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Notion of *L*-fuzzy soft subhemirings of a hemiring

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Abstract

The purpose of this paper, is to study the algebraic nature of an *L*-fuzzy soft subhemirings of a hemiring. Here we have defined the structure L- fuzzy soft subhemiring of a hemiring and developed some of its basic properties as theorems.

Keywords

Soft set, Fuzzy soft set, *L*-fuzzy soft set, L-fuzzy soft subhemiring, anti-L-fuzzy soft subhemiring and pseudo *L*-fuzzy soft coset.

AMS Subject Classification 05C38, 15A15, 05A15, 15A18.

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1. Introduction

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. In 1999 Molodtsov presented the soft set and estrablished the crucial consequences of the new thorey. Many analysis have contributed towards the fuzzification of the idea of the Fuzzy soft set. In specific, nearrings and a few sorts of semirings have been demonstrated extremely helpful. Semirings called Hemiring which are regarded as a generalization of Rings have been found useful in solving problems in different areas of applied Mathematics and information sciences because a semiring provides an algebraic framework for modeling and studying the factor in these applied areas. They play important role in studying of automata theory formal language theory. After the introduction of fuzzy sets by L.A. Zadeh [13], several researchers explored on the generalization of the concept of fuzzy sets .M. Borah, T. J. Neog and D. K. Sut, [5] were developed some operations of fuzzy soft sets, On operations of soft sets was developed by A.Sezgin and A. O. Atagun, [11] and Kumud Borgohain and Chittaranjan Gohain, [7] was

developed some New operations on Fuzzy Soft Sets,

The main purpose of this paper is to introduce the basic notion of *L*-Fuzzy soft sets and the author investigate the characterizations of *L*-Fuzzy soft sets and basic Properties are introduced.

2. Preliminaries

In this session R is said to be a hemiring if it is an addition have defined some basic definitions which is needed for our further studies.

Definition 2.1. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)(\varepsilon \in E)$ from this family maybe considered as the set of ε -elements of the soft sets (F, E) or as the set of ε -approximate elements of the soft set.

Definition 2.2. Let (U, E) be a soft universe and $A \subseteq E$. Let $\mathscr{F}(U)$ be the set of all fuzzy subsets in U. A pair (\widetilde{F}, A) is called a fuzzy soft set over U, where \widetilde{F} , is a mapping given by $\widetilde{F} : A \to \mathscr{F}(U)$.

Definition 2.3. *Let X be a non-empty set and* $L = (L, \leq)$ *be a lattice with least element 0 and greatest element 1.*

Definition 2.4. Let R be a hemiring. An L-Fuzzy soft subset (F,A) of R is said to be an Fuzzy soft subhemiring (FSHR) of R if it satisfies the following conditions:

(*i*)
$$\mu_{(F,A)}(x+y) \ge \{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\}$$

(*ii*) $\mu_{(F,A)}(xy) \ge \{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\}$ for all x and y in *R*.

Definition 2.5. Let $(R, +, \cdot)$ be a hemiring. An L-fuzzy soft set subhemiring (F,A) of R is said to be an L-fuzzy soft set normal subhemiring (FSNSHR) of R if it satisfies the following conditions:

 $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$ for all x and y in R.

Definition 2.6. If $(R, +, \cdot)$ and $(R', +, \cdot)$ are any two hemirings, then the function $f : R \to R'$ is called a homomorphism if f(x+y) = f(x) + f(y) and f(xy) = f(x)f(y), for all x and y in R.

Definition 2.7. If $(R, +, \cdot)$ and $(R', +, \cdot)$ are any two hemirings, then the function $f : R \to R'$ is called a anti-homomorphism if f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R.

Definition 2.8. Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f : R \to R'$ be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a hemiring anti-isomorphism.

Definition 2.9. Let *R* and *R'* be any two hemirings. Let *f* : $R \rightarrow R'$ be any function and let *A* be an Fuzzy soft subhemiring in *R*, *V* be an Fuzzy soft subhemiring in f(R) = R', defined by $\mu_{V(y)} = \sup_{x \in f^{-1}(y)} (\mu_{(F,A)})(x)$ for all *x* in *R* and *y* in *R'*. Then *A*

is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 2.10. Let (F,A) be an Fuzzy soft subhemiring of a hemiring $(R, +, \cdot)$ and a in R. Then the pseudo Fuzzy soft coset $(a(F,A)^p)$ is defined by $((a\mu_{(F,A)})^p)(x) = p(a)\mu_{(F,A)}(x)$, for every x in R.

3. Fuzzy soft subhemirings of a hemiring

Theorem 3.1. If (F,A) is an L-Fuzzy soft subhemiring of a hemiring $(R,+,\cdot)$, then (F,A) is an L-Fuzzy soft subhemiring of R.

Proof. Let (F,A) be an L-fuzzy soft set subhemiring of a hemiring R. Consider $(F,A) = \{\langle x, \mu_{(F,A)}(x) \rangle\}$, for all x in R, we take $(F,A) = (F,B) = \{\langle x, \mu_{(F,B)}(x), \rangle\}$, where $\mu_{(F,B)}(x) = \mu_{(F,A)}(x)$. Clearly $\mu_{(F,B)}(x+y) \ge \{\mu_{(F,B)}(x) \land \mu_{(F,B)}(y)\}$, for all x and y in R and $\mu_{(F,B)}(xy) \ge \{\mu_{(F,B)}(x) \land \mu_{(F,B)}(y)\}$, for all x and y in R. Since A is an fuzzy soft subhemiring of R, we have $\mu_{(F,A)}(x+y) \ge \{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\}$ for all x and y in R, and $\mu_{(F,A)}(xy) \ge \{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\}$ for all x and y in R. Hence (F,B) = (F,A) is an L-fuzzy soft subhemiring of a hemiring R.

Theorem 3.2. If (F,A) is an L-Fuzzy soft subhemiring of a hemiring $(R, +, \cdot)$, then (F,A) is an L-Fuzzy soft subhemiring of R.

Proof. Let (F,A) be an L-fuzzy soft set subhemiring of a hemiring R. That is $(F,A) = \{\langle x, \mu_{(F,A)}(x) \rangle\}$, for all x in R. Let $(F,A) = (F,B) = \{\langle x, \mu_{(F,B)}(x), \rangle\}$, for all x and y in R. Since (F,A) is an fuzzy soft subhemiring of R, which implies that $1 - \mu_{(F,B)}(xy) \ge \{(1 - \mu_{(F,B)}(x)) \lor (1 - \mu_{(F,B)}(y))\}$ which implies that

$$\begin{split} & \mu_{(F,B)}(xy) \\ & \geq 1 - \{ (1 - \mu_{(F,B)}(x)) \lor (1 - \mu_{(F,B)}(y)) \} \\ & = \{ \mu_{(F,B)}(x) \land \mu_{(F,B)}(y) \}. \end{split}$$

Therefore

$$\mu_{(F,B)}(xy) \ge \{\mu_{(F,B)}(x) \land \mu_{(F,B)}(y)\}$$

for all x and y in R. Hence (F,B) = (F,A) is an L-fuzzy soft subhemiring of a hemiring R.

Theorem 3.3. Let $(R, +, \cdot)$ be a hemiring and (F, A) be a non empty subset of R. Then (F, A) is a subhemiring of R if and only if $(F, B) = \langle \chi_{(F,A)}, \overline{\chi}_{(F,A)} \rangle$ is a L=fuzzy soft subhemiring of R, where $\chi_{(F,A)}$ is the characteristic function.

Proof. Let $(R, +, \cdot)$ be a hemiring and (F, A) be a non empty subset of *R*. First let (F, A) be a subhemiring of *R*. Take *x* and *y* in *R*.

Case:1 If x and y in (F,A), then x + y, xy in (F,A), since (F,A) is a subhemiring of R,

$$\chi_{(F,A)}(x)$$

$$= \chi_{(F,A)}(y)$$

$$= \chi_{(F,A)}(x+y)$$

$$= \chi_{(F,A)}(xy)$$

$$= 1$$

and

$$\begin{aligned} \chi_{(F,A)}(x) \\ &= \chi_{(F,A)}(y) \\ &= \chi_{(F,A)}(x+y) \\ &= \chi_{(F,A)}(xy) \\ &= 0. \end{aligned}$$

So

$$\boldsymbol{\chi}_{(F,A)}(x+y) \geq \min\{(x), \boldsymbol{\mu}_{(F,A)}\boldsymbol{\chi}(y)\}$$

for all x and y in R,

$$\begin{aligned} \boldsymbol{\chi}_{(F,A)}(xy) \\ \geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\} \end{aligned}$$

for all x and y in R. So

$$\boldsymbol{\chi}_{(F,A)}(x+y) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$



for all *x* and *y* in *R*.

$$\boldsymbol{\chi}_{(F,A)}(xy) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all x and y in R.

Case:2 If x in (F,A), y not in (F,A) (or x not in (F,A), y in (F,A)), then x + y, xy may or may not be in (F,A),

$$\begin{split} \chi_{(F,A)}(x) &= 1, \\ \chi_{(F,A)}(y) &= 0 \quad \text{(or)} \\ x_{(F,A)}(x) &= 0, \\ \chi_{(F,A)}(y) &= 1, \\ \chi_{(F,A)}(x+y) &= \chi_{(F,A)}(xy) = 1 \text{ (or) } 0 \end{split}$$

and

$$\chi_{(F,A)}(x) = 0, \chi_{(F,A)}(y) = 1 \quad \text{(or)} \chi_{(F,A)}(x) = 1, \chi_{(F,A)}(y) = 0 = \chi_{(F,A)}(x+y) = \chi_{(F,A)}(xy) = 0 \text{ (or)} 1.$$

Clearly

$$\boldsymbol{\chi}_{(F,A)}(x+y) \geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\},\$$

for all x and y in R.

$$\boldsymbol{\chi}_{(F,A)}(xy) \geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all *x* and *y* in *R* and

$$\boldsymbol{\chi}_{(F,A)}(x+y) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all *x* and *y* in *R*.

$$\boldsymbol{\chi}_{(F,A)}(xy) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all x and y in R.

Case:3 If x and y not in (F,A), then x + y, xy in (F,A), since (F,A) is a subhemiring of R,

$$\begin{aligned} \boldsymbol{\chi}_{(F,A)}(x) \\ &= \boldsymbol{\chi}_{(F,A)}(y) \\ &= \boldsymbol{\chi}_{(F,A)}(x+y) \\ &= \boldsymbol{\chi}_{(F,A)}(xy) \\ &= 1 \quad \text{or } 0. \end{aligned}$$

and

 $\chi_{(F,A)}(x)$ $= \chi_{(F,A)}(y) = 1$ $= \chi_{(F,A)}(x+y)$ $= \chi_{(F,A)}(xy)$ = 0 or 1.

Clearly

$$\boldsymbol{\chi}_{(F,A)}(x+y) \geq \min\{(x), \boldsymbol{\mu}_{(F,A)}\boldsymbol{\chi}(y)\}$$

for all x and y in R,

$$\boldsymbol{\chi}_{(F,A)}(xy) \geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all x and y in R. So

$$\boldsymbol{\chi}_{(F,A)}(x+y) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all x and y in R.

$$\boldsymbol{\chi}_{(F,A)}(xy) \leq \{\boldsymbol{\chi}_{(F,A)}(x) \lor \boldsymbol{\chi}_{(F,A)}(y)\}$$

for all x and y in R. So in all the three cases, we have B is a fuzzy soft subhemiring of (F,A). Conversely, let x and y in (F,A), since (F,A) is (F,A) non empty subset of R. And,

$$\begin{split} \chi_{(F,A)}(x) \\ &= \chi_{(F,A)}(y) \\ &= 1, \\ \chi_{(F,A)}(x) \\ &= \chi_{(F,A)}(y) \\ &= 0. \end{split}$$

Since

$$B = \left\langle \chi_{(F,A)}, \overline{\chi}_{(F,A)} \right\rangle$$

is a fuzzy soft subhemiring of R, we have

$$\begin{aligned} \boldsymbol{\chi}_{(F,A)}(x+y) \\ &\geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\} \\ &= \{1 \land 1\} \\ &= 1, \\ \boldsymbol{\chi}_{(F,A)}(xy) \\ &\geq \{\boldsymbol{\chi}_{(F,A)}(x) \land \boldsymbol{\chi}_{(F,A)}(y)\} \\ &= \{1 \land 1\} \\ &= 1. \end{aligned}$$

Therefore

$$\boldsymbol{\chi}_{(F,A)}(x+y) = \boldsymbol{\chi}_{(F,A)}(xy) = 1$$

and,

$$\begin{split} \chi_{(F,A)}(x+y) \\ &\leq \{\chi_{(F,A)}(x) \lor \chi_{(F,A)}(y)\} \\ &= \{0 \lor 0\} \\ &= 0, \\ \chi_{(F,A)}(xy) \\ &\leq \{\chi_{(F,A)}(x) \lor \chi_{(F,A)}(y)\} \\ &= \{0 \lor 0\} \\ &= 0. \end{split}$$

Therefore

$$\boldsymbol{\chi}_{(F,A)}(x+y) = \boldsymbol{\chi}_{(F,A)}(xy) = 0.$$

Therefore, x + y and xy in (F,A). Hence (F,A) is a subhemiring of R.

We denote the composition of operations by \circ for the following:

Theorem 3.4. Let (F,A) be an L-fuzzy soft subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then $(F,A) \circ f$ is an L-fuzzy soft subhemiring of R.

Proof. Let x and y in R and (F,A) be an fuzzy soft subhemiring of a hemiring H. Then we have,

$$\begin{aligned} &(\mu_{(F,A)} \circ f)(x+y) \\ &= \mu_{(F,A)}(f(x+y)) \\ &= \mu_{(F,A)}(f(x)+f(y)), \end{aligned}$$

as f is an homomorphism

$$\geq \{ \mu_{(F,A)}(f(x)) \land \mu_{(F,A)}(f(y)) \}$$

= $\{ (\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y) \}$

which implies that

$$\begin{aligned} &(\boldsymbol{\mu}_{(F,A)} \circ f)(x+y) \\ &\geq \{(\boldsymbol{\mu}_{(F,A)} \circ f)(x) \land (\boldsymbol{\mu}_{(F,A)} \circ f)(y)\}. \end{aligned}$$

And

$$\begin{aligned} &(\mu_{(F,A)} \circ f)(xy) \\ &= \mu_{(F,A)}(f(xy)) \\ &= \mu_{(F,A)}(f(x)f(y)), \end{aligned}$$

as f is an isomorphism

$$\geq \{ \mu_{(F,A)}(f(x)) \land \mu_{(F,A)}(f(y)) \}$$

= $\{ (\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y) \},$

which implies that

$$(\mu_{(F,A)} \circ f)(xy)$$

$$\geq \{(\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y)\}$$

Therefore $(F,A) \circ f$ is an *L*-Fuzzy soft subhemiring of a hemiring *R*.

Theorem 3.5. Let (F,A) be an L-fuzzy soft subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. then $(F,A) \circ f$ is an L-fuzzy soft subhemiring of R.

Proof. Let x and y in R and (F,A) be an L-fuzzy soft set subhemiring of a hemiring H. Then we have

$$\begin{aligned} &(\mu_{(F,A)} \circ f)(x+y) \\ &= \mu_{(F,A)}(f(x+y)) \\ &= \mu_{(F,A)}(f(y)+f(x)), \end{aligned}$$

as f is an anti-isomorphism

$$\geq \{ \mu_{(F,A)}(f(x)) \land \mu_{(F,A)}(f(y)) \} \\= \{ (\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y) \}$$

which implies that

$$(\mu_{(F,A)} \circ f)(x+y)$$

$$\geq \{(\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y)\}.$$

And

$$(\mu_{(F,A)} \circ f)(xy)$$

= $\mu_{(F,A)}(f(xy))$
= $\mu_{(F,A)}(f(y)f(x)),$

as f is an anti-isomorphism

$$\geq \{ \mu_{(F,A)}(f(x)) \land \mu_{(F,A)}(f(y)) \} \\ = \{ (\mu_{(F,A)} \circ f)(x) \land (\mu_{(F,A)} \circ f)(y) \},$$

which implies that

$$\begin{aligned} &(\boldsymbol{\mu}_{(F,A)} \circ f)(xy) \\ &\geq \{(\boldsymbol{\mu}_{(F,A)} \circ f)(x) \land (\boldsymbol{\mu}_{(F,A)} \circ f)(y)\} \end{aligned}$$

Therefore $(F,A) \circ f$ is an *L*-Fuzzy soft subhemiring of a hemiring *R*.

Theorem 3.6. Let (F,A) be an L-fuzzy soft subhemiring of a hemiring $(R, +, \cdot)$, then the pseudo fuzzy soft coset $(a(F,A))^p$ is an L-fuzzy soft subhemiring of a hemiring R for every a in R.

Proof. Let (F,A) be an *L*-fuzzy soft subhemiring of a hemiring *R*. For every *x* and *y* in *R*, we have

$$((a\mu_{(F,A)})^{p})(x+y) = p(a)\mu_{(F,A)}(x+y) \geq p(a)\{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\} = \{p(a)\mu_{(F,A)}(x) \land p(a)\mu_{(F,A)}(y)\} = \{((a\mu_{(F,A)})^{p})(x) \land ((a\mu_{(F,A)})^{p})(y)\}.$$

Therefore,

$$((a\mu_{(F,A)})^p)(x+y) \geq \{((a\mu_{(F,A)})^p)(x) \land ((a\mu_{(F,A)})^p)(y)\}$$



Now,

$$\begin{aligned} &((a\mu_{(F,A)})^{p})(xy) \\ &= p(a)\mu_{(F,A)}(xy) \\ &\geq p(a)\{\mu_{(F,A)}(x) \land \mu_{(F,A)}(y)\} \\ &= \{p(a)\mu_{(F,A)}(x) \land p(a)\mu_{(F,A)}(y)\} \\ &= \{((a\mu_{(F,A)})^{p})(x) \land ((a\mu_{(F,A)})^{p})(y)\}. \end{aligned}$$

Therefore,

$$((a\mu_{(F,A)})^p)(xy) \ge \{((a\mu_{(F,A)})^p)(x) \land ((a\mu_{(F,A)})^p)(y)\}.$$

Hence $(a(F,A))^p$ is an *L*-fuzzy soft subhemiring of a hemiring *R*.

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