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A new idea on interval valued T-fuzzy soft subhemirings of a hemiring

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Abstract

In this paper, we study some operations of interval valued T-fuzzy soft sets and give fundamental properties of interval valued T-fuzzy soft sets. Then, we illustrate properties of homomorphism and anti-homomorphism, normal operations by giving theroems.

Keywords

Interval valued fuzzy subset, interval valued T-fuzzy soft subhemiring, and interval valued T-soft normal subhemiring.

AMS Subject Classification

03F55, 08A72, 20N25.

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Contents

1. Introduction

Many fields deal daily with the uncertain data that may not be successfully modeled by the classical mathematics. There are some mathematical tools for dealing with uncertainties; two of them are fuzzy set theory, developed by Zadeh (1965), and soft set theory, introduced by Molodtsov (1999), that are related to this work. Molodtsov showed the applications of soft sets in various fields like stability and regularization, game theory, operations research and analysis. Maji and Roy presented a theoretical study [9] and defined several operations on soft sets. In 2001, they proposed the concept of "Fuzzy Soft Sets" [10] and later on applied the theories in decision making problem [11, 15]. Different algebraic structures and their applications have also been studied in soft and fuzzy soft context [2, 3, 5].The paper is organized by two sections. First have given preliminaries on the theories of T-fuzzy soft sets. After accomplishing an account of algebraic properties of interval valued T-fuzzy soft sets, we study the overall algebraic structures of interval valued T-fuzzy soft sets subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

2. Preliminaries

Definition 2.1. *A T-norm is a binary operation $T : [0,1] \times [0,1] \to [0,1]$ satisfying the following requirements:*

(i) $1Tx = x$ *(boundary conditions)*

(ii) $xTy = yTx$ *(commutativity)*

(iii) $xT(yTz) = (xTy)Tz$ *(associativity)*

(iv) If $x \in y$ *and* $w \in z$*, then* $xTw \in yTz$ *(monotonicity).*

Definition 2.2. *Let $(R,+,.)$ be a hemiring. A interval valued T-fuzzy soft subset $[(F,A)]$ of R is said to be an interval valued T-fuzzy soft subhemiring (IVTFHR) of R if the following conditions are satisified:*

(i) $[\mu_{(F,A)}](x+y) \leq T\left([\mu_{(F,A)}](x), [\mu_{(F,A)}](y)\right)$

(ii) $[\mu_{(F,A)}](xy) \leq T\left([\mu_{(F,A)}](x), [\mu_{(F,A)}](y)\right)$, *for all x and y in R.*

Definition 2.3. *Let $(R,+,\cdot)$ be a hemiring. A interval valued T-fuzzy soft subhemiring $[(F,A)]$ of R is said to be an interval valued T-fuzzy soft normal subhemiring (IVTFNSHR) of R if $([\mu_{(F,A)}](xy)) = ([\mu_{(F,A)}](yx))$, for all x and y in R.*

Definition 2.4. *Let X and X' be any two sets. Let $f: X \to X'$ be any function and $[A]$ be an interval valued T-fuzzy soft subset in X, $[G,V]$ be an interval valued T-fuzzy soft subset in $f(X) = X'$, $[\mu_{(G,V)}](y) = \sup_{x \in f^{-1}(y)} [\mu_{(F,A)}](x)$ for all x in X and y in X'. Then $[F,A]$ is called a pre-image of $[G,V]$ under f and is denoted by $f^{-1}([G,V])$.*

3. Properties of interval valued T-fuzzy soft subhemirings

Theorem 3.1. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The homomorphic image of an interval valued T-fuzzy soft subhemiring of R is an interval valued T-fuzzy soft subhemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f: R \to R'$ be a homomorphism. Then, $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $[(G,V)] = f([(F,A)])$, where $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. We have to prove that $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R'. Now,for $f(x), f(y)$ in R'.

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))+(f(y)))\\&=[\mu_{(G,V)}](f(x+y))\\&\geq[\mu_{(F,A)}](x+y)\\&\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}](y)\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))+(f(y)))\\&\leq T\left([\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))\right).\end{aligned}$$

Again,

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))(f(y)))\\&=[\mu_{(G,V)}]((f(xy)))\\&\geq[\mu_{(F,A)}](xy)\\&\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}](y)\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))(f(y)))\\&\leq T\left([\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))\right).\end{aligned}$$

Hence $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R'. □

Theorem 3.2. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The homomorphic preimage of an interval valued T-fuzzy soft subhemiring of R' is an interval valued T-fuzzy soft subhemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f: R \to R'$ be a homomorphism. Then, $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $[(G,V)] = f([(F,A)])$, where $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R'. We have to prove that $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R. Now,for x,y in R. Then

$$\begin{aligned}&[\mu_{(F,A)}](x+y)\\&=[\mu_{(F,A)}](f(x+y)\\&\geq[\mu_{(G,V)}](f(x)+f(y))\\&\leq T\{[\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))\}\\&=T\{[\mu_{(F,A)}](x),\mu_{(F,A)}(y)\},\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(F,A)}](x+y)\\&\leq T\{[\mu_{(F,A)}](x),\mu_{(F,A)}(y)\}.\end{aligned}$$

Again,

$$\begin{aligned}&[\mu_{(F,A)}](xy)\\&=[\mu_{(G,V)}](f(xy))\\&\geq[\mu_{(G,V)}](f(x)f(y))\\&\leq T\{[\mu_{(G,V)}](f((x)),[\mu_{(G,V)}](f(y))\}\\&\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}(y)](y)\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(F,A)}](xy)\\&\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}(y)](y)\}.\end{aligned}$$

Hence $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. □

Theorem 3.3. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The anti-homomorphic image of an interval valued T-fuzzy soft subhemiring of R is an interval valued T-fuzzy soft subhemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f: R \to R'$ be a homomorphism. Then, $f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let $[(G,V)] = f([(F,A)])$, where $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. We have to prove that $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R'. Now,for $f(x), f(y)$ in R'.

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))+(f(y)))\\&=[\mu_{(G,V)}]((f(y))+(f(x)))\\&=[\mu_{(G,V)}](f(y+x)\\&\geq[\mu_{(F,A)}](x+y)\\&\leq T\{[\mu_{(F,A)}](y),[\mu_{(F,A)}](x))\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))+(f(y)))\\ &\leq T([\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))).\end{aligned}$$

Again,

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))(f(y)))\\ &=[\mu_{(G,V)}]((f(yx)))\\ &\geq[\mu_{(F,A)}](yx)\\ &\leq T\{[\mu_{(F,A)}](y),[\mu_{(F,A)}](x))\}\\ &\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}](y))\}.\end{aligned}$$

Which implies that

$$\begin{aligned}&[\mu_{(G,V)}]((f(x))(f(y)))\\ &\leq T([\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))).\end{aligned}$$

Hence $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R'. □

Theorem 3.4. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The anti-homomorphic preimage of an interval valued T-fuzzy soft subhemiring of R' is an interval valued T-fuzzy soft subhemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f:R\rightarrow R'$ be a homomorphism. Then, $f(x+y)=f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all x and y in R. Let $[(G,V)]=f([(F,A)])$, where $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of R'. We have to prove that $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. Now,for x,y in R. Then

$$\begin{aligned}&[\mu_{(F,A)}](x+y)\\ &=[\mu_{(G,V)}](f(x+y))\\ &=[\mu_{(G,V)}](f(y+x))\\ &\leq T\{[\mu_{(G,V)}](f(y)),[\mu_{(G,V)}](f(x))\}\\ &\leq T\{[\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))\}\\ &\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}](y)\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(F,A)}](x+y)\\ &\leq T\{[\mu_{(F,A)}](x),[\mu_{(F,A)}](y)\}.\end{aligned}$$

Again,

$$\begin{aligned}&[\mu_{(F,A)}](xy)\\ &=[\mu_{(G,V)}]((f(xy)))\\ &\geq[\mu_{(G,V)}](f(x)f(y))\\ &\leq T\{[\mu_{(G,V)}](f(y)),[\mu_{(G,V)}](f(x))\}\\ &\leq T\{[\mu_{(G,V)}](f(x)),[\mu_{(G,V)}](f(y))\}\\ &=T\{[\mu_{(F,A)}(x).[\mu_{(F,A)}](y)\}\end{aligned}$$

which implies that

$$\begin{aligned}&[\mu_{(F,A)}](xy)\\ &\leq T\{[\mu_{(F,A)}(x).[\mu_{(F,A)}](y)\}.\end{aligned}$$

Hence $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. □

Theorem 3.5. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The homomorphic image of an interval valued T-fuzzy soft normal subhemiring of R is an interval valued T-fuzzy soft subhemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f:R\rightarrow R'$ be a homomorphism. Let $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of R. We have to prove that $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of $f(R)=R'$. Now,for $f(x),f(y)$ in R'. Clearly, $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of $f(R)=R'$. Since $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. Now,

$$\begin{aligned}&([\mu_{(G,V)}]f(x)f(y))\\ &=T([\mu_{(G,V)}](f(xy))\\ &\leq T([\mu_{(F,A)}](xy))\\ &=T([\mu_{(F,A)}](yx))\\ &\leq T([\mu_{(G,V)}]\{f(yx))\})\\ &=T([\mu_{(G,V)}]((f(y))(f(x))))\end{aligned}$$

which implies that

$$\begin{aligned}&T([\mu_{(G,V)}]((f(x))(f(y))))\\ &=T([\mu_{(G,V)}]((f(y))(f(x)))).\end{aligned}$$

Hence $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of the hemiring R'.

□

Theorem 3.6. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The homomorphic preimage of an interval valued T-fuzzy soft normal subhemiring of R' is an interval valued T-fuzzy soft subhemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f:R\rightarrow R'$ be a homomorphism. Let $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of $f(R)=R'$. We have to prove that $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of R. Let x and y in R. Then clearly $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. Now,

$$\begin{aligned}&T([\mu_{(F,A)}](xy))\\ &=T([\mu_{(G,V)}](f(xy))\\ &=T([\mu_{(G,V)}]\{(f(x))(f(y))\})\\ &=T([\mu_{(G,V)}]((f(y))(f(x))))\\ &=T([\mu_{(F,A)}](f(yx)))\\ &=T([\mu_{(F,A)}](yx))\end{aligned}$$

which implies that

$$T([\mu_{(F,A)}](xy) \\ = T([\mu_{(F,A)}](yx)).$$

for all x and y in R. Hence $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of the hemiring R.

□

Theorem 3.7. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The antihomomorphic image of an interval valued T-fuzzy soft normal subhemiring of R is an interval valued T-fuzzy soft subhemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f: R \to R'$ be a homomorphism. Let $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of R. We have to prove that $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of $f(R) = R'$. Now,for $f(x), f(y)$ in R'. Clearly, $[(G,V)]$ is an interval valued T-fuzzy soft subhemiring of $f(R) = R'$. Since $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. Now,

$$\begin{aligned} & T([\mu_{(G,V)}]f(x)f(y)) \\ & = T([\mu_{(G,V)}](f(xy)) \\ & \leq T([\mu_{(F,A)}](xy)) \\ & = T([\mu_{(F,A)}](yx)) \\ & \leq T([\mu_{(G,V)}]\{f(yx))\}) \\ & = T([\mu_{(G,V)}]((f(y))(f(x)))) \end{aligned}$$

which implies that

$$T([\mu_{(G,V)}]((f(x))(f(y)))) \\ = T([\mu_{(G,V)}]((f(y))(f(x)))).$$

Hence $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of the hemiring R'.

□

Theorem 3.8. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The antihomomorphic preimage of an interval valued T-fuzzy soft normal subhemiring of R' is an interval valued T-fuzzy soft subhemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f: R \to R'$ be a homomorphism. Let $[(G,V)]$ is an interval valued T-fuzzy soft normal subhemiring of $f(R) = R'$. We have to prove that $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of R. Let x and y in R. Then clearly $[(F,A)]$ is an interval valued T-fuzzy soft subhemiring of R. Since $[(G,V)]$ is an interval valued T- fuzzy soft subhemiring of the hemiring R'. Now,

$$\begin{aligned} & T([\mu_{(F,A)}](xy)) \\ & = T([\mu_{(G,V)}](f(xy)) \\ & = T([\mu_{(G,V)}]\{f(y)f(x)\}) \\ & = T([\mu_{(G,V)}]((f(x))(f(y)))) \\ & = ([\mu_{(G,V)}](f(yx))) \\ & = T([\mu_{(F,A)}](yx)) \end{aligned}$$

which implies that

$$T([\mu_{(F,A)}](xy)) \\ = T([\mu_{(F,A)}](yx)),$$

for all x and y in R. Hence $[(F,A)]$ is an interval valued T-fuzzy soft normal subhemiring of the hemiring R.

□

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