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Exact solutions for linear systems of local fractional partial differential equations

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Abstract

The basic motivation of the present study is to apply the local fractional Sumudu decomposition method to solve linear system of local fractional partial differential equations. The local fractional Sumudu decomposition methodl (LFSDM) can easily be applied to many problems and it's capable of reducing the size of computational work to find non-differentiable solutions for similar problems. Some illustrative examples are given, revealing the effectiveness and convenience of this method.

Keywords

Local fractional derivative operator, local fractional Sumudu decomposition method, linear systems of local fractional partial differential equations.

AMS Subject Classification

44A05, 26A33, 44A20, 34K37.

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1. Introduction

Systems of partial differential equations have attracted much attention in a variety of applied sciences. The general ideas and the essential features of these systems are of wide applicability. These systems were formally derived to describe wave propagation, to control the shallow water waves, and to examine the chemical reaction-diffusion model of Brusselator [1]. To solve these equations or systems, researchers use many methods, among them, we find an Adomian decomposition method (ADM) [2], homotopy perturbation method (HPM) [3], variational iteration method (VIM) [4], Fourier transform method [5], Fourier series method [6], Laplace transform method [7], and Sumudu transform method [8], and then extended it to solve differential equations of fractional orders. Recently, there appeared a large part of scientific research concerning local fractional differential equations or local fractional partial differential, adopted in its entirety on the above mentioned methods to solve this new types of equations. For example, among these research we find, local fractional Adomian decomposition method ([9], [10], [15]), local fractional homotopy perturbation method ([11], [12]), local fractional homotopy perturbation Sumudu transform method [13], local fractional variational iteration method ([14], [15]), local fractional variational iteration transform method ([16]-[18]), local fractional Fourier series method ([19]-[21]), Laplace transform series expansion method [22], local fractional Sumudu transform method ([23], [24]), local fractional Sumudu transform series expansion method ([25], [26]), local fractional Sumudu decomposition method for linear partial differential equations with local fractional derivative [27].

The basic motivation of the present study is to extend the application of the local fractional Sumudu decomposition method suggested by D. Ziane et al.[27] to solve linear system of local fractional partial differential equations. The advantage of this method is its ability to combine two powerful methods to obtain exact solutions for linear system of local fractional partial differential equations. Three examples are given to re-confirm the effectiveness of this method.

The present paper has been organized as follows: In Section 2 some basic definitions and properties of the local fractional calculus and local fractional Sumudu transform method.

In section 3 We present an analysis of the proposed method. In section 4 We apply the modified method (LFSDM) to solve the proposed systems. Finally, the conclusion follows.

2. Preliminaries

In this section, we present the basic theory of local fractional calculus and we focus specifically on the definitions of the following concepts: Local fractional derivative, local fractional integral, some important results and local fractional Sumudu transform.

Definition 2.1. *The local fractional derivative of $\Psi(r)$ of order η at $r=r_0$ is defined ([28],[29])*

$$\Psi^{(\eta)}(r)=\left.\frac{d^\eta\Phi}{dr^\eta}\right|_{r=r_0}=\frac{\Delta^\eta(\Psi(r)-\Psi(r_0))}{(r-r_0)^\eta}, \tag{2.1}$$

where

$$\Delta^\eta(\Psi(r)-\Psi(r_0))\cong\Gamma(1+\eta)\left[(\Psi(r)-\Psi(r_0))\right]. \tag{2.2}$$

For any $r\in(\alpha,\beta)$, there exists

$$\Psi^{(\eta)}(r)=D_r^\eta\Psi(r),$$

denoted by

$$\Psi(r)\in D_r^\eta(\alpha,\beta).$$

Local fractional derivative of high order is written in the form

$$\Psi^{(m\eta)}(r)=\overbrace{D_r^{(\eta)}\cdots D_r^{(\eta)}}^{m\ times}\Psi(r), \tag{2.3}$$

and local fractional partial derivative of high order $\ln 3$

$$\frac{\partial^{m\eta}\Psi(r)}{\partial r^{m\eta}}=\overbrace{\frac{\partial^\eta}{\partial r^\eta}\cdots\frac{\partial^\eta}{\partial r^\eta}}^{m\ times}\Psi(r). \tag{2.4}$$

Definition 2.2. *The local fractional integral of $\Psi(r)$ of order η in the interval $[\alpha,\beta]$ is defined as ([28],[29])*

$$\begin{aligned}{}_\alpha I_\beta^{(\eta)}\Psi(r) &= \frac{1}{\Gamma(1+\eta)}\int_\alpha^\beta\Psi(s)(ds)^\eta\\ &= \frac{1}{\Gamma(1+\eta)}\lim_{\Delta s\longrightarrow 0}\sum_{i=0}^{N-1}f(s_i)(\Delta s_i)^\eta, \end{aligned}\tag{2.5}$$

where $\Delta s_i=s_{i+1}-s_i$, $\Delta s=\max\{\Delta s_0,\Delta s_1,\Delta s_2,\cdots\}$ and $[s_i,s_{i+1}]$, $s_0=\alpha$, $s_N=\beta$, is a partition of the interval $[\alpha,\beta]$. For any $r\in(\alpha,\beta)$, there exists

$${}_\alpha I_r^{(\eta)}\Psi(r),$$

denoted by

$$\Psi(r)\in I_r^{(\eta)}(\alpha,\beta).$$

Definition 2.3. *In fractal space, the Mittage Leffler function, sine function and cosine function are defined as ([28],[29])*

$$E_\eta(r^\eta)=\sum_{m=0}^{+\infty}\frac{r^{m\eta}}{\Gamma(1+m\eta)},\quad 0<\eta\leqslant 1, \tag{2.6}$$

$$\sin_\eta(r^\eta)=\sum_{m=0}^{+\infty}(-1)^\eta\frac{r^{(2m+1)\eta}}{\Gamma(1+(2m+1)\eta)},\quad 0<\eta\leqslant 1, \tag{2.7}$$

$$\cos_\eta(r^\eta)=\sum_{m=0}^{+\infty}(-1)^\eta\frac{r^{2m\eta}}{\Gamma(1+2m\eta)},\ 0<\eta\leqslant 1, \tag{2.8}$$

The properties of local fractional derivatives and integral of nondifferentiable functions are given by ([28],[29])

$$\frac{d^\eta}{dr^\eta}\frac{r^{m\eta}}{\Gamma(1+m\eta)}=\frac{r^{(m-1)\eta}}{\Gamma(1+(m-1)\eta)}. \tag{2.9}$$

$$\frac{d^\eta}{dr^\eta}E_\eta(r^\eta)=E_\eta(r^\eta). \tag{2.10}$$

$$\frac{d^\eta}{dr^\eta}\sin_\eta(r^\eta)=\cos_\eta(r^\eta). \tag{2.11}$$

$$\frac{d^\eta}{dr^\eta}\cos_\eta(r^\eta)=-\sin_\eta(r^\eta).$$

$$\frac{d^\eta}{dr^\eta}\sinh_\eta(r^\eta)=\cosh_\eta(r^\eta). \tag{2.12}$$

$$\frac{d^\eta}{dr^\eta}\cosh_\eta(r^\eta)=\sinh_\eta(r^\eta). \tag{2.13}$$

Definition 2.4. *[30] The local fractional Sumudu transform of $\Psi(r)$ of order η is defined as*

$$\begin{aligned} LFS_\eta\{\Psi(r)\} &= F_\eta(u),\ 0<\eta\leqslant 1 \qquad (2.14)\\ &= \frac{1}{\Gamma(1+\eta)}\int_0^\infty E_\eta(-u^{-\eta}r^\eta)\frac{\Psi(r)}{u^\eta}(dr)^\eta \end{aligned}$$

Following (2.14), its inverse formula is defined as

$$LFS_\eta^{-1}\{F_\eta(u)\}=\Psi(r),\ \ 0<\eta\leqslant 1. \tag{2.15}$$

Theorem 2.5. *(1) (local fractional Sumudu transform of local fractional derivative). If $LFS_\eta\{\Psi(r)\} = F_\eta(u)$, then one has*

$$LFS_\eta\left\{\frac{d^\eta \Psi(r)}{dr^\eta}\right\} = \frac{F_\eta(u) - F(0)}{u^\eta}. \tag{2.16}$$

As the direct result of (2.16), we have the following results. If $LFS_\eta\{\Psi(r)\} = F_\eta(u)$, we obtain

$$LFS_\eta\left\{\frac{d^{n\eta}\Psi(r)}{dr^{n\eta}}\right\} = \frac{1}{u^{n\eta}}\left[F_\eta(u) - \sum_{k=0}^{n-1} u^{k\eta}\Psi^{(k\eta)}(0)\right]. \tag{2.17}$$

When $n = 2$, from (2.17), we get

$$LFS_\eta\left\{\frac{d^{2\eta}\Psi(r)}{dr^{2\eta}}\right\} = \frac{1}{u^{2\eta}}\left[F_\eta(u) - \Psi(0) - u^\eta\Psi^{(\eta)}(0)\right]. \tag{2.18}$$

(2) (local fractional Sumudu transform of local fractional integral). If $LFS_\eta\{\Psi(r)\} = F_\eta(u)$, then we have

$$LFS_\eta\left\{{}_0I_r^{(\eta)}\,\Psi(r)\right\} = u^\eta F_\eta(u). \tag{2.19}$$

3. Analysis of the Method

To illustrate the basic idea of this method, we consider a general linear operator with local fractional derivative

$$\begin{cases} \frac{\partial^\eta V}{\partial s^\eta} + \frac{\partial^\eta W}{\partial r^\eta} + R_1(V,W) = f(r,s), \\ \frac{\partial^\eta W}{\partial s^\eta} + \frac{\partial^\eta V}{\partial r^\eta} + R_2(V,W) = g(r,s), \end{cases} \tag{3.1}$$

where $\frac{\partial^\eta}{\partial(\cdot)^\eta}$ denotes linear local fractional derivative operator of order η, R_1, R_2 are the linear operators, and $f(r,s)$, $g(r,s)$ are the nondifferentiable source terms.

Taking the local fractional Sumudu transform (denoted in this paper by LFS_η) on both sides of (3.1), we get

$$\begin{cases} LFS_\eta\left[\frac{\partial^\eta V}{\partial s^\eta}\right] + LFS_\eta\left[\frac{\partial^\eta W}{\partial r^\eta}\right] + LFS_\eta\left[R_1(V,W)\right] = LFS_\eta\left[f(r,s)\right] \\ LFS_\eta\left[\frac{\partial^\eta W}{\partial s^\eta}\right] + LFS_\eta\left[\frac{\partial^\eta V}{\partial r^\eta}\right] + LFS_\eta\left[R_2(V,W)\right] = LFS_\eta\left[g(r,s)\right] \end{cases} \tag{3.2}$$

Using the property of the local fractional Sumudu transform, we have

$$\begin{cases} LFS_\eta\left[V(r,s)\right] = V(r,0) + u^\eta\left(LFS_\eta\left[f(r,s)\right]\right) \\ \quad -u^\eta\left(LFS_\eta\left[\frac{\partial^\eta W}{\partial r^\eta} + R_1(V,W)\right]\right) \\ LFS_\eta\left[W(r,s)\right] = W(r,0) + u^\eta\left(LFS_\eta\left[g(r,s)\right]\right) \\ \quad -u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V}{\partial r^\eta} + R_2(V,W)\right]\right) \end{cases} \tag{3.3}$$

Taking the inverse local fractional Sumudu transform on both sides of (3.3), gives

$$\begin{cases} V(r,s) = V(r,0) + LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[f(r,s)\right]\right)\right) \\ \quad -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta W}{\partial r^\eta} + R_1(V,W)\right]\right)\right) \\ W(r,s) = W(r,0) + LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[g(r,s)\right]\right)\right) \\ \quad -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V}{\partial r^\eta} + R_2(V,W)\right]\right)\right) \end{cases} \tag{3.4}$$

According to the Adomian decomposition method [2], we decompose the two unknown functions V and W as an infinite series given by

$$\begin{matrix} V(r,s) = \sum_{n=0}^\infty V_n(r,s) \\ W(r,s) = \sum_{n=0}^\infty W_n(r,s) \end{matrix}. \tag{3.5}$$

Substituting (3.5) in (3.4), we get

$$\begin{cases} V(r,s) = V(r,0) + LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[f(r,s)\right]\right)\right) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^\infty W_n\right) + R_1\left(\sum_{n=0}^\infty V_n, \sum_{n=0}^\infty W_n\right)\right]\right)\right) \\ W(r,s) = W(r,0) + LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[g(r,s)\right]\right)\right) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^\infty V_n\right) + R_2\left(\sum_{n=0}^\infty V_n, \sum_{n=0}^\infty W_n\right)\right]\right)\right) \end{cases} \tag{3.6}$$

On comparing both sides of (3.6), we have

$$\begin{aligned} V_0(r,s) &= V(r,0) + LFS_\eta^{-1}\left(u^\eta\, LFS_\eta\left[f(r,s)\right]\right), \\ V_1(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta W_0}{\partial r^\eta} + R_1(V_0,W_0)\right]\right)\right), \\ V_2(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta W_1}{\partial r^\eta} + R_1(V_1,W_1)\right]\right)\right), \\ V_3(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta W_2}{\partial r^\eta} + R_1(V_2,W_2)\right]\right)\right), \\ &\vdots \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} W_0 &= W(r,0) + LFS_\eta^{-1}\left(u^\eta\, LFS_\eta\left[g(r,s)\right]\right), \\ W_1 &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_0}{\partial r^\eta} + R_2(V_0,W_0)\right]\right)\right), \\ W_2 &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_1}{\partial r^\eta} + R_2(V_1,W_1)\right]\right)\right), \\ W_3 &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_2}{\partial r^\eta} + R_2(V_2,W_2)\right]\right)\right), \\ &\vdots \end{aligned} \tag{3.8}$$

Finally, we approximate the analytical nondifferentiable solution (V,W) of the system (3.1) by

$$\begin{cases} V(r,s) = \lim_{N\to\infty}\sum_{n=0}^N V_n(x,t) \\ W(r,s) = \lim_{N\to\infty}\sum_{n=0}^N W_n(x,t) \end{cases} \tag{3.9}$$

4. Applications

In this section, we will implement the proposed method local fractional Sumudu decomposition method (LFSDM) for solving some exemples.

Example 4.1. *First, we consider the homogeneous linear system of local fractional partial differential equations*

$$\begin{cases} \frac{\partial^\eta U}{\partial s^\eta} - \frac{\partial^\eta V}{\partial r^\eta} + U + V = 0 \\ \frac{\partial^\eta V}{\partial s^\eta} - \frac{\partial^\eta U}{\partial r^\eta} + U + V = 0 \end{cases}, 0 < \eta \leqslant 1, \tag{4.1}$$

with initial conditions

$$U(r,0) = \sinh_\eta(r^\eta),\ V(r,0) = \cosh_\eta(r^\eta). \tag{4.2}$$

Taking the local fractional Sumudu transform on both sides of each equation of the system (4.1), we have

$$\begin{cases} LFS_\eta\left[U(r,s)\right] = U(r,0) \\ -u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s)}{\partial r^\eta} + U(r,s) + V(r,s)\right]\right) \\ LFS_\eta\left[V(r,s)\right] = V(r,0) \\ -u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U(r,s)}{\partial r^\eta} + U(r,s) + V(r,s)\right]\right) \end{cases} \tag{4.3}$$

Taking the inverse local fractional Sumudu transform on both sides of each equation of the system (4.3) subject to the initial conditions (4.2), we obten

$$\begin{cases} U(r,s) = \sinh_\eta(r^\eta) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s)}{\partial r^\eta} + U(r,s) + V(r,s)\right]\right)\right) \\ V(r,s) = \cosh_\eta(r^\eta) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U(r,s)}{\partial r^\eta} + U(r,s) + V(r,s)\right]\right)\right) \end{cases} \tag{4.4}$$

According to the Adomian decomposition method [2], we decompose the unknown functions U and V as two infinite series given by

$$\begin{aligned} U(r,s) &= \textstyle\sum_{n=0}^{\infty} U_n(r,s), \\ V(r,s) &= \textstyle\sum_{n=0}^{\infty} V_n(r,s). \end{aligned} \tag{4.5}$$

Substituting (4.5) in (4.4), we get

$$\begin{cases} \sum_{n=0}^{\infty} U_n(r,s) = \sinh_\eta(r^\eta) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\begin{array}{c} -\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^{\infty} V_n(r,s)\right) \\ +\sum_{n=0}^{\infty} U_n(r,s) + \sum_{n=0}^{\infty} V_n(r,s) \end{array}\right]\right)\right) \\ \sum_{n=0}^{\infty} V_n(r,s) = \cosh_\eta(r^\eta) \\ -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\begin{array}{c} -\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^{\infty} U_n(r,s)\right) \\ +\sum_{n=0}^{\infty} U_n(r,s) + \sum_{n=0}^{\infty} V_n(r,s) \end{array}\right]\right)\right) \end{cases} \tag{4.6}$$

On comparing both sides of (4.6), we have

$$\begin{aligned} U_0(r,s) &= \sinh_\eta(r^\eta), \\ U_1(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_0(r,s)}{\partial r^\eta} + U_0(r,s) + V_0(r,s)\right]\right)\right), \\ U_2(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_1(r,s)}{\partial r^\eta} + U_1(r,s) + V_1(r,s)\right]\right)\right), \\ U_3(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_2(r,s)}{\partial r^\eta} + U_2(r,s) + V_2(r,s)\right]\right)\right), \\ &\vdots \end{aligned} \tag{4.7}$$

$$\begin{aligned} V_0(r,s) &= \cosh_\eta(r^\eta), \\ V_1(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_0(r,s)}{\partial r^\eta} + U_0(r,s) + V_0(r,s)\right]\right)\right), \\ V_2(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_1(r,s)}{\partial r^\eta} + U_1(r,s) + V_1(r,s)\right]\right)\right), \\ V_3(r,s) &= -LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_2(r,s)}{\partial r^\eta} + U_1(r,s) + V_2(r,s)\right]\right)\right), \\ &\vdots \end{aligned} \tag{4.8}$$

and so on.

From the equations (4.7)-(4.8), the first solution terms of local fractional sumudu decomposition method of the system (4.1), is given by

$$\begin{aligned} U_0(r,s) &= \sinh_\eta(r^\eta), \\ U_1(r,s) &= -\cosh_\eta(r^\eta)\frac{s^\eta}{\Gamma(1+\eta)}, \\ U_2(r,s) &= \sinh_\eta(r^\eta)\frac{s^{2\eta}}{\Gamma(1+2\eta)}, \\ U_3(r,s) &= -\cosh_\eta(r^\eta)\frac{s^{3\eta}}{\Gamma(1+3\eta)}, \\ &\vdots \end{aligned} \tag{4.9}$$

and

$$\begin{aligned} V_0(r,s) &= \cosh_\eta(r^\eta), \\ V_1(r,s) &= -\sinh_\eta(r^\eta)\frac{s^\eta}{\Gamma(1+\eta)}, \\ V_2(r,s) &= \cosh_\eta(r^\eta)\frac{s^{2\eta}}{\Gamma(1+2\eta)}, \\ V_3(r,s) &= -\sinh_\eta(r^\eta)\frac{s^{3\eta}}{\Gamma(1+3\eta)}, \\ &\vdots \end{aligned} \tag{4.10}$$

Then the local fractional solution (U,V) in series form is given by

$$\begin{aligned} U(r,s) &= \sinh_\eta(r^\eta)\left(1 + \frac{s^{2\eta}}{\Gamma(1+2\eta)} + \cdots\right) \\ &\quad -\cosh_\eta(r^\eta)\left(\frac{s^\eta}{\Gamma(1+\eta)} + \frac{s^{3\eta}}{\Gamma(1+3\eta)} + \cdots\right), \\ V(r,s) &= \cosh_\eta(r^\eta)\left(1 + \frac{s^{2\eta}}{\Gamma(1+2\eta)} + \cdots\right) \\ &\quad -\sinh_\eta(r^\eta)\left(\frac{s^\eta}{\Gamma(1+\eta)} + \frac{s^{3\eta}}{\Gamma(1+3\eta)} + \cdots\right), \end{aligned} \tag{4.11}$$

and the solution (U,V) in the close form is

$$\begin{aligned} U(r,s) &= \sinh_\eta(r^\eta - s^\eta). \\ V(r,s) &= \cosh_\eta(r^\eta - s^\eta). \end{aligned} \tag{4.12}$$

Substituting $\eta = 1$ into (4.12), we obtain

$$\begin{aligned} U(r,s) &= \sinh(r-s). \\ V(r,s) &= \cosh(r-s). \end{aligned} \tag{4.13}$$

This obtained result is the same as in the article [31] in the case $\alpha = \beta = 1$, as well as the same result presented in the article [32] in the same case.

Example 4.2. *Second, we consider the nonhomogeneous linear system of local fractional partial differential equations*

$$\begin{cases} \frac{\partial^\eta U}{\partial s^\eta} - \frac{\partial^\eta V}{\partial r^\eta} - U + V = -2 \\ \frac{\partial^\eta V}{\partial s^\eta} + \frac{\partial^\eta U}{\partial r^\eta} - U + V = -2 \end{cases} ,0 < \eta \leqslant 1, \tag{4.14}$$

subject to the initial conditions

$$U(r,0) = 1 + E_\eta(r^\eta),\ V(r,0) = -1 + E_\eta(r^\eta). \tag{4.15}$$

Taking the local fractional Sumudu transform on both sides of each equation of the system (4.14), we have

$$\begin{cases} LFS_\eta\left[U(r,s)\right] = U(r,0) - 2u^\eta \\ +u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V(r,s)}{\partial r^\eta} + U(r,s) - V(r,s)\right]\right) \\ LFS_\eta\left[V(r,s)\right] = V(r,0) - 2u^\eta \\ +u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U(r,s)}{\partial r^\eta} + U(r,s) - V(r,s)\right]\right) \end{cases} \tag{4.16}$$

Taking the inverse local fractional Sumudu transform on both sides of each equation of the system (4.16) subject to the initial conditions (4.15), we obtain

$$\begin{cases} U(r,s) = 1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)} \\ +LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V(r,s)}{\partial r^\eta} + U(r,s) - V(r,s)\right]\right)\right) \\ V(r,s) = -1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)} \\ +LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U(r,s)}{\partial r^\eta} + U(r,s) - V(r,s)\right]\right)\right) \end{cases} \tag{4.17}$$

According to the Adomian decomposition method [2], we decompose the unknown functions U and V as two infinite series given by

$$\begin{aligned} U(r,s) &= \textstyle\sum_{n=0}^\infty U_n(r,s), \\ V(r,s) &= \textstyle\sum_{n=0}^\infty V_n(r,s). \end{aligned} \tag{4.18}$$

Substituting (4.18) in (4.17), we get

$$\begin{cases} \sum_{n=0}^\infty U_n(r,s) = 1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)} \\ +LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^\infty V_n\right) + \sum_{n=0}^\infty U_n - \sum_{n=0}^\infty V_n\right]\right)\right) \\ \sum_{n=0}^\infty V_n(r,s) = -1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)} \\ +LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta}{\partial r^\eta}\left(\sum_{n=0}^\infty U_n\right) + \sum_{n=0}^\infty U_n - \sum_{n=0}^\infty V_n\right]\right)\right) \end{cases} \tag{4.19}$$

On comparing both sides of (4.19), we have

$$\begin{aligned} U_0(r,s) &= 1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ U_1(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_0(r,s)}{\partial r^\eta} + U_0(r,s) - V_0(r,s)\right]\right)\right), \\ U_2(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_1(r,s)}{\partial r^\eta} + U_1(r,s) - V_1(r,s)\right]\right)\right), \\ U_3(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[\frac{\partial^\eta V_2(r,s)}{\partial r^\eta} + U_2(r,s) - V_2(r,s)\right]\right)\right), \\ &\vdots \end{aligned} \tag{4.20}$$

$$\begin{aligned} V_0(r,s) &= -1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ V_1(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_0(r,s)}{\partial r^\eta} + U_0(r,s) - V_0(r,s)\right]\right)\right), \\ V_2(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_1(r,s)}{\partial r^\eta} + U_1(r,s) - V_1(r,s)\right]\right)\right), \\ V_3(r,s) &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta U_2(r,s)}{\partial r^\eta} + U_2(r,s) - V_2(r,s)\right]\right)\right), \\ &\vdots \end{aligned} \tag{4.21}$$

and so on.

From the equations (4.20)-(4.21), the first solution terms of local fractional sumudu decomposition method of the system (4.14), is given by

$$\begin{aligned} U_0(r,s) &= 1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ U_1(r,s) &= E_\eta(r^\eta)\frac{s^\eta}{\Gamma(1+\eta)} + 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ U_2(r,s) &= E_\eta(r^\eta)\frac{s^{2\eta}}{\Gamma(1+2\eta)}, \\ U_3(r,s) &= E_\eta(r^\eta)\frac{s^{3\eta}}{\Gamma(1+3\eta)}, \\ &\vdots \end{aligned} \tag{4.22}$$

and

$$\begin{aligned} V_0(r,s) &= -1 + E_\eta(r^\eta) - 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ V_1(r,s) &= -E_\eta(r^\eta)\frac{s^\eta}{\Gamma(1+\eta)} + 2\frac{s^\eta}{\Gamma(1+\eta)}, \\ V_2(r,s) &= E_\eta(r^\eta)\frac{s^{2\eta}}{\Gamma(1+2\eta)}, \\ V_3(r,s) &= -E_\eta(r^\eta)\frac{s^{3\eta}}{\Gamma(1+3\eta)}, \\ &\vdots \end{aligned} \tag{4.23}$$

Then the local fractional solution (U,V) in series form is given by

$$
\begin{aligned}
U(r,s) &= 1+E_\eta(r^\eta)(1+\tfrac{s^\eta}{\Gamma(1+\eta)}+\tfrac{s^{2\eta}}{\Gamma(1+2\eta)}+\tfrac{s^{3\eta}}{\Gamma(1+3\eta)}+\cdots),\\
V(r,s) &= -1+E_\eta(r^\eta)(1-\tfrac{s^\eta}{\Gamma(1+\eta)}+\tfrac{s^{2\eta}}{\Gamma(1+2\eta)}-\tfrac{s^{3\eta}}{\Gamma(1+3\eta)}+\cdots),
\end{aligned} \tag{4.24}
$$

and the solution (U,V) in the close form is

$$
\begin{aligned}
U(r,s) &= 1+E_\eta((r+s)^\eta).\\
V(r,s) &= -1+E_\eta((r-s)^\eta).
\end{aligned} \tag{4.25}
$$

Substituting $\eta=1$ into (4.25), we obtain

$$
\begin{aligned}
U(r,s) &= 1+e^{r+s}.\\
V(r,s) &= -1+e^{r-s}.
\end{aligned} \tag{4.26}
$$

This obtained result is the same as in the article [1], as well as the same work presented in [33].

Example 4.3. *Finaly, we consider the homogeneous linear system of local fractional partial differential equations*

$$
\begin{cases}
\frac{\partial^\eta U}{\partial t^\eta}+\frac{\partial^\eta V}{\partial r^\eta}-\frac{\partial^\eta W}{\partial s^\eta}=W\\
\frac{\partial^\eta V}{\partial t^\eta}+\frac{\partial^\eta W}{\partial r^\eta}+\frac{\partial^\eta U}{\partial s^\eta}=U\\
\frac{\partial^\eta W}{\partial t^\eta}+\frac{\partial^\eta V}{\partial r^\eta}-\frac{\partial^\eta V}{\partial r^\eta}=V
\end{cases} ,0<\eta\leqslant 1, \tag{4.27}
$$

subject to the initial conditions

$$
\begin{aligned}
U(r,s,0) &= \sin_\eta((r+s)^\eta),\\
V(r,s,0) &= \cos_\eta((r+s)^\eta),\\
W(r,s,0) &= -\sin_\eta((r+s)^\eta).
\end{aligned} \tag{4.28}
$$

Taking the local fractional Sumudu transform on both sides of each equation of the system (4.27), we have

$$
\begin{cases}
LFS_\eta[U(r,s,t)] = U(r,s,0)\\
+u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s,t)}{\partial r^\eta}+\frac{\partial^\eta W(r,s,t)}{\partial s^\eta}+W(r,s,t)\right]\right)\\
LFS_\eta[V(r,s,t)] = V(r,s,0)\\
+u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta W(r,s,t)}{\partial r^\eta}-\frac{\partial^\eta U(r,s,t)}{\partial s^\eta}+U(r,s,t)\right]\right)\\
LFS_\eta[W(r,s,t)] = W(r,s,0)\\
+u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s,t)}{\partial r^\eta}+\frac{\partial^\eta V(r,s,t)}{\partial s^\eta}+V(r,s,t)\right]\right)
\end{cases} \tag{4.29}
$$

Taking the inverse local fractional Sumudu transform on both sides of each equation of the system (4.29) subject to the initial conditions (4.28), we obtain

$$
\begin{cases}
U(r,s,t) = \sin_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s,t)}{\partial r^\eta}+\frac{\partial^\eta W(r,s,t)}{\partial s^\eta}+W(r,s,t)\right]\right)\right)\\
V(r,s,t) = \cos_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta W(r,s,t)}{\partial r^\eta}-\frac{\partial^\eta U(r,s,t)}{\partial s^\eta}+U(r,s,t)\right]\right)\right)\\
W(r,s,t) = -\sin_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V(r,s,t)}{\partial r^\eta}+\frac{\partial^\eta V(r,s,t)}{\partial s^\eta}+V(r,s,t)\right]\right)\right)
\end{cases} \tag{4.30}
$$

According to the Adomian decomposition method [2], we decompose the unknown functions U and V as two infinite series given by

$$
\begin{aligned}
U(r,s,t) &= \textstyle\sum_{n=0}^\infty U_n(r,s,t),\\
V(r,s,t) &= \textstyle\sum_{n=0}^\infty V_n(r,s,t),\\
W(r,s,t) &= \textstyle\sum_{n=0}^\infty W_n(r,s,t),
\end{aligned} \tag{4.31}
$$

Substituting (4.31) in (4.30), we get

$$
\begin{cases}
\sum_{n=0}^\infty U_n(r,s,t) = \sin_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta}{\partial r^\eta}(\sum_{n=0}^\infty V_n)+\frac{\partial^\eta}{\partial s^\eta}(\sum_{n=0}^\infty W_n)+\sum_{n=0}^\infty W_n\right]\right)\right)\\
\sum_{n=0}^\infty V_n(r,s,t) = \cos_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta}{\partial r^\eta}(\sum_{n=0}^\infty W_n)-\frac{\partial^\eta}{\partial s^\eta}(\sum_{n=0}^\infty U_n)+\sum_{n=0}^\infty U_n\right]\right)\right)\\
\sum_{n=0}^\infty V_n(r,s,t) = -\sin_\eta((r+s)^\eta)\\
+LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta}{\partial r^\eta}(\sum_{n=0}^\infty V_n)+\frac{\partial^\eta}{\partial s^\eta}(\sum_{n=0}^\infty V_n)+\sum_{n=0}^\infty V_n\right]\right)\right)
\end{cases} \tag{4.32}
$$

On comparing both sides of (4.32), we have

$$
\begin{aligned}
U_0 &= \sin_\eta((r+s)^\eta),\\
U_1 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_0}{\partial r^\eta}+\frac{\partial^\eta W_0}{\partial s^\eta}+W_0\right]\right)\right),\\
U_2 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_1}{\partial r^\eta}+\frac{\partial^\eta W_1}{\partial s^\eta}+W_1\right]\right)\right),\\
U_3 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_2}{\partial r^\eta}+\frac{\partial^\eta W_2}{\partial s^\eta}+W_2\right]\right)\right),\\
&\vdots
\end{aligned} \tag{4.33}
$$

$$
\begin{aligned}
V_0 &= \cos_\eta((r+s)^\eta),\\
V_1 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta W_0}{\partial r^\eta}-\frac{\partial^\eta U_0}{\partial s^\eta}+U_0\right]\right)\right),\\
V_2 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta W_1}{\partial r^\eta}-\frac{\partial^\eta U_1}{\partial s^\eta}+U_1\right]\right)\right),\\
V_3 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta W_2}{\partial r^\eta}-\frac{\partial^\eta U_2}{\partial s^\eta}+U_2\right]\right)\right),\\
&\vdots
\end{aligned} \tag{4.34}
$$

and

$$\begin{aligned}
W_0 &= -\sin_\eta((r+s)^\eta),\\
W_1 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_0}{\partial r^\eta}+\frac{\partial^\eta V_0}{\partial s^\eta}+V_0\right]\right)\right),\\
W_2 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_1}{\partial r^\eta}+\frac{\partial^\eta V_1}{\partial s^\eta}+V_1\right]\right)\right),\\
W_3 &= LFS_\eta^{-1}\left(u^\eta\left(LFS_\eta\left[-\frac{\partial^\eta V_2}{\partial r^\eta}+\frac{\partial^\eta V_2}{\partial s^\eta}+V_2\right]\right)\right),\\
&\vdots
\end{aligned}\tag{4.35}$$

and so on.

From the equations (4.33)-(4.35), the first solution terms of local fractional sumudu decomposition method of the system (4.27), is given by

$$\begin{aligned}
U_0(r,s,t) &= \sin_\eta((r+s)^\eta),\\
U_1(r,s,t) &= -\cos_\eta((r+s)^\eta)\frac{t^\eta}{\Gamma(1+\eta)},\\
U_2(r,s,t) &= -\sin_\eta((r+s)^\eta)\frac{t^{2\eta}}{\Gamma(1+2\eta)},\\
U_3(r,s,t) &= \cos_\eta((r+s)^\eta)\frac{t^{3\eta}}{\Gamma(1+3\eta)},\\
U_4(r,s,t) &= \sin_\eta((r+s)^\eta)\frac{t^{4\eta}}{\Gamma(1+4\eta)},\\
&\vdots
\end{aligned}\tag{4.36}$$

$$\begin{aligned}
V_0(r,s,t) &= \cos_\eta((r+s)^\eta),\\
V_1(r,s,t) &= \sin_\eta((r+s)^\eta)\frac{t^\eta}{\Gamma(1+\eta)},\\
V_2(r,s,t) &= -\cos_\eta((r+s)^\eta)\frac{t^{2\eta}}{\Gamma(1+2\eta)},\\
V_3(r,s,t) &= -\sin_\eta((r+s)^\eta)\frac{t^{3\eta}}{\Gamma(1+3\eta)},\\
V_4(r,s,t) &= \cos_\eta((r+s)^\eta)\frac{t^{4\eta}}{\Gamma(1+4\eta)},\\
&\vdots
\end{aligned}\tag{4.37}$$

and

$$\begin{aligned}
W_0(r,s,t) &= -\sin_\eta((r+s)^\eta),\\
W_1(r,s,t) &= \cos_\eta((r+s)^\eta)\frac{t^\eta}{\Gamma(1+\eta)},\\
W_2(r,s,t) &= \sin_\eta((r+s)^\eta)\frac{t^{2\eta}}{\Gamma(1+2\eta)},\\
W_3(r,s,t) &= -\cos_\eta((r+s)^\eta)\frac{t^{3\eta}}{\Gamma(1+3\eta)},\\
W_4(r,s,t) &= -\sin_\eta((r+s)^\eta)\frac{t^{4\eta}}{\Gamma(1+4\eta)},\\
&\vdots
\end{aligned}\tag{4.38}$$

Then, the local fractional solution (U,V,W) in series form, is given by

$$\begin{aligned}
U(r,s,t) &= \sin_\eta((r+s)^\eta)\left(1-\frac{t^{2\eta}}{\Gamma(1+2\eta)}+\frac{t^{4\eta}}{\Gamma(1+4\eta)}+\cdots\right)\\
&\quad -\cos_\eta((r+s)^\eta)\left(\frac{t^\eta}{\Gamma(1+\eta)}-\frac{t^{3\eta}}{\Gamma(1+3\eta)}+\cdots\right),\\
V(r,s,t) &= \cos_\eta((r+s)^\eta)\left(1-\frac{s^{2\eta}}{\Gamma(1+2\eta)}+\frac{s^{4\eta}}{\Gamma(1+4\eta)}+\cdots\right)\\
&\quad +\sin_\eta((r+s)^\eta)\left(\frac{t^\eta}{\Gamma(1+\eta)}-\frac{t^{3\eta}}{\Gamma(1+3\eta)}+\cdots\right),\\
W(r,s,t) &= -\sin_\eta((r+s)^\eta)\left(1-\frac{t^{2\eta}}{\Gamma(1+2\eta)}+\frac{t^{4\eta}}{\Gamma(1+4\eta)}+\cdots\right)\\
&\quad +\cos_\eta((r+s)^\eta)\left(\frac{t^\eta}{\Gamma(1+\eta)}-\frac{t^{3\eta}}{\Gamma(1+3\eta)}+\cdots\right),
\end{aligned}\tag{4.39}$$

and the solution (U,V,W) in the close form is

$$\begin{aligned}
U(r,s,t) &= \sin_\eta((r+s-t)^\eta).\\
V(r,s,t) &= \cos_\eta((r+s-t)^\eta).\\
W(r,s,t) &= -\sin_\eta((r+s-t)^\eta).
\end{aligned}\tag{4.40}$$

In the case of $\eta=1$, we get

$$\begin{aligned}
U(r,s,t) &= \sin(r+s-t).\\
V(r,s,t) &= \cos(r+s-t).\\
W(r,s,t) &= -\sin(r+s-t).
\end{aligned}\tag{4.41}$$

This result is the exact solution of the system (4.27) in the case of $\eta=1$

5. Conclusion

In this work, we have seen that the composite method of Adomian decomposition method and Sumudu transform method in the sense of local fractional derivative, proved very effective to solve linear systems of local fractional partial differential equations. The local fractional Sumudu decomposition method (LFSDM) is suitable for such problems and is very user friendly. The advantage of this method is its ability to combine two powerful methods for obtaining exact solutions of linear systems of local fractional partial differential equations. The results obtained in the examples presented, showed that this method is capable of solving other problems of these types.

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