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T-fuzzy TL -ideal of Γ-near ring

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Abstract

In this paper, we establish and revise the concept of T-fuzzy TL– ideal of Γ- near ring. Also the notations of TL– ideal of Γ near ring were introduced with some related properties.

Keywords

Γ-near ring, TL-ideal, TL-Fuzzy ideal, Direct Product of TL– Fuzzy Sub Γ- near ring, Quotient Γ-near ring with t-norm.

AMS Subject Classification

16Y30, 16Y99, 03E72.

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Contents

1. Introduction

The theory of fuzzy sets and their related properties was introduced by Zadeh.L.A [15] in 1965. In 1991, Abou Zaid. S [1] defined a Fuzzy subnear-rings and ideals. In 1996, Seung dong kim and hee sikkim was defined as the homomorphic image of fuzzy ideals and some related properties. The notation of Γ-near ring was introduced by Bhavanari Satyanarayana and Syam Prasad. K [11]. In 2007, Akram. M was introduced the T- fuzzy ideals in near rings. In 2012, Srinivas.T and Nagaiah.T [13] was presented T-fuzzy ideal of Γ-near ring has several properties of Γ-near rings. In this paper, by using T- fuzzy ideal and TL-ideal of Γ-near ring all the above are use them. Further, additionally we introduce homomorphic images and direct product of T- fuzzy TL-ideal of Γ-near ring. We may expand to this paper as the Γ- near ring from a theoretical portion.

2. Preliminaries

In this section, we review the some definitions that will be required in this paper.

Definition 2.1. *A non empty set N with two binary operations "$+$ "(addition) and "$\cdot$" (Multiplication) is called a near ring if it satisfies the following axioms:*

(i) $(N,+)$ *is a group.*

(ii) $(N,\cdot)$ *is a semi group.*

(iii) $x\cdot(y+z) = x\cdot y+x\cdot z$ *for all* $x,y,z\in N$.

precisely speaking it is a left near ring because it satisfies the left distributive law. We will use the word near ring instead of "Left near-ring". We denote xy instead of $x\cdot y$. Note that $x0=0$ and $x(-y)=-xy$, but $0x\neq 0$ for $x,y\in N$.

Definition 2.2. *Let $(R,+)$ be a group and Γ be a nonempty set. Then R is said to be a Γ-near ring if their exist a mapping $R\times\Gamma\times R\to R$ satisfies the following conditions:*

(i) $(x+y)\alpha z = x\alpha z+y\alpha z.$

(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$

for all $x,y,z\in R$ *and* $\alpha,\beta\in\Gamma$.

Definition 2.3. *Let R be a Γ- near ring. A normal subgroup $(I,+)$ of $(R,+)$ is called*

(i) a left ideal if $x\alpha(y+i)$–$x\alpha y\in I$ *for all* $x,y\in R,\alpha\in\Gamma,i\in I$.

(ii) a right ideal if $i\alpha x\in I$ *for all* $x\in R,\alpha\in\Gamma,i\in I$

(iii) an ideal if it is both a left ideal and a right ideal of R.

A Γ- near ring R is said to be a zero – symmetric if $a\alpha 0=0$ for all $a\in R$ and $\alpha\in\Gamma$, where 0 is additive identity in R.

Definition 2.4. *A subset M of a Γ- near ring R is said to be a sub Γ- near ring if there exist a mapping $M\times\Gamma\times M\rightarrow M$ such that*

(i) $(M,+)$ *be a subgroup of* $(R,+)$.

(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$ *for every* $x,y,z\in M$ *and* $\alpha\in\Gamma$.

(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ *for every* $x,y,z\in M$ *and* $\alpha,\beta\in\Gamma$.

Definition 2.5. *A fuzzy sub near ring A of R is called a fuzzy ideal if it satisfies the following conditions:*

(i) $A(y+x-y)\geq A(x)$ *for all* $x,y,z\in R$.

(ii) $A(xy)\geq A(y)$ *for all* $x,y\in R,\alpha\in\Gamma$.

(iii) $A((x+i)y–xy)\geq A(i)$ *for all* $x,y,i\in R$.

if $\mu(0)\geq\mu(x)$ *for all* $x\in R$.

Definition 2.6. *A binary operation T on a lattice L is called a t-norm if it satisfies the following conditions:*

(i) $T(T(a,b),c)=T(a,T(b,c))$,

(ii) $T(a,b)=T(b,a)$,

(iii) $b\leq c\Rightarrow T(a,b)\leq T(a,c)$,

(iv) $T(a,1)=a$, *for all* $a,b,c\in L$.

Definition 2.7. *A function $\mu : R\rightarrow L$ is called an L-subset of R. The set of all L-subsets of R is called the L-power set of R and is denoted by L^R.*

Definition 2.8. *A fuzzy set A of a Γ- near ring R is called a T-fuzzy sub Γ- near ring with respect to a t-norm of R if,*

(i) $A(x–y)\geq A(x)TA(y)$

(ii) $A(x\alpha y)\geq A(x)TA(y)$

for all $x,y\in R$ *and* $\alpha\in\Gamma$.

Definition 2.9. *A fuzzy set A of a Γ- near ring R is called a T-fuzzy sub Γ- near ring A of R is called a $T-$fuzzy ideal of R it satisfies the following conditions,*

(i) $A(y+x-y)\geq A(x)$ *for all* $x,y,z\in R$.

(ii) $A(x\alpha y)\geq A(y)$ *for all* $x,y\in R,\alpha\in\Gamma$.

(iii) $A(x\alpha(z+y)–x\alpha y)\geq A(z)$ *for all* $x,y,z\in R,\alpha\in\Gamma$.

Definition 2.10. *An L- subset $A\in L^R$ of a Γ- near ring R is called a $TL–$ ideal of R if*

(i) $A(0)=1$

(ii) $A(-x)\geq A(x)$

(iii) $A(x+y)\geq A(x)TA(y)$

(iv) $A(y+x-y)\geq A(x)$

(v) $A(xy)\geq A(y)$

(vi) $A((x+i)y–xy)\geq A(i)$

for all $x,y,i\in R$.

Definition 2.11. *An L- subset $A\in L^R$ of a Γ- near ring R is called a T-fuzzy TL-ideal of R if*

(i) $A(0)=1$

(ii) $A(-x)\geq A(x)$

(iii) $A(x+y)\geq A(x)TA(y)$

(iv) $A(y+x-y)\geq A(x)$

(v) $A(x\alpha y)\geq A(y)$

(vi) $A(x\alpha(z+y)–x\alpha y))\geq A(z)$

for all $x,y,z\in R,\alpha\in\Gamma$.

Definition 2.12. *Let A and B be the T-fuzzy subsets of the Γ-near ring R has defined as $(A\cap B)(x)=A(x)TB(x)$.*

Definition 2.13. *([6]) Let L- subset A and $B\in L^R$ be $T-$fuzzy TL- ideal of Γ- near ring R. Then the direct product of T-fuzzy ideals is defined by, $(A\times B)(x,y)=A(x)TB(y)$ for all $x,y\in R$.*

Definition 2.14. *A L- subset T-fuzzy TL-ideal $A\in L^R$ of a Γ-near ring R is said to be normal if their exist an element $a\in R$ such that $A(a)=1$.*

If A is normal of a Γ-near ring R if and only if $A(1)=1$.

Definition 2.15. *Let R be a $\Gamma-$ near ring. Let ρ be a fuzzy set of a T-fuzzy ideal of R and f be a function defined on R, then the fuzzy set $\rho^f(y)=\sup_{x\in f^{-1}(y)}\rho(x)$ for all $y\in f(R)$ and is called as the image of ρ under f. Similarly, if λ is a fuzzy set in $f(R)$, then $\rho=\lambda\circ f$ in R is defined as $\rho(x)=\lambda(f(x))$ for all $x\in R$ and is called the pre image of λ under f.*

3. T-fuzzy TL-ideal of Γ-near ring

Let R be a near-ring and L be a complete lattice.

Theorem 3.1. *If L- subset of $A\in L^R$ is a T-fuzzy TL-ideal of R then, $A(x-y)\geq A(0)\Rightarrow A(x)=A(y)$ for all $x,y\in R$.*

Proof. Let $L-$ subset of A is a T-fuzzy TL-ideal of R then we have, $A(x-y)\geq A(0)$. But, $A(0)\leq A(x-y)$. Thus , $A(x-y)=A(0)$. Now consider,

$$\begin{aligned}A(x)&=A(y+x-y)\\&=A((y+(x-y))\\&\geq A(y)TA(x-y)\\&\geq A(y)TA(0)\\&\geq A(y).\end{aligned}$$

Similarly we can prove that $A(y)\geq A(x)$. Hence $A(x)=A(y)$ for all $x,y\in R$. □

Theorem 3.2. *If $L-$ subset of A and $B \in L^R$ are T-fuzzy TL-ideal of a Γ- near ring R then $A \cap B$ is a T-fuzzy TL-ideal of R.*

Proof. (i)

$$(A\cap B)(0) \geq A(0)TB(0) = 1$$

(ii)

$$\begin{aligned}&(A\cap B)(-x)\\ &\geq A(-x)TB(-x)\\ &\geq A(0-x)TB(0-x)\\ &\geq A(0)TA(x)TB(0)TB(x)\\ &\geq A(x)TB(x)\\ &\geq (A\cap B)(x)\end{aligned}$$

(iii)

$$\begin{aligned}&(A\cap B)(x-y)\\ &\geq A(x-y)TB(x-y)\\ &\geq A(x)TA(y)TB(x)TB(y)\\ &\geq (A(x)TB(x))(A(y)TB(y))\\ &\geq (A\cap B)(x)T(A\cap B)(y)\end{aligned}$$

(iv)

$$\begin{aligned}&(A\cap B)(y+x-y)\\ &\geq A(y+x-y)TB(y+x-y)\\ &\geq A(x)TB(x)\\ &\geq (A\cap B)(x)\end{aligned}$$

(v)

$$\begin{aligned}&(A\cap B)(x\alpha y)\\ &\geq A(x\alpha y)TB(x\alpha y)\\ &\geq A(y)TB(y)\\ &\geq (A\cap B)(y)\end{aligned}$$

(vi)

$$\begin{aligned}&(A\cap B)[(x\alpha(z+y)\text{–}x\alpha y)]\\ &\geq A[(x\alpha(z+y)\text{–}x\alpha y)]TB[(x\alpha(z+y)\text{–}x\alpha y)]\\ &\geq A(z)TB(z) \geq (A\cap B)(z).\end{aligned}$$

Hence $A\cap B$ is a T-fuzzy TL-ideal of R. This completes the proof. □

Theorem 3.3. *Let L- subset of $A \in L^R$ be a T-fuzzy TL-ideal of a Γ- near ring R and L- subset of $A^* \in L^R$ be a fuzzy set in R then A^* defined by, $A^* = \frac{A(x)}{A(1)} \forall x \in R$. Then A^* is a normal T-fuzzy TL-ideal of R contains A.*

Proof. Let L- subset of $A \in L^R$ be a T-fuzzy TL-ideal of a Γ-near ring R. For any $x, y, z \in R$ and $\alpha \in \Gamma$.We have,

(i)

$$A^*(0) = \frac{A(0)}{A(1)} \geq 1.$$

(ii)

$$\begin{aligned}A^*(-x) &= \frac{A(-x)}{A(1)}\\ &= \frac{A(0-x)}{A(1)}\\ &\geq \frac{A(0)TA(x)}{A(1)}\\ &\geq \frac{A(0)}{A(1)}T\frac{A(x)}{A(1)}\\ &\geq A^*(0)TA^*(x)\\ &\geq A^*(x).\end{aligned}$$

(iii)

$$\begin{aligned}A^*(x-y) &= \frac{A(x-y)}{A(1)}\\ &\geq \frac{A(x)TA(y)}{A(1)}\\ &\geq A(x)A(1)T\frac{A(y)}{A(1)}\\ &\geq A^*(x)TA^*(y).\end{aligned}$$

(iv)

$$\begin{aligned}A^*(y+x-y) &= \frac{A(y+x-y)}{A(1)}\\ &\geq \frac{A(x)}{A(1)}\\ &\geq A^*(x).\end{aligned}$$

(v)

$$\begin{aligned}A^*(x\alpha y) &= \frac{A(x\alpha y)}{A(1)}\\ &\geq \frac{A(y)}{A(1)}\\ &\geq A^*(y).\end{aligned}$$

(iv)

$$\begin{aligned}A^*[x\alpha(z+y)\text{–}x\alpha y] &= \frac{A[x\alpha(z+y)\text{–}x\alpha y]}{A(1)}\\ &\geq \frac{A(z)}{A(1)}\\ &\geq A^*(z).\end{aligned}$$

Hence A^* is a normal T-fuzzy TL-ideal of R contains A. □

Theorem 3.4. *Let L- subset of $A \in L^R$ be a T-fuzzy TL-ideal of a Γ- near ring R and let $L-$ subset of $A^+ \in L^R$ be a fuzzy set in R then A^+ is defined by, $A^+(x) = A(x) + 1 + A(1)$ for all $x \in R$. Then A^+ is a T-fuzzy TL-ideal of Γ-near ring R containing A.*

Proof. Let $L-$ subset of $A \in L^R$ be a $T-$ fuzzy $TL-$ ideal of a Γ-near ring R. For any $x, y, z \in R$ and $\alpha \in \Gamma$.

(i)

$$A^+(0) = A(0) + 1 + A(1) = 1.$$

(ii)

$$\begin{aligned} & A^+(-x) \\ & = A(0-x) + 1 + A(1) \\ & \geq (A(0) + 1 + A(1))T(A(x) + 1 + A(1)) \\ & \geq A(x) + 1 + A(1) \\ & \geq A^+(x) \end{aligned}$$

(iii)

$$\begin{aligned} & A^+(x-y) \\ & = A(x-y) + 1 + A(1) \\ & \geq (A(x)TA(y)) + 1 + A(1) \\ & \geq (A(x) + 1 + A(1))T(A(y) + 1 + A(1)) \\ & \geq A^+(x)TA^+(y). \end{aligned}$$

(iv)

$$\begin{aligned} & A^+(y+x-y) \\ & = A(y+x-y) + 1 + A(1) \\ & \geq A(x) + 1 + A(1) \\ & \geq A^+(x). \end{aligned}$$

(v)

$$\begin{aligned} & A^+(x\alpha y) \\ & = A(x\alpha y) + 1 + A(1) \\ & \geq A(y) + 1 + A(1) \\ & \geq A^+(y). \end{aligned}$$

(vi)

$$\begin{aligned} & A^+[x\alpha(z+y) – x\alpha y] \\ & = A[x\alpha(z+y) – x\alpha y] + 1 + A(1) \\ & \geq A(z) + 1 + A(1) \\ & \geq A(z). \end{aligned}$$

Hence A^+ is a T-fuzzy TL-ideal of Γ- near ring R containing A.

□

Theorem 3.5. *An onto homomorphic image of a T-fuzzy TL-ideal with sup property is a T-fuzzy TL-ideal.*

Proof. Let M and N are Γ- near rings. Let $f : M \to N$ be epimorphism and L- subset of $A \in L^R$.A be a T-fuzzy TL-ideal of R with sub property. Let $x, y \in N, x_0 \in f'(x), y_0 \in f'(y)$ and $z_0 \in f'(z)$ be such that $A(x_0) = \sup_{n \in f'(x)} A(n), A(y_0) = \sup_{n \in f'(y)} A(n)$ and $A(z_0) = \sup_{n \in f'(z)} A(n)$ respectively. Then for any $\alpha \in A$, we have,

(i)

$$A^f(0) = \sup_{z \in f'(0)} A(z) \geq A(0) = 1.$$

(ii)

$$\begin{aligned} & A^f(-x) \\ & = \sup_{z \in f'(0)} A(z) \\ & \geq A(x_0) \\ & \geq [\sup_{n \in f'(x)} A(n)] \\ & = A^f(x). \end{aligned}$$

(iii)

$$\begin{aligned} & A^f(x-y) \\ & = \sup_{z \in f'(x-y)} A(z) \\ & \geq A(x_0 - y_0) \\ & \geq A(x_0)TA(y_0) \\ & \geq [\sup_{n \in f'(x)} A(n)]T[\sup_{n \in f'(y)} A(n)] \\ & = A^f(x)TA^f(y). \end{aligned}$$

(iv)

$$\begin{aligned} & A^f(y+x-y) \\ & = \sup_{z \in f'(y+x-y)} A(z) \\ & \geq A(x_0) \\ & \geq [\sup_{n \in f'(x)} A(n)] \\ & = A^f(x). \end{aligned}$$

(v)

$$\begin{aligned} & A^f(x\alpha y) \\ & = \sup_{z \in f'(x\alpha y)} A(z) \\ & \geq A(y_0) \\ & \geq [\sup_{n \in f'(y)} A(n)] \\ & = A^f(y). \end{aligned}$$

(vi)

$$A^f[x\alpha(z+y)\text{–}x\alpha y]$$
$$=\sup_{[x\alpha(z+y)-x\alpha y]} A(z)$$
$$\geq A(z_0)$$
$$\geq [\sup_{n\in f'(z)} A(n)]$$
$$= A^f(z).$$

This completes the proof. □

Theorem 3.6. *An epimorphic pre image of a T-fuzzy TL-ideal of a Γ-near ring is a T-fuzzy TL-ideal of R.*

Proof. Let M and N be Γ near rings. Let $f: M \to N$ is an epimorphism. Let $L-$ subset of $\lambda \in L^R$ be the T-fuzzy TL-ideal of N and ρ be the pre image of λ under f. Then for any $x,y,z \in M$ and $\alpha \in \Gamma$. We have,

(i)

$$\rho(0) = (\lambda \circ f)(0) = \lambda(f(0)) = 1.$$

(ii)

$$\rho(-x) = (\lambda \circ f)(-x)$$
$$= \lambda(f(-x))$$
$$\geq \lambda(f(x))$$
$$\geq (\lambda \circ f)(x)$$
$$\geq \rho(x)$$

(iii)

$$\rho(x-y) = (\lambda \circ f)(x-y)$$
$$= \lambda(f(x-y))$$
$$\geq \lambda(f(x)Tf(y))$$
$$\geq \lambda(f(x))T\lambda(f(y))$$
$$\geq (\lambda \circ f)(x)T(\lambda \circ f)(y)$$
$$\geq \rho(x)T\rho(y).$$

(iv)

$$\rho(y+x-y) = (\lambda \circ f)(y+x-y)$$
$$= \lambda(f(y+x-y)$$
$$\geq \lambda(f(x))$$
$$\geq (\lambda \circ f)(x)$$
$$\geq \rho(x).$$

(v)

$$\lambda(x\alpha y) = (\lambda \circ f)(x\alpha y)$$
$$= \lambda(f(x\alpha y))$$
$$\geq \lambda(f(y))$$
$$\geq (\lambda \circ f)(y)$$
$$\geq \rho(y).$$

(vi)

$$\rho[x\alpha(z+y)-x\alpha y]$$
$$= (\lambda \circ f)[x\alpha(z+y)-x\alpha y]$$
$$= \lambda(f[x\alpha(z+y)-x\alpha y])$$
$$\geq \lambda(f(z))$$
$$\geq (\lambda \circ f)(z)$$
$$\geq \rho(x).$$

Hence ρ is a $T-$fuzzy TL-ideal of Γ- near ring.

□

Theorem 3.7. *Let M and N be Γ- near rings. If $L-$subset of A_1 and $A_2 \in L^R$ be T-fuzzy TL-ideal of Γ-near rings of M and N respectively, then $A = A_1XA_2$ is a T-fuzzy TL-ideal of the direct product of MXN.*

Proof. Let $L-$ subset of A_1 and $A_2 \in L^R$ be $T-$fuzzy $TL-$ideal of Γ-near rings of M and N respectively. Let $(x_1,x_2),(y_1,y_2),(z_1,z_2) \in MXN$ and $\alpha \in \Gamma$. Then,

(i)

$$A(0) = A_1XA_2(0) = 1.$$

(ii)

$$A(-(x_1,x_2))$$
$$= A_1XA_2(-(x_1,x_2))$$
$$= A_1(-(x_1,x_2))TA_2(-(x_1,x_2))$$
$$\geq A_1(x_1,x_2)TA_2(x_1,x_2)$$
$$\geq A_1XA_2(x_1,x_2)$$
$$\geq A(x_1,x_2).$$

(iii)

$$A((x_1,x_2)-(y_1,y_2))$$
$$= A_1XA_2(x_1-y_1,x_2-y_2)$$
$$= A_1(x_1-y_1)TA_2(x_2-y_2)$$
$$\geq A_1(x_1)TA_1(y_1)TA_2(x_2)TA_2(y_2)$$
$$\geq A_1(x_1)TA_2(x_2)TA_1(y_1)TA_2(y_2)$$
$$\geq (A_1XA_2)(x_1,x_2)T(A_1XA_2)(y_1,y_2)$$
$$\geq A(x_1,x_2)TA(y_1,y_2).$$

(iv)

$$\begin{aligned}&A((x_1,x_2)\alpha(y_1,y_2))\\&=A_1XA_2((x_1,x_2)\alpha(y_1,y_2))\\&=A_1((x_1,x_2)\alpha(y_1,y_2))TA_2((x_1,x_2)\alpha(y_1,y_2))\\&\geq A_1(y_1,y_2)TA_2(y_1,y_2)\\&\geq(A_1XA_2)(y_1,y_2)\\&\geq A((y_1,y_2)+(x_1,x_2)-(y_1,y_2))\\&=A_1XA_2((y_1,y_2)+(x_1,x_2)-(y_1,y_2))\\&=A_1((y_1,y_2)+(x_1,x_2)-(y_1,y_2))\\&\quad(\times)TA_2((y_1,y_2)+(x_1,x_2)-(y_1,y_2))\\&\geq A_1(x_1,x_2)TA_2(x_1,x_2)\\&\geq A_1XA_2(x_1,x_2)\\&\geq A(x_1,x_2).\end{aligned}$$

(vi)

$$\begin{aligned}&A[(x_1,x_2)\alpha((z_1,z_2)+(y_1,y_2))\\&-((x_1,x_2)\alpha(y_1,y_2))\\&=A_1XA_2[(x_1,x_2)\alpha((z_1,z_2)+(y_1,y_2))\\&-((x_1,x_2)\alpha(y_1,y_2))]\\&=A_1[(x_1,x_2)\alpha((z_1,z_2)+(y_1,y_2))\\&\quad-((x_1,x_2)\alpha(y_1,y_2))]TA_2[(x_1,x_2)\alpha((z_1,z_2)\\&\quad+(y_1,y_2))-((x_1,x_2)\alpha(y_1,y_2))]\\&\geq A_1(z_1,z_2)TA_2(z_1,z_2)\\&\geq A_1XA_2(z_1,z_2)\\&\geq A(z_1,z_2).\end{aligned}$$

Hence $A=A_1XA_2$ is a T-fuzzy TL-ideal of R. □

Notation ([11]) Let L-subset $\mu\in L^R$ be a T-fuzzy TL-ideal of a Γ- near ring R. We define $\varphi_\mu=\frac{R}{\mu}\to[0,1]$ by $\varphi_\mu(x+\mu)=\mu(x)$ for all $x\in R$.

Theorem 3.8. *If L-subset $\mu\in L^R\mu$ is a T-fuzzy TL-ideal of a Γ- near ring R, then φ_μ is a T-fuzzy TL-ideal of a Γ- near ring.*

Proof. Let $L-$ subset $\mu\in L^R$ be a T-fuzzy TL-ideal of a Γ-near ring R and $x,y\in R$. Suppose that, $(x+\mu)=(y+\mu)$. Then $\mu(x-y)=0$. This implies that, $\mu(x)=\mu(y).\varphi_\mu(x+\mu)=\varphi_\mu(y+\mu)$. Let $x+\mu,y+\mu,z+\mu\in\frac{R}{\mu}$ and $\alpha\in\Gamma$. Then,

(i)

$$\varphi_\mu(0+\mu)=\mu(0)=1.$$

(ii)

$$\begin{aligned}\varphi_\mu((-x)+\mu)&=\mu(-x)\\&\geq\mu(x)\\&\geq\varphi_\mu(x+\mu)\end{aligned}$$

(iii)

$$\begin{aligned}&\varphi_\mu((x+\mu)–(y+\mu))\\&=\varphi_\mu((x-y)+\mu)\\&=\mu(x-y)\\&\geq\mu(x)T\mu(y)\\&\geq\varphi_\mu(x+\mu)T\varphi_\mu(y+\mu).\end{aligned}$$

(iv)

$$\begin{aligned}&\varphi_\mu[(x+\mu)\alpha(y+\alpha)]\\&=\varphi_\mu(x\alpha y+\alpha)\\&=\mu(x\alpha y+\mu)\\&\geq\mu(y)\\&\geq\varphi_\mu(y+\mu).\end{aligned}$$

(v)

$$\begin{aligned}&\varphi_\mu((y+\mu)+(x+\mu)-(y+\mu))\\&=\varphi_\mu[(y+x-y)+\mu]\\&=\mu(y+x-y)\geq\mu(x)\\&\geq\varphi_\mu(x+\mu).\end{aligned}$$

(vi)

$$\begin{aligned}&\varphi_\mu[((x+\mu)\alpha(z+\mu)+(y+\mu))-((x+\mu)\alpha(y+\mu))]\\&=\varphi_\mu[(x\alpha(z+y)-x\alpha y)]\\&=\mu[(x\alpha(z+y)-x\alpha y)]\\&\geq\mu(z)\\&\geq\varphi_\mu(z+\mu).\end{aligned}$$

Hence φ_μ is a T-fuzzy TL-ideal of a Γ-near ring $\frac{R}{\mu}$. □

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