44	55
5	49	34
6	49	48

4. Conclusion

The two machine fuzzy flow shop problem is solved with fuzzy Johnson algorithm.The performance criteria are evaluated using fuzzy subtraction and fuzzy maximization .The results are compared to show the effectiveness of fuzzy maximum operator.The fuzzy Johnson algorithm is extended to 3-machines and are able to find the optimum solutions to certain problems.Because the extension requires the 2^{nd} machine to be dominated by either /both of the 1^{st} and/or 3^{rd} machines.Similarly ,this procedure could be extended to m-machines under the successful satisfaction of dominance property between the machines.

Table 12. Evaluation of Fuzzy Finish Times

Job	Machine1	Machine2	Machine3
1	(11,13,15,17,19,21,23,25)	(11,15,19,23,27,31,35,39)	(23,29,35,41,47,53,59,65)
4	(40,44,48,52,56,60,64,68)	(40,46,52,58,66,72,78,84)	(80,88,96,104,118,122,130,138)
6	(70,76,82,88,94,100,106,112)	(75,83,91,98,107,115,124,131)	(108,118,128,156,162,172,182)
5	(106,114,122,130,138,146,154,162)	(106,115,125,135,145,155,165,174)	(129,141,153,165,185,193,205,217)
3	(128,138,148,158,168,178,188,198)	(131,143,155,167,181,193,205,217)	(147,161,175,189,205,219,233,247)
2	(132,144,156,168,182,194,206,218)	(138,151,166,179,194,209,222,237)	(152,168,184,200,218,234,250,266)

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