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Lower level subhemirings of an anti-fuzzy soft subhemirings of hemiring

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Abstract

Hemiring show up in a characteristic way, in few applications to the theory of automata, the theory of formal dialects, chart theory, plan theory and combinatorial geometry. As of late, the ideas of delicate subhemiring of a hemiring with unique structures were presented. In this paper we try to investigate some logarithmic thought of lower level subhemiring of an anti-fuzzy soft subhemiring of a hemiring. This is done by introducing some properties of hemiring.

Keywords

Fuzzy soft subset, fuzzy soft subhemiring, anti-fuzzy soft subhemiring, pseudo fuzzy soft coset.

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1. Introduction

In this segment, we exhibit lower level subhemiring of an anti-fuzzy soft sub hemiring of a hemiring and set up a couple of results on these. We moreover made an attempt to consider the properties of lower level subhemiring of an anti-fuzzy soft subhemiring of a hemiring under homomorphism and anti-homomorphism.

2. Preliminaries

Definition 2.1. *Let R be a hemiring. An intuitionistic fuzzy soft subset (F,A) of R is said to be an intuitionistic fuzzy soft subhemiring (IFSSHR) of R in case it satisfies the going with conditions:*

(i) $\mu_{(F,A)}(x_{(F,A)}+y_{(F,A)}) \geq \min\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}$,

(ii) $\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \geq \min\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}$,

(iii) $\nu_{(F,A)}(x_{(F,A)}+y_{(F,A)}) \leq \max\{\nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)})\}$,

(iv) $\nu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \max\{\nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)})\}$,

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

Definition 2.2. *Let (F,A) be an intuitionistic fuzzy soft subset in a set S, the most grounded intuitionistic fuzzy Soft relation on S, that is an intuitionistic fuzzy soft relation on (F,A) is (G,V) given by $\mu_{(G,V)}(x_{(G,V)}, y_{(G,V)}) = \min\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}$ and $\nu_{(G,V)}(x_{(G,V)}, y_{(G,V)}) = \max\{\nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)})\}$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in S.*

Definition 2.3. *An intuitionistic fuzzy soft subhemiring A of a hemiring R is presently as an intuitionistic fuzzy soft characteristic $x_{(F,A)}(x_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}))$ and $\nu_{(F,A)}(x_{(F,A)}) = \nu_{(F,A)}(f(x_{(F,A)}))$, for all $x_{(F,A)}$ in R and f in $Aut(R)$.*

Definition 2.4. *$(R,+,\cdot)$ and $(R',+,\cdot)$ be any two Hemirings. Let $f : R \to R'$ be any function and (F,A) be an intuitionistic fuzzy soft subhemiring in R, (G,V) be an intuitionistic fuzzy soft subhemiring in $f(R) = R'$, portrayed by $\mu_{(G,V)}(y_{(G,V)}) = \sup_{x \in f^{-1}(y)} \mu_{(F,A)}(x_{(F,A)})$ and $\nu_{(G,V)}(y_{(G,V)}) = \inf_{x \in f^{-1}(y)} \nu_{(F,A)}(x_{(F,A)})$, for all $x_{(F,A)}$ in R and $y_{(G,V)}$ in R'. At the point (F,A) is presently as a preimage of (G,V) under f and is indicated by $f^{-1}((G,V))$.*

Note: This definition is used all through this part for image and preimage in capacities.

Definition 2.5. *An intuitionistic fuzzy soft subset A of a set X is said to be standardized if there exist x in X with the true objective that* $\mu_{(F,A)}(x_{(F,A)}) = 1$ *and* $\nu_{(F,A)}(x_{(F,A)}) = 0$.

Definition 2.6. *Let* (F,A) *be an intuitionistic fuzzy soft subhemiring of a hemiring* $(R,+,\cdot)$ *and an in R. By at the point, the pseudo intuitionistic fuzzy soft coset* $(a(F,A))^p$ *is described by* $((a\mu_{(F,A)})^p)(x_{(F,A)}) = p(a)\mu_{(F,A)}(x_{(F,A)})$ *and* $((a\nu_{(F,A)})^p)(x_{(F,A)}) = p(a)\nu_{(F,A)}(x_{(F,A)})$*, for every* $x_{(F,A)}$ *in R and some p in P.*

Definition 2.7. *Let* (F,A) *be an intuitionistic fuzzy soft subset of X. For* α,β *in* $[0,1]$*, the level soft subset of* (F,A) *is the set,* $(F,A)_{(\alpha,\beta)} = \{x_{(F,A)} \in X : \mu_{(F,A)}(x_{(F,A)}) \geq \alpha,\ \nu_{(F,A)}(x_{(F,A)}) \leq \beta\}$*. This is called an intuitionistic fuzzy soft level subset of A.*

Definition 2.8. *Let* (F,A) *be a fuzzy soft subset of X. For* α *in* $[0,1]$*, the lower level soft subset of* (F,A) *is the set* $A_\alpha = \{x_{(F,A)} \in X : \mu_{(F,A)}(x_{(F,A)}) \leq \alpha\}$.

3. Lower level subhemirings of an anti-fuzzy soft subhemiring of a hemiring

Theorem 3.1. *Let* (F,A) *be an anti-fuzzy soft subhemiring of a hemiring R. At that point* α *in* $[0,1], (F,A)_\alpha$ *is a lower level soft subhemiring of R.*

Proof. For all $x_{(F,A)}$ and y in $(F,A)_\alpha$, we have $\mu_{(F,A)}(x_{(F,A)}) \leq \alpha$ and $\mu_{(F,A)}(y_{(F,A)}) \leq \alpha$. Presently,

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)})\\ &\leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}\\ &\leq \max\{\alpha,\alpha\}\\ &= \alpha,\end{aligned}$$

which infers that $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \alpha$. And

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)}y_{(F,A)})\\ &\leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}\\ &\leq \max\{\alpha,\alpha\}\\ &= \alpha,\end{aligned}$$

which infers that $\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha$. In this manner, $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \alpha$ and $\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha$. In this manner, $\mu_{(F,A)} + y_{(F,A)}$ and $x_{(F,A)}y_{(F,A)}$ in $(F,A)_\alpha$. Consequently $(F,A)_\alpha$ is a lower level subhemiring of a hemiring R. □

Theorem 3.2. *Let* (F,A) *be a fuzzy soft subhemiring of a hemiring R. At that point two lower level soft subhemiring* $(F,A)_{\alpha 1}, (F,A)_{\alpha 2}$ *and* α_1,α_2 *are in* $[0,1]$ *with* $\alpha_1 < \alpha_2$ *of* (F,A) *are equivalent if and just if there is no x in R to such an extent that* $\alpha_2 > \mu_{(F,A)}(x_{(F,A)}) > \alpha_1$.

Proof. Assume that $(F,A)_{\alpha 1} = (F,A)_{\alpha_2}$. Assume there exists $x_{(F,A)}$ in R with the end goal that $\alpha_2 > \mu_{(F,A)}(x_{(F,A)}) > \mu_1$. At that point, $(F,A)_{\alpha 1} \subseteq (F,A)_\alpha$ infers x belongs to $(F,A)_{\mu 2}$, but not in $(F,A)_{\alpha 1}$ This is logical inconsistency to $(F,A)_{\alpha 1} = (F,A)_{\alpha 2}$. Along these lines there is no $x_{(F,A)} \in R$ with the end goal that $\alpha_2 > \mu_{(F,A)}(x_{(F,A)}) > \alpha_1$. On the other hand if there is no $x_{(F,A)} \in R$ with the end goal that $\alpha_2 > \mu_{(F,A)}(x_{(F,A)}) > \alpha_1$. At that point $(F,A)_{\alpha 1} = (F,A)_{\alpha 2}$. (By the meaning of lower level soft set). □

Theorem 3.3. *Let R be a hemiring and* (F,A) *be a fuzzy soft subset of R with the end goal that* $(F,A)_\alpha$ *be a subhemiring of R. On the off chance that* α *in* $[0,1]$*, at the point* (F,A) *is an anti-fuzzy soft subhemiring of R.*

Proof. Let R be a hemiring and $x_{(F,A)}$ and $y_{(F,A)}$ in R. Let $\mu_{(F,A)}(x_{(F,A)}) = \alpha_1$ and $\mu_{(F,A)}(y_{(F,A)}) = \alpha_2$.

Case 1: If $\alpha_1 < \alpha_2$ at the point $\mu_{(F,A)}, y_{(F,A)} \in (F,A)_{\alpha 2}$. As $(F,A)_{\alpha 2}$ is a subhemiring of $R, x_{(F,A)} + y_{(F,A)}$ and $x_{(F,A)}y_{(F,A)}$ in $(F,A)_{\alpha 2}$. Presently,

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \alpha_2\\ &= \max\{\alpha_1,\alpha_2\}\\ &= \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},\end{aligned}$$

which infers that

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)})\\ &\leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},\end{aligned}$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Likewise,

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha_2\\ &= \max\{\alpha_2,\alpha_2\}\\ &= \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},\end{aligned}$$

which infers that

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

Case 2: If $\alpha_1 > \alpha_2$ at the point $\mu_{(F,A)}, y_{(F,A)} \in (F,A)_{\alpha 1}$. As $(F,A)_{\alpha 1}$ is a subhemiring of $R, x_{(F,A)} + y_{(F,A)}$ and $x_{(F,A)}y_{(F,A)}$ in $(F,A)_{\alpha 1}$. Addtionaly,

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)})\\ &= \max\{\alpha_2,\alpha_1\}\\ &= \max\{\mu_{(F,A)}(x_{(F,A)}), \max\{\mu_{(F,A)}(y_{(F,A)})\},\end{aligned}$$

which infers that

$$\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Likewise,

$$\begin{aligned}&\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha_2\\ &= \max\{\alpha_2,\alpha_2\}\\ &= \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},\end{aligned}$$

which infers that

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

Case 3: If $\alpha_1 = \alpha_2$.

It is unimportant. In all the cases, (F,A) is an anti-fuzzy soft subhemiring of a hemiring R.

□

Theorem 3.4. *Let (F,A) be an anti-fuzzy soft subhemiring of a hemiring R. On the off chance that any two lower level soft subhemiring of (F,A) belongs to R, at that point their intersection is likewise lower level soft subhemiring of A in R.*

Proof. Let $\alpha_1, \alpha_2 \in [0,1]$.

Case:1 If $\alpha_1 < \mu_{(F,A)}(x_{(F,A)}) < \alpha_2$ at that point $(F,A)_{\alpha 1} \subseteq (F,A)_{\alpha 2}$. Along these lines, $(F,A)_{\alpha 1} \cap (F,A)_{\alpha 2} = (F,A)_{\alpha 1}$, but $(F,A)_{\alpha 1}$ is a lower level soft subhemiring of (F,A).

Case:2 If $\alpha_1 > \mu_{(F,A)}(x_{(F,A)}) > \alpha_2$ at that point $(F,A)_{\alpha 2} \subseteq (F,A)_{\alpha 1}$. Along these lines, $(F,A)_{\alpha 1} \cap (F,A)_{\alpha 2} = (F,A)_{\alpha 2}$, but $(F,A)_{\alpha 2}$ is a lower level soft subhemiring of (F,A).

Case:3 If $\alpha_1 = \alpha_2$, at that point $(F,A)_{\alpha 1} = (F,A)_{\alpha 2}$. In all cases, intersection of any two lower level soft subhemiring is a lower level soft subhemiring of (F,A).

□

Theorem 3.5. *Let (F,A) be an anti-fuzzy soft subhemiring of a hemiring R. In the event that $\alpha_i \in [0,1]$, and $(F,A)_{\alpha i}, i \in I$ is an accumulation of lower level soft subhemiring of (F,A), at that point their intersection is likewise a lower level soft subhemiring of (F,A).*

Proof. It is trifling. □

Theorem 3.6. *Let (F,A) be an anti-fuzzy soft subhemiring of a hemiring R. If any two lower level soft subhemiring of (F,A) belongs to R, at that point their union is likewise a lower level soft subhemiring of (F,A) in R.*

Proof. Let $\alpha_1, \alpha_2 \in [0,1]$.

Case:1 If $\alpha_1 < \mu_{(F,A)}(x_{(F,A)}) < \alpha_2$ at that point $(F,A)_{\alpha 1} \subseteq (F,A)_{\alpha 2}$. Along these lines, $(F,A)_{\alpha 1} \cup (F,A)_{\alpha 2} = (F,A)_{\alpha 2}$, but $(F,A)_{\alpha 2}$ is a lower level soft subhemiring of (F,A).

Case:2 If $\alpha_1 > \mu_{(F,A)}(x_{(F,A)}) > \alpha_2$ at that point $(F,A)_{\alpha 2} \subseteq (F,A)_{\alpha 1}$. Along these lines, $(F,A)_{\alpha 1} \cup (F,A)_{\alpha 2} = (F,A)_{\alpha 1}$, yet $(F,A)_{\alpha 1}$ is a lower level soft subhemiring of (F,A).

Case:3 If $\alpha_1 = \alpha_2$, at that point $(F,A)_{\alpha 1} = (F,A)_{\alpha 2}$. In all cases, intersection of any two lower level soft subhemiring is a lower level soft subhemiring of (F,A).

□

Theorem 3.7. *Let (F,A) be an anti-fuzzy soft subhemiring of a hemiring R. If $\alpha_i \in [0,1]$, and $(F,A)_{\alpha i}, i \in I$ is a collection of lower level soft subhemiring of (F,A), at the point their union is likewise a lower level soft subhemiring of A.*

Proof. It is paltry. □

Theorem 3.8. *The homomorphic image of a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R is a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two Hemirings and $f : R \to R'$ be a homomorphism. That is, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $(G,V) = f((F,A))$, where (F,A) is an anti-fuzzy soft subhemiring of a hemiring R. Plainly (G,V) is an anti- fuzzy soft subhemiring of a hemiring R'. Let $x_{(F,A)}$ and $y_{(F,A)}$ in R, infers $f(x_{(G,V)})$ and $f(y_{(G,V)})$ in R'. Let $(F,A)_\alpha$ is a lower level subhemiring of (F,A). That is, $\mu_{(F,A)}(x_{(F,A)}) \leq \alpha$ and $\mu_{(F,A)}(y_{(F,A)}) \leq \alpha; \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \alpha, \mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha$. We need to demonstrate that $f((F,A)_\alpha)$ is a lower level soft subhemiring of V. Presently, $\mu_{(G,V)}(f(x_{(G,V)})) \leq \mu_{(F,A)}(x_{(F,A)}) \leq \alpha$, which infers that $\mu_{(G,V)}(f(x_{(G,V)})) \leq \alpha$; and $\mu_{(G,V)}(f(y_{(G,V)})) \leq \alpha$, which suggests that $\mu_{(G,V)}(f(y_{(G,V)})) \leq \alpha$ and $\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) = \mu_{(G,V)}(f(x_{(G,V)} + y_{(G,V)}))$, as f is a homomorphism $\leq \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \alpha$, which infers that $\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \leq \alpha$. Additionaly,

$$\begin{aligned}&\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)}))\\&= \mu_{(G,V)}(f(x_{(G,V)}y_{(G,V)})),\end{aligned}$$

as f is a homomorphism $\leq \mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha$, which infers that

$$\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \leq \alpha.$$

Accordingly,

$$\begin{aligned}&\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \leq \alpha,\\&\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \leq \alpha.\end{aligned}$$

Consequently $f((F,A)_\alpha)$ is a lower level soft subhemiring of an anti-fuzzy soft subhemiring (G,V) of a hemiring R'.

□

Theorem 3.9. *The homomorphic pre-image of a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R' is a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two Hemirings and $f : R \to R'$ be a homomorphism. That is, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $(G,V) = f((F,A))$, where (G,V) is an anti-fuzzy soft subhemiring of a hemiring R'. Unmistakably (F,A) is an anti- fuzzy soft subhemiring of a hemiring R. Let $f(x_{(G,V)})$ and $f(y_{(G,V)})$ in R', infers $x_{(F,A)}$ and $y_{(F,A)}$ in R. Let $f((F,A)_\alpha)$ is a lower level subhemiring of V. That is, $\mu_{(G,V)}(f(y_{(G,V)})) \leq \alpha$ and

$$\begin{aligned}&\mu_{(G,V)}(f(y_{(G,V)})) \leq \alpha;\\&\mu_{(G,V)}(f(x_{(G,V)})) + (f(y_{(G,V)})) \leq \alpha,\\&\mu_{(G,V)}(f(x_{(G,V)}))(f(y_{(G,V)})) \leq \alpha.\end{aligned}$$

We need to demonstrate that $(F,A)_\alpha$ is a lower level soft subhemiring of (F,A). Presently,

$$\mu_{(F,A)}(x_{(F,A)}) \\ \le \mu_{(G,V)}(f(x_{(G,V)})) \\ \le \alpha,$$

which infers that

$$\mu_{(F,A)}(x_{(F,A)}) \le \alpha;$$

and

$$\mu_{(F,A)}(y_{(F,A)})) = \mu_{(G,V)}(f(y_{(G,V)})) \le \alpha,$$

which suggests that

$$\mu_{(F,A)}(y_{(F,A)}) \le \alpha$$

and

$$\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \\ = \mu_{(G,V)}(f(x_{(G,V)} + y_{(G,V)})),$$

as f is a homomorphism $\le \alpha$, which infers that

$$\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \ge \alpha.$$

Additionaly,

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) = \mu_{(G,V)}(f(x_{(G,V)}y_{(G,V)}))$$

, as f is a homomorphism $\le \alpha$, which infers that

$$\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \le \alpha.$$

Accordingly,

$$\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \le \alpha, \\ \mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \le \alpha.$$

Consequently $f((F,A)_\alpha)$ is a lower level soft subhemiring of an anti-fuzzy soft subhemiring (G,V) of a hemiring R.

□

Theorem 3.10. *The anti-homomorphic image of a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R is a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two Hemirings and $f : R \to R'$ be a homomorphism. That is, $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Let $(G,V) = f((F,A))$, where (F,A) is an anti-fuzzy soft subhemiring of a hemiring R. Plainly (G,V) is an anti- fuzzy soft subhemiring of a hemiring R'. Let $x_{(F,A)}$ and $y_{(F,A)}$ in R, infers $f(x_{(G,V)})$ and $f(y_{(G,V)})$ in R'. Let $(F,A)_\alpha$ is a lower level subhemiring of (F,A). That is, $\mu_{(F,A)}(x_{(F,A)}) \le \alpha$ and

$$\mu_{(F,A)}(y_{(F,A)}) \le \alpha; \\ \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \le \alpha, \\ \mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \le \alpha.$$

We need to demonstrate that $f((F,A)_\alpha)$ is a lower level soft subhemiring of V. Presently,

$$\mu_{(G,V)}(f(x_{(G,V)})) \\ \le \mu_{(F,A)}(x_{(F,A)}) \\ \le \alpha,$$

which infers that

$$\mu_{(G,V)}(f(x_{(G,V)})) \le \alpha;$$

and

$$\mu_{(G,V)}(f(y_{(G,V)})) \le \alpha,$$

which suggests that

$$\mu_{(G,V)}(f(y_{(G,V)})) \le \alpha.$$

Presently,

$$\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \\ = \mu_{(G,V)}(f(x_{(G,V)} + y_{(G,V)})),$$

as f is an anti-homomorphism $\le \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \le \alpha$, which suggest that

$$\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \le \alpha.$$

Likewise,

$$\mu_{(G,V)}(f(x_{(G,V)}f(y_{(G,V)})) \\ = \mu_{(G,V)}(f(x_{(G,V)}y_{(G,V)})),$$

as f is an anti-homomorphism $\le \mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \le \alpha$, which suggest that

$$\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \le \alpha.$$

Along these lines,

$$\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \le \alpha, \\ \mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \le \alpha.$$

Thus $f((F,A)_\alpha)$ is a lower level soft subhemiring of an anti-fuzzy soft subhemiring (G,V) of a hemiring R'.

□

Theorem 3.11. *The anti-homomorphic pre-image of a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R' is a lower level soft subhemiring of an anti-fuzzy soft subhemiring of a hemiring R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two Hemirings and $f : R \to R'$ be a homomorphism. That is, $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let $(G,V) = f((F,A))$, where (G,V) is an anti-fuzzy soft subhemiring of a hemiring R'. Unmistakably, (F,A) is an anti-fuzzy soft subhemiring of a hemiring R. Let $f(x_{(G,V)})$ and $f(y_{(G,V)})$ in R', infers $x_{(F,A)}$ and $y_{(F,A)}$ in R. Let $f((F,A)_\alpha)$ is a lower level soft subhemiring of (G,V). That is, $\mu_{(G,V)}(f(x_{(G,V)})) \leq \alpha$ and

$$\mu_{(G,V)}(f(y_{(G,V)})) \leq \alpha;$$
$$\mu_{(G,V)}(f(x_{(G,V)})+f(y_{(G,V)})) \leq \alpha,$$
$$\mu_{(G,V)}(f(y_{(G,V)})f(x_{(G,V)})) \leq \alpha.$$

We need to demonstrate that A_α is a lower level soft subhemiring of A. Presently,

$$\mu_{(F,A)}(x_{(F,A)}))$$
$$= \mu_{(G,V)}(f(x_{(G,V)}))$$
$$\leq \alpha,$$

which infers that

$$\mu_{(F,A)}(x_{(F,A)}) \leq \alpha;$$

and

$$\mu_{(F,A)}(y_{(F,A)}) = \mu_{(G,V)}(f(x_{(G,V)})) \leq \alpha,$$

which suggests that

$$\mu_{(F,A)}(y_{(F,A)}) \leq \alpha$$

and

$$\mu_{(F,A)}(f(x_{(G,V)})+f(y_{(G,V)}))$$
$$= \mu_{(G,V)}(f(y_{(G,V)})+f(x_{(G,V)})),$$

as f is a anti-homomorphism $\leq \alpha$, which infers that

$$\mu_{(F,A)}(x_{(F,A)}+y_{(F,A)}) \leq \alpha.$$

Likewise,

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)})$$
$$= \mu_{(G,V)}(f(y_{(G,V)}x_{(G,V)})),$$

as f is an anti-homomorphism $\leq \alpha$, which infers that

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \alpha.$$

Along these lines,

$$\mu_{(G,V)}(f(x_{(G,V)})+f(y_{(G,V)})) \leq \alpha,$$
$$\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \leq \alpha.$$

Thus $(F,A)_\alpha$ is a lower level soft subhemiring of an anti-fuzzy soft subhemiring (F,A) of a hemiring R.

□

Theorem 3.12. *Let $(R,+,\cdot)$ be a hemiring and (F,A) be a non-empty subset of R. At the point (F,A) is a subhemiring of R if and just if $(H,B) = \langle \overline{\chi_{(F,A)}} \rangle$ is an anti-fuzzy soft subhemiring of R, where $\overline{\chi_{(F,A)}}$ is the characteristic function.*

Proof. Let $(R,+,\cdot)$ be a hemiring and (F,A) be a non-empty subset of R. To begin with let (F,A) be a subhemiring of R. Take x and y in R.

Case:1 If $x_{(F,A)}$ and $y_{(F,A)}$ in (F,A), at the point $x_{(F,A)}+y_{(F,A)}, x_{(F,A)}y_{(F,A)}$ in (F,A), since (F,A) is a subhemiring of R,

$$\overline{\chi_{(F,A)}}(x_{(F,A)})$$
$$= \overline{\chi_{(F,A)}}(y_{(F,A)})$$
$$= \overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})$$
$$= \overline{\chi_{(F,A)}}(\overline{\chi_{(F,A)}}y_{(F,A)})$$
$$= 0.$$

So,

$$\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})$$
$$\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}), \overline{\chi_{(F,A)}}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R,

$$\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})$$
$$\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}), \overline{\chi_{(F,A)}}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

Case:2 If $x_{(F,A)}$ in (F,A), $y_{(F,A)}$ not in (F,A) (or $x_{(F,A)}$ not in (F,A) , $y_{(F,A)}$ in (F,A)), then $x_{(F,A)}+y_{(F,A)}, x_{(F,A)}y_{(F,A)}$ may or may not be in (F,A),

$$\overline{\chi_{(F,A)}}(x_{(F,A)}) = 0,$$
$$\overline{\chi_{(F,A)}}(y_{(F,A)}) = 1 \quad \text{or}$$
$$\overline{\chi_{(F,A)}}(x_{(F,A)}) = 1,$$
$$\overline{\chi_{(F,A)}}(y_{(F,A)}) = 0,$$
$$\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})$$
$$= \overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})$$
$$= 0(\text{or}1).$$

Obviously,

$$\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})$$
$$\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}), \overline{\chi_{(F,A)}}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R,

$$\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})$$
$$\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}), \overline{\chi_{(F,A)}}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

Case:3 If $x_{(F,A)}$ and $y_{(F,A)}$ not in (F,A), at the point

$x_{(F,A)} + y_{(F,A)}, x_{(F,A)}y_{(F,A)}$ may or may not be in (F,A),

$$\begin{aligned}&\overline{\chi_{(F,A)}}(x_{(F,A)})\\&=\overline{\chi_{(F,A)}}(y_{(F,A)})\\&=1,\\&\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})\\&=\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})\\&=0 \quad \text{or } 1.\end{aligned}$$

Obviously

$$\begin{aligned}&\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})\\&\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}),\overline{\chi_{(F,A)}}(y_{(F,A)})\},\end{aligned}$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R

$$\begin{aligned}&\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})\\&\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}),\overline{\chi_{(F,A)}}(y_{(F,A)})\},\end{aligned}$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. So in all the three cases, we have (H,B) is an anti-fuzzy soft subhemiring of a hemiring R. Then again, let $x_{(F,A)}$ and $y_{(F,A)}$ in (F,A), since (F,A) is a non-empty subset of R, in this way,

$$\overline{\chi_{(F,A)}}(x_{(F,A)}) = \overline{\chi_{(F,A)}}(y_{(F,A)}) = 0.$$

Since $(H,B) = \langle\overline{\chi_{(F,A)}}\rangle$ is an anti-fuzzy soft sub hemiring of R, we have

$$\begin{aligned}&\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})\\&\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}),\overline{\chi_{(F,A)}}(y_{(F,A)})\}\\&=\max\{0,0\}\\&=0,\\&\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})\\&\leq \max\{\overline{\chi_{(F,A)}}(x_{(F,A)}),\overline{\chi_{(F,A)}}(y_{(F,A)})\}\\&=\max\{0,0\}\\&=0.\end{aligned}$$

Along these lines

$$\begin{aligned}&\overline{\chi_{(F,A)}}(x_{(F,A)}+y_{(F,A)})\\&=\overline{\chi_{(F,A)}}(x_{(F,A)}y_{(F,A)})\\&=0.\end{aligned}$$

Hence $x_{(F,A)} + y_{(F,A)}$ and $x_{(F,A)}y_{(F,A)}$ in (F,A), so (F,A) is a soft subhemiring of R.

□

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