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Level subsets of bipolar valued fuzzy subhemiring of a hemiring

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Abstract

In this paper, we study some of the properties of (α,β)-level subsets of bipolar valued fuzzy subhemiring of a hemiring and prove some results on these.

Keywords

Bipolar valued fuzzy subset, bipolar valued fuzzy subhemiring, level subset.

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Contents

1. Introduction

In 1965, Zadeh [15] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc [7]. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0,1]$ to $[-1,1]$. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree $(0,1]$ indicates that elements somewhat satisfy the property and the membership degree $[-1,0)$ indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [9, 10]. Anitha.M.S., Muruganantha Prasad & K.Arjunan [1] defined as bipolar valued fuzzy subgroups of a group. We introduce the concept of (α,β)-level subsets of bipolar valued fuzzy subhemirings of a hemiring are discussed. Using these concepts, some results are established.

2. Prelimaries

Definition 2.1. *A bipolar valued fuzzy set (BVFS) of X is defined as an object of the form $A=\{<x,A^+(x),A^-(x)>/x\in X\}$, where $A^+:X\to[0,1]$ and $A^-:X\to[-1,0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If $A^+(x)\neq 0$ and $A^-(x)=0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x)=0$ and $A^-(x)\neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x)\neq 0$ and $A^-(x)\neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.*

Example 2.2. $A=\{<a,0.7,-0.5>,<b,0.6,-0.3>,<c,0.5,-0.9>\}$ *is a bipolar valued fuzzy subset of* $X=\{a,b,c\}$.

Definition 2.3. *Let R be a hemiring. A bipolar valued fuzzy subset A of R is said to be a bipolar valued fuzzy subhemiring of R (BVFSHR) if the following conditions are satisfied,*

(i) $A^+(x+y)\geq\min\{A^+(x),A^+(y)\}$

(ii) $A^+(xy)\geq\min\{A^+(x),A^+(y)\}$

(iii) $A^-(x+y) \le \max\{A^-(x), A^-(y)\}$

(iv) $A^-(xy) \le \max\{A^-(x), A^-(y)\}$ *for all* x *and* y *in* R.

Example 2.4. *Let* $R = Z_3 = \{0,1,2\}$ *be a hemiring with respect to the ordinary addition and multiplication. Then* $A = \{<0, 0.6, -0.7>, <1, 0.5, -0.6>, <2, 0.5, -0.6>\}$ *is a bipolar valued fuzzy subhemiring of* R.

Definition 2.5. *Let* $A = \langle A^+, A^- \rangle$ *be a bipolar valued fuzzy subset of* X. *For* α *in* $[0,1]$ *and* β *in* $[-1,0]$, *the* (α,β)*-level subset of* A *is the set* $A_{(\alpha,\beta)} = \{x \in X : A^+(x) \ge \alpha$ *and* $A^-(x) \le \beta\}$.

Example 2.6. *Consider the set* $X = \{0,1,2,3,4\}$. *Let* $A = \{(0,0.7,-0.3),(1,0.6,-0.5),(2,0.8,-0.25),(3,0.65,-0.4),(4,0.4,-0.7)\}$ *be a bipolar valued fuzzy subset of* X *and* $\alpha = 0.6, \beta = -0.3$. *Then* $(0.6,-0.3)$*-level subset of* A *is* $A_{(0.6,-0.3)} = \{0,1,3\}$.

Definition 2.7. *Let* $A = \langle A^+, A^- \rangle$ *be a bipolar valued fuzzy subset of* X. *For* α *in* $[0,1]$, *the* A^+ *-level* α*-cut of* A *is the set* $P(A^+,\alpha) = \{x \in X : A^+(x) \ge \alpha\}$.

Example 2.8. *Consider the set* $X = \{0,1,2,3,4\}$. *Let* $A = \{(0,0.5,-0.1),(1,0.4,-0.3),(2,0.6,-0.05),(3,0.45,-0.2),(4,0.2,-0.5)\}$ *be a bipolar valued fuzzy subset of* X *and* $\alpha = 0.4$. *Then* A^+ *-level* 0.4*-cut of* A *is* $P(A^+,0.4) = \{0,1,2,3\}$.

Definition 2.9. *Let* $A = \langle A^+, A^- \rangle$ *be a bipolar valued fuzzy subset of* X. *For* β *in* $[-1,0]$, *the* A^- *-level* β*-cut of* A *is the set* $N(A^-,\beta) = \{x \in X : A^-(x) \le \beta\}$.

Example 2.10. *Consider the set* $X = \{0,1,2,3,4\}$. *Let* $A = \{(0,0.6,-0.2),(1,0.5,-0.4),(2,0.7,-0.15),(3,0.55,-0.3),(4,0.3,-0.6)\}$ *be a bipolar valued fuzzy subset of* X *and* $\beta = -0.2$. *Then* A^- *-level* -0.2*-cut of* A *is* $N(A^-,-0.2) = \{0,1,3,4\}$.

Definition 2.11. *Let* X *and* X' *be any two sets. Let* $f : X \to X'$ *be any function and let* A *be a bipolar valued fuzzy subset in* X, V *be a bipolar valued fuzzy subset in* $f(X) = X'$, *defined by* $V^+(y) = \sup_{x \in f^{-1}(y)} A^+(x)$ *and* $V^-(y) = \inf_{x \in f^{-1}(y)} A^-(x)$, *for all* x *in* X *and* y *in* X'. A *is called a preimage of* V *under* f *and is defined as* $A^+(x) = V^+(f(x)), A^-(x) = V^-(f(x))$ *for all* x *in* X *and is denoted by* $f^{-1}(V)$.

3. Properties

Theorem 3.1. *Let* R *and* R' *be any two hemirings. The homomorphic image of a bipolar valued fuzzy subhemiring of* R *is a bipolar valued fuzzy subhemiring of* R'.

Theorem 3.2. *Let* R *and* R' *be any two hemirings. The homomorphic preimage of a bipolar valued fuzzy subhemiring of* R' *is a bipolar valued fuzzy subhemiring of* R.

Theorem 3.3. *Let* R *and* R' *be any two hemirings. The anti-homomorphic image of a bipolar valued fuzzy subhemiring of* R *is a bipolar valued fuzzy subhemiring of* R'.

Theorem 3.4. *Let* R *and* R' *be any two hemirings. The anti-homomorphic preimage of a bipolar valued fuzzy subhemiring of* R' *is a bipolar valued fuzzy subhemiring of* R.

Theorem 3.5. *Let* $A = \langle A^+, A^- \rangle$ *be a bipolar valued fuzzy subhemiring of a hemiring* R. *Then for* α *in* $[0,1]$ *and* β *in* $[-1,0]$ *such that* $\alpha \le A^+(e)$ *and* $\beta \ge A^-(e)$, *is a* (α,β)*-level subhemiring of* R.

Proof. For all x and y in $A_{(\alpha,\beta)}$, we have, $A^+(x) \ge \alpha$ and $A^-(x) \le \beta$ and $A^+(y) \ge \alpha$ and $A^-(y) \le \beta$. Now

$$\begin{aligned} A^+(x+y) &\ge \min\{A^+(x), A^+(y)\} \\ &\ge \min\{\alpha,\alpha\} \\ &= \alpha, \end{aligned}$$

which implies that

$$A^+(x+y) \ge \alpha.$$

And

$$\begin{aligned} A^+(xy) &\ge \min\{A^+(x), A^+(y)\} \\ &\ge \min\{\alpha,\alpha\} \\ &= \alpha, \end{aligned}$$

which implies that

$$A^+(xy) \ge \alpha.$$

Also

$$\begin{aligned} A^-(x+y) &\le \max\{A^-(x), A^-(y)\} \\ &\le \max\{\beta,\beta\} \\ &= \beta, \end{aligned}$$

which implies that

$$A^-(x+y) \le \beta.$$

And

$$\begin{aligned} A^-(xy) &\le \max\{A^-(x), A^-(y)\} \\ &\le \max\{\beta,\beta\} \\ &= \beta, \end{aligned}$$

which implies that

$$A^-(xy) \le \beta.$$

Therefore $x+y, xy$ in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of R. □

Theorem 3.6. *Let* $A = \langle A^+, A^- \rangle$ *be a bipolar valued fuzzy subhemiring of a hemiring* R. *Then for* α, δ *in* $[0,1], \beta, \phi$ *in* $[-1,0], \alpha \le A^+(e), \delta \le A^+(e), \beta \ge A^-(e), \phi \ge A^-(e), \delta < \alpha$ *and* $\beta < \phi$, *the two* (α,β)*-level subhemirings* $A_{(\alpha,\beta)}$ *and* $A_{(\delta,\phi)}$ *of* A *are equal if and only if there is no* x *in* R *such that* $\alpha > A^+(x) > \delta$ *and* $\beta < A^-(x) < \phi$.

Proof. Assume that $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$. Suppose there exists x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$. Then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$ implies x belongs to $A_{(\delta,\phi)}$, but not in $A_{(\alpha,\beta)}$. This is contradiction to $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$. Therefore there is no x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$. Conversely, if there is no x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$. Then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$ (By the definition of (α,β)-level subset). □

Theorem 3.7. *Let $A = \langle A^+, A^-\rangle$ be a bipolar valued fuzzy subhemiring of a hemiring R. If any two (α,β)-level subhemirings of A belongs to R, then their intersection is also (α,β)-level subhemiring of A in R.*

Proof. Let α,β in $[0,1], \beta,\phi$ in $[-1,0], \alpha \leq A^+(e)$, $\delta \leq A^+(e), \beta \geq A^-(e), \phi \geq A^-(e)$.

Case (i): If $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$. Therefore $A_{(\alpha,\beta)} \cap A_{(\delta,\phi)} = A_{(\alpha,\beta)}$, but $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of A.

Case (ii): If $\alpha < A^+(x) < \delta$ and $\beta > A^-(x) > 0$, then $A_{(\delta,\phi)} \subset A_{(\alpha,\beta)}$. Therefore $A_{(\alpha,\beta)} \cap A_{(\delta,\phi)} = A_{(\delta,\phi)}$, but $A_{(\delta,\phi)}$ is a (α,β)-level subhemiring of A.

Case (iii): If $\alpha < A^+(x) < \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\delta,\beta)} \subseteq A_{(\alpha,\phi)}$. Therefore $A_{(\delta,\beta)} \cap A_{(\alpha,\phi)} = A_{(\delta,\beta)}$, but $A_{(\delta,\beta)}$ is a (α,β)-level subhemiring of A.

Case (iv): If $\alpha > A^+(x) > \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\alpha,\phi)} \subseteq A_{(\delta,\beta)}$. Therefore $A_{(\alpha,\phi)} \cap A_{(\delta,\beta)} = A_{(\alpha,\phi)}$, but $A_{(\alpha,\phi)}$ is a (α,β)-level subhemiring of A.

Case (v): If $\alpha = \alpha$ and $\beta = \beta$, then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$. In other cases are true, so, in all the cases, intersection of any two (α,β)-level subhemirings is a (α,β)-level subhemiring of A. □

Theorem 3.8. *Let $A = \langle A^+, A^-\rangle$ be a bipolar valued fuzzy subhemiring of a hemiring R. The intersection of a collection of (α,β)-level subhemirings of A is also a (α,β)-level subhemiring of A.*

Proof. It is easily proved by Theorem 3.7. □

Theorem 3.9. *Let $A = \langle A^+, A^-\rangle$ be a bipolar valued fuzzy subhemiring of a hemiring R. If any two (α,β)-level subhemirings of A belongs to R, then their union is also (α,β)-level subhemiring of A in R.*

Proof. Let α,δ in $[0,1], \beta,\phi$ in $[-1,0], \alpha \leq A^+(e)$, $\delta \leq A^+(e), \beta \geq A^-(e), \phi \geq A^-(e)$.

Case (i): If $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$. Therefore $A_{(\alpha,\beta)} \cup A_{(\delta,\phi)} = A_{(\delta,\phi)}$, but $A_{(\delta,\phi)}$ is a (α,β)-level subhemiring of A.

Case (ii): If $\alpha < A^+(x) < \delta$ and $\beta > A^-(x) > 0$, then $A_{(\delta,\phi)} \subset A_{(\alpha,\beta)}$. Therefore $A_{(\alpha,\beta)} \cup A_{(\delta,\phi)} = A_{(\alpha,\beta)}$, but $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of A.

Case (iii): If $\alpha < A^+(x) < \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\delta,\beta)} \subseteq A_{(\alpha,\phi)}$. Therefore $A_{(\delta,\beta)} \cup A_{(\alpha,\phi)} = A_{(\alpha,\phi)}$, but $A_{(\alpha,\phi)}$ is a (α,β)-level subhemiring of A.

Case (iv): If $\alpha > A^+(x) > \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\alpha,\phi)} \subseteq A_{(\delta,\beta)}$. Therefore $A_{(\alpha,\phi)} \cup A_{(\delta,\phi)} = A_{(\delta,\beta)}$, but $A_{(\delta,\beta)}$ is a (α,β)-level subhemiring of A.

Case (v): If $\alpha = \delta$ and $\beta = \phi$, then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$. In other cases are true, so, in all the cases, intersection of any two (α,β)-level subhemirings is a (α,β)-level subhemiring of A. □

Theorem 3.10. *Let $A = \langle A^+, A^-\rangle$ be a bipolar valued fuzzy subhemiring of a hemiring R. The union of a collection of (α,β)-level subhemirings of A is also a (α,β)-level subhemiring of A.*

Proof. It is easily proved by Theorem 3.9. □

Theorem 3.11. *The homomorphic image of a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R'.*

Proof. Let $V = f(A)$. Here $A = \langle A^+, A^-\rangle$ is a bipolar valued fuzzy subhemiring of R. By Theorem 3.1, $V = \langle V^+, V^-\rangle$ is a bipolar valued fuzzy subhemiring of R'. Let x and y in R. Then $f(x)$ and $f(y)$ in R'. Let $A_{(\alpha,\beta)}$ be a (α,β)-level subhemiring of A. That is, $A^+(x) \geq \alpha$ and

$$A^-(x) \leq \beta; A^+(y) \geq \alpha$$

and

$$\begin{aligned} &A^-(y) \leq \beta; \\ &A^+(x+y) \geq \alpha, \\ &A^-(x+y) \leq \beta, \\ &A^+(xy) \geq \alpha, \\ &A^-(xy) \leq \beta. \end{aligned}$$

We have to prove that $f(A_{(\alpha,\beta)})$ is a (α,β)-level subhemiring of V. Now $V^+(f(x)) \geq A^+(x) \geq \alpha$ which implies that $V^+(f(x)) \geq \alpha$; and $V^+(f(y)) \geq A^+(y) \geq \alpha$ which implies that $V^+(f(y)) \geq \alpha$. Then

$$\begin{aligned} &V^+(f(x)+f(y)) \\ &= V^+(f(x+y)) \\ &\geq A^+(x+y) \\ &\geq \alpha, \end{aligned}$$

which implies that

$$V^+(f(x)+f(y)) \geq \alpha.$$

And

$$\begin{aligned} &V^+(f(x)f(y)) \\ &= V^+(f(xy)) \\ &\geq A^+(xy) \\ &\geq \alpha, \end{aligned}$$

which implies that

$$V^+(f(x)f(y)) \geq \alpha.$$

And

$$V^-(f(x)) \leq A^-(x) \leq \beta$$

which implies that

$$V^-(f(x)) \leq \beta;$$

and

$$V^-(f(y)) \leq A^-(y) \leq \beta$$

which implies that

$$V^-(f(y)) \leq \beta.$$

Then

$$\begin{aligned}&V^-(f(x)+f(y))\\&=V^-(f(x+y))\\&\leq A^-(x+y)\\&\leq \beta,\end{aligned}$$

which implies that

$$V^-(f(x)+f(y)) \leq \beta.$$

And

$$\begin{aligned}&V^-(f(x)f(y))\\&=V^-(f(xy))\\&\leq A^-(xy)\\&\leq \beta,\end{aligned}$$

which implies that

$$V^-(f(x)f(y)) \leq \beta.$$

Hence $f(A_{(\alpha,\beta)})$ is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring V of R'. □

Theorem 3.12. *The homomorphic pre-image of a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R' is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R.*

Proof. Let $V = f(A)$. Here $V = \langle V^+, V^-\rangle$ is a bipolar valued fuzzy subhemiring of R'. By Theorem 3.2, $A = \langle A^+, A^-\rangle$ is a bipolar valued fuzzy subhemiring of R. Let $f(x)$ and $f(y)$ in R'. Then x and y in R. Let $f(A_{(\alpha,\beta)})$ be a (α,β)-level subhemiring of V. That is, $V^+(f(x)) \geq \alpha$ and $V^-(f(x)) \leq \beta; V^+(f(y)) \geq \alpha$ and

$$\begin{aligned}&V^-(f(y)) \leq \beta;\\&V^+(f(x)+f(y)) \geq \alpha,\\&V^-(f(x)+f(y)) \leq \beta,\\&V^+((fx)f(y)) \geq \alpha,\\&V^-((fx)f(y)) \leq \beta.\end{aligned}$$

We have to prove that $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of A. Now $A^+(x) = V^+(f(x)) \geq \alpha$ which implies that $A^+(x) \geq \alpha$; and $A^+(y) \geq V^+(f(y)) \geq \alpha$ which implies that $A^+(y) \geq \alpha$. Then $A^+(x+y) = V^+(f(x+y)) = V^+(f(x)+f(y)) \geq \alpha$, which implies that $A^+(x+y) \geq \alpha$. And $A^+(xy) = A^+(f(xy)) \geq V^+f((x)f(y)) \geq \alpha$, which implies that $A^+(xy) \geq \alpha$. And $A^-(x) \leq V^-(f(x)) \leq \beta$ which implies that $A^-(x) \leq \beta$; and $A^-(y) \leq V^-(f(y)) \leq \beta$ which implies that $A^-(y) \leq \beta$. Then

$$\begin{aligned}&A^-(x+y)\\&=V^-(f(x+y))\\&\leq V^-(f(x)+f(y))\\&\leq \beta,\end{aligned}$$

which implies that

$$A^-(x+y) \leq \beta.$$

And

$$A^-(xy) = V^-(f(xy)) = V^-(f(x)f(y)) \leq \beta,$$

which implies that $A^-(xy) \leq \beta$. Hence $f(A_{(\alpha,\beta)})$ is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring A of R. □

Theorem 3.13. *The anti-homomorphic image of a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R'.*

Proof. Let $V = f(A)$. Here $A = \langle A^+, A^-\rangle$ is a bipolar valued fuzzy subhemiring of R. By Theorem 3.3, $V = \langle V^+, V^-\rangle$ is a bipolar valued fuzzy subhemiring of R'. Let x and y in R. Then $f(x)$ and $f(y)$ in R'. Let $A_{(\alpha,\beta)}$ be a (α,β)-level subhemiring of A. That is, $A^+(x) \geq \alpha$ and $A^-(x) \leq \beta; A^+(y) \geq \alpha$ and $A^-(y) \leq \beta$ and $A^+(x+y) \geq \alpha, A^-(x+y) \leq \beta, A^+(xy) \geq \alpha, A^-(xy) \leq \beta$. We have to prove that $f(A_{(\alpha,\beta)})$ is a (α,β)-level subhemiring of V. Now $V^+(f(x)) \geq A^+(x) \geq \alpha$ which implies that $V^+(f(x)) \geq \alpha$; and $V^+(f(y)) \geq A^+(y) \geq \alpha$ which implies that $V^+(f(y)) \geq \alpha$. also

$$\begin{aligned}&V^+(f(x)+f(y))\\&=V^+(f(y+x))\\&\geq A^+(x+y)\\&\geq \alpha,\end{aligned}$$

which implies that

$$V^+(f(x)+f(y)) \geq \alpha.$$

And

$$V^+(f(x)f(y)) = V^+(f(yx)) \geq A^+(yx) \geq \alpha,$$

which implies that

$$V^+(f(x)f(y)) \geq \alpha.$$

And

$$V^-(f(x)) \leq A^-(x) \leq \beta$$

which implies that

$$V^-(f(x)) \leq \beta;$$

and

$$V^-(f(y)) \leq A^-(y) \leq \beta$$

which implies that

$$V^-(f(y)) \leq \beta.$$

Then

$$\begin{aligned} &V^-(f(x)+f(y)) \\ &= V^-(f(y+x)) \\ &\leq A^-(x+y) \\ &\leq \beta, \end{aligned}$$

which implies that

$$V^-(f(x)+f(y)) \leq \beta.$$

And

$$V^-(f(x)f(y)) = V^-(f(yx)) \leq A^-(yx) \leq \beta,$$

which implies that

$$V^-(f(x)f(y)) \leq \beta.$$

Hence $f(A_{(\alpha,\beta)})$ is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring V of R'. □

Theorem 3.14. *The anti-homomorphic pre-image of a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R' is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring of a hemiring R.*

Proof. Let $V = f(A)$. Here $V = \langle V^+, V^-\rangle$ is a bipolar valued fuzzy subhemiring of R'. By Theorem 3.4, $A = \langle A^+, A^-\rangle$ is a bipolar valued fuzzy subhemiring of R. Let $f(x)$ and $f(y)$ in R'. Then x and y in R. Let $f(A_{(\alpha,\beta)})$ be a (α,β)-level subhemiring of V. That is, $V^+(f(x)) \geq \alpha$ and $V^-(f(x)) \leq \beta; V^+(f(y)) \geq \alpha$ and

$$\begin{aligned} &V^-(f(y)) \leq \beta; \\ &V^+(f(y)+f(x)) \geq \alpha, \\ &V^-(f(y)+f(x)) \leq \beta, \\ &V^+(f(y)f(x)) \geq \alpha, \\ &V^-(f(y)f(x)) \leq \beta. \end{aligned}$$

We have to prove that $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of A. Now $A^+(x) = V^+(f(x)) \geq \alpha$ which implies that $A^+(x) \geq \alpha$; and $A^+(y) \geq V^+(f(y)) \geq \alpha$ which implies that $A^+(y) \geq \alpha$. Then $A^+(x+y) = V^+(f(x+y)) = V^+(f(x)+f(y)) \geq \alpha$, which implies that $A^+(x+y) \geq \alpha$. And $A^+(xy) = V^+(f(xy)) \geq V^+f((x)f(y)) \geq \alpha$, which implies that $A^+(xy) \geq \alpha$. And $A^-(x) \leq V^-(f(x)) \leq \beta$ which implies that $A^-(x) \leq \beta$; and $A^-(y) \leq V^-(f(y)) \leq \beta$ which implies that $A^-(y) \leq \beta$. Then $A^-(x+y) = V^-(f(x+y)) \leq V^-(f(x)+f(y)) \leq \beta$, which implies that $A^-(x+y) \leq \beta$. And $A^-(f(x)f(y)) = A^-(xy) \leq V^-(f(xy)) \leq \beta$, which implies that $A^-(xy) \leq \beta$. Hence $A_{(\alpha,\beta)}$ is a (α,β)-level subhemiring of a bipolar valued fuzzy subhemiring A of R. □

Theorem 3.15. *Let A be a bipolar valued fuzzy subhemiring of a hemiring R. Then for α in $[0,1]$, A^+-level α-cut $P(A^+,\alpha)$ is a A^+-level α-cut subhemiring of R.*

Proof. For all x and y in $P(A^+,\alpha)$, we have $A^+(x) \geq \alpha$ and $A^+(y) \geq \alpha$. Now

$$\begin{aligned} A^+(x+y) &\geq \min\{A^+(x), A^+(y)\} \\ &\geq \min\{\alpha,\alpha\} \\ &= \alpha, \end{aligned}$$

which implies that

$$A^+(x+y) \geq \alpha.$$

And

$$\begin{aligned} A^+(xy) &\geq \min\{A^+(x), A^+(y)\} \\ &\geq \min\{\alpha,\alpha\} \\ &= \alpha, \end{aligned}$$

which implies that $A^+(xy) \geq \alpha$. Therefore $x+y, xy$ in $P(A^+,\alpha)$. Hence $P(A^+,\alpha)$ is a A^+-level α-cut subhemiring of R. □

Theorem 3.16. *Let A be a bipolar valued fuzzy subhemiring of a hemiring R. Then for β in $[-1,0]$, A^--level β-cut $N(A^-,\beta)$ is a A^--level β-cut subhemiring of R.*

Proof. For all x and y in $N(A^-,\beta)$, we have $A^-(x) \leq \beta$ and $A^-(y) \leq \beta$. Now

$$\begin{aligned} A^-(x+y) &\leq \max\{A^-(x), A^-(y)\} \\ &\leq \max\{\beta,\beta\} \\ &= \beta, \end{aligned}$$

which implies that

$$A^-(x+y) \leq \beta.$$

And

$$\begin{aligned} A^-(xy) &\leq \max\{A^-(x), A^-(y)\} \\ &\leq \max\{\beta,\beta\} \\ &= \beta, \end{aligned}$$

which implies that

$$A^-(xy) \leq \beta.$$

Therefore $x+y, xy$ in $N(A^-,\beta)$. Hence $N(A^-,\beta)$ is a A^--level β-cut subhemiring of R. □

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