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# Interval valued anti-fuzzy soft subhemiring of a hemiring

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**Abstract**
In this paper, we have studied the various operations of interval valued anti fuzzy soft subhemirings of a hemiring to establish their basic properties. We have discussed different algebraic structures of interval valued anti-fuzzy soft subhemirings of a hemiring under the restricted and extended operations. Product and strongest relation have also been made in order to give a complete overview of these structures.

**Keywords**
Interval valued anti-fuzzy soft subhemiring of a hemiring , product of Interval valued anti-fuzzy soft subhemiring of a hemiring, strongest relation of Interval valued anti-fuzzy soft subhemiring of a hemiring.

**AMS Subject Classification**
03E72, 06D72, 08A72.

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## Contents



## 1. Introduction

While modeling most of the real world problems, vagueness or fuzziness of data creates difficulties and we do not get the required results. This inaccuracy and imprecision cause problems in the successful implementation of a mathematical model. To handle these uncertainties, different theories and approaches have been adapted so far. L.A. Zadeh [19] introduced his revolutionary concept of a fuzzy set in 1965.Fuzzy set theory is applied in almost all the branches of mathematics. Molodtsov [12] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [9] described the application of soft set theory to a decision making problem. Maji et al. [11] also studied several operations on the theory of soft sets. Chen et al. [6] presented a new definition of soft set parameterizations reduction, and compared this definition to the related concept of attributes reduction in rough set theory. In this paper, we introduce the concept of interval valued anti-fuzzy soft subhemiring of a hemiring and establish some results on these. We also made an attempt to study the some properties of interval valued anti-fuzzy soft subhemiring of hemiring.

## 2. Preliminaries

**Definition 2.1.** *Let $(R,+,\cdot)$ be a hemiring. A interval valued fuzzy soft subset $[F,A]$ of Ris said to be an interval valued anti-fuzzy soft subhemiring (IAFSSHR) of R if it satisfies the following conditions:*

*(i)*

$$\mu_{(F,A)}(x_{(F,A)}+y_{(F,A)}) \\ \leq \max\{\mu_{(F,A)}(x_{(F,A)}),\mu_{(F,A)}(y_{(F,A)})\},$$

*(ii)*

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \\ \leq \max\{\mu_{(F,A)}(x_{(F,A)}),\mu_{(F,A)}(y_{(F,A)})\},$$

*for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.*

**Definition 2.2.** *Let $(R,+,\cdot)$ be a hemiring. An interval valued anti-fuzzy soft subhemiring $[F,A]$ of $R$ is said to be an interval valued anti-fuzzy interval valued soft normal subhemiring(IAFSNSHR) of $R$ if*

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) = \mu_{(F,A)}(y_{(F,A)}x_{(F,A)})$$

*for all $x_{(F,A)}$ and $y_{(F,A)}$ in $R$.*

**Definition 2.3.** *Let $[F,A]$ and $[G,B]$ be fuzzy interval valued soft subsets of sets $G$ and $H$, respectively. The interval valued anti-product of $[F,A]$ and $[G,B]$, denoted by $[F,A]) \times [G,B]$, is defined as*

$$[F,A] \times [G,B] \\ = \{\langle(x_{(F,A)}, y_{(F,A)}), \mu_{[F,A]\times[G,B]}(x_{(F,A)}, y_{(G,B)})\rangle / \text{ for}$$

*all $x_{(F,A)}$ in $G$ and in $H$, where*

$$\mu_{[F,A]\times[G,B]} = \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,B)}(y_{(G,B)})\}.$$

**Definition 2.4.** *Let $[F,A]$ be a interval valued fuzzy soft subset in a set $S$, the interval valued anti-strongest fuzzy soft relation on $S$, that is a interval valued fuzzy soft relation on $[F,A]$ is $[G,V]$ given by*

$$\mu_{[G,V]}(x_{(F,A)}, y_{(F,A)}) = \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

*for all $x_{(F,A)}$ and $y_{(F,A)}$ in $S$.*

**Definition 2.5.** *An interval valued anti-fuzzy soft subhemiring $[F,A]$ of a hemiring $R$ is called an interval valued anti-fuzzy soft characteristic subhemiring of $R$ if $\mu_{(F,A)}(x_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}))$, for all $x_{(F,A)}$ in $R$ and $f$ in $Aut(R)$.*

**Definition 2.6.** *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. Let $f : R \to R'$ be any function and $[F,A]$ be an interval valued anti-fuzzy soft subhemiring in $R$, $[G,V]$ be an interval valued anti-fuzzy soft subhemiring in $f(R) = R'$, defined by $\mu_{[G,V]}(y) = \inf_{x\in f^{-1}(y)} \mu_{(F,A)}(x_{(F,A)})$, for all $x_{(F,A)}$ in $R$ and $y$ in $R'$. Then $[F,A]$ is called a preimage of $[G,V]$ under $f$ and is denoted by $f^{-1}([G,V])$.*

**Definition 2.7.** *Let $[F,A]$ and $[G,B]$ be two soft sets over a common universe $U$. The union of $[F,A]$ and $[G,B]$ is defined as the soft set $[H,C]$ satisfying the following conditions:*

*(i)* $C = A \cup B$

*(ii)* *For all* $x \in C$,

$$H(x) = \begin{cases} (F(x) & if x \in A-B, \\ G(x) & if x \in B-A, \\ F(x)\cup G(x) & if x \in A\cap B, \end{cases}$$

*This is denoted by $[F,A]\cup[G,B] = [H,C]$.*

**Definition 2.8.** *Let $[F,A]$ be a an interval valued anti –fuzzy soft subset of $X$. For $\alpha$ in $[0,1]$, the lower level soft subset of $[F,A]$ is the set $[F,A]_\alpha = \{x \in X : \mu_{(F,A)}(x_{(F,A)}) \le \alpha\}$.*

**Definition 2.9.** *Let $[F,A]$ be an interval valued anti –fuzzy soft subhemiring ofa hemiring $R$.Then $[F,A]^0$ is defined as $[F,A]^0(x_{(F,A)}) = [F,A](x_{(F,A)})/[F,A](0)$, for all $x_{(F,A)} \in R$.*

## 3. Properties of interval valued anti-fuzzy soft subhemiring of a hemiring

**Theorem 3.1.** *Union of any two interval valued anti-fuzzy softsubhemiring of a hemiring $R$ is an interval valued anti-fuzzy soft subhemiring of $R$.*

*Proof.* Let $[F,A]$ and $[F,B]$ be any two interval valued anti-fuzzy soft subhemirings of a hemiring $R$ and $x$ and $y$ in $R$. Let $[F,A] = \{(\mu_{(F,A)}, \mu_{(F,A)}(x_{(F,A)}))/x_{(F,A)} \in R\}$ and $[G,B] = \{(x_{[G,B]}, \mu_{[G,B]}(x_{[G,B]}))/x_{[G,B]} \in R\}$ and also let

$$[H,C] = [F,A]\cup[G,B] \\ = \{(x_{[H,C]}, \mu_{[H,C]}(x_{[H,C]}))/x_{[H,C]} \in R\},$$

where

$$\max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{[G,B]}(x_{[G,B]})\} \\ = \mu_{[H,C]}(x_{[H,C]}).$$

Where

$$\max\{x_{(F,A)}(x_{(F,A)}), \mu_{[G,B]}(x_{[G,B]})\} \\ = \mu_{[H,C]}(x_{[H,C]}).$$

It is clear the

$$[F,A]\cup[G,B](x) = [F,A](x)$$

When $\mu_{(F,A)}(x_{(F,A)}) \ne 0$ and

$$\mu_{[G,B]}(x_{[G,B]}) = 0.$$

Also,

$$[[F,A]\cup[G,B]](x) = [G,B](x)$$

When $\mu_{(F,A)}(x_{(F,A)}) = 0$ and

$$\mu_{[G,B]}(x_{[G,B]}) \ne 0.$$

It is enough to prove that

$$[[F,A]\cup[G,B]](x) \\ = \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{[G,B]}(x_{[G,B]}\}.$$

Now,

$$[\mu_{[H,C]}](x_{[H,C]} + y_{[H,C]}) \\ = \max\{\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}), \mu_{[G,B]}(x_{[G,B]} + y_{[G,B]})\} \\ \le \max\{\max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}, \\ \max\{\mu_{[G,B]}(x_{[G,B]}), \mu_{[G,B]}(y_{[G,B]})\}\} \\ = \max\{\max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{[G,B]}(x_{[G,B]})\}, \\ \max\{\mu_{(F,A)}(y_{(F,A)}), \mu_{[G,B]}(y_{[G,B]})\} \\ = \max\{\mu_{[H,C]}(x_{[H,C]}), \mu_{[H,C]}(y_{[H,C]})\}.$$

Therefore,

$$\mu_{[H,C]}(x_{[H,C]} + y_{[H,C]}) \\ \leq \max\{\mu_{[H,C]}(x_{[H,C]}), \mu_{[H,C]}(y_{[H,C]})\},$$

for all $x_{[H,C]}$ and $y_{[H,C]}$ in $R$. And,

$$\mu_{[H,C]}(x_{[H,C]}y_{[H,C]}) \\ = \max\{\mu_{(F,A)}(x_{[F,A]}y_{[F,A]}), \\ \mu_{[G,B]}(x_{[G,B]}y_{[G,B]})\} \\ \leq \max\{\max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}, \\ \max\{\mu_{[G,B]}(x_{[G,B]}), \mu_{[G,B]}(y_{[G,B]})\}\} \\ = \max\{\max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{[G,B]}(x_{[G,B]})\}, \\ \max\{\mu_{(F,A)}(y_{(F,A)}), \mu_{[G,B]}(y_{[G,B]})\}\} \\ = \max\{\mu_{[H,C]}(x_{[H,C]}), \mu_{[H,C]}(y_{[H,C]})\}.$$

Therefore,

$$(\mu_{[H,C]})(x_{[H,C]}y_{[H,C]}) \\ \leq \max\{\mu_{[H,C]}(x_{[H,C]}), \mu_{[H,C]}(y_{[H,C]})\},$$

for all $x_{[H,C]}$ and $y_{[H,C]}$ in $R$. Therefore $[H,C]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring $R$. Hence the union of any two interval valued anti-fuzzy softsubhemirings of a hemiring $R$ is an interval valued anti-fuzzy soft subhemiring of $R$. □

**Theorem 3.2.** *The union of a family of interval valued anti-fuzzy soft subhemirings of hemiring $R$ is an interval valued anti-fuzzy soft subhemiring of $R$.*

*Proof.* Let $\{[F,V_i] : i \in I\}$ be a family of interval valued anti-fuzzysoft subhemirings of a hemiring $R$ and let $[F,A] = i \in IV_i$ Let $x$ and $y$ in $R$. Then,

$$\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \\ = \sup_{i\in I} \mu_{[F,V_i]}(x_{(F,A)} + y_{(F,A)}) \\ \leq \sup_{i\in I} \max\{\mu_{[F,V_i]}(x_{(F,A)}), \\ \mu_{[F,V_i]}(y_{(F,A)})\} \\ = \max\{\sup_{i\in I} \mu_{[F,V_i]}(x_{(F,A)}), \\ \sup_{i\in I} \mu_{[F,V_i]}(y_{(F,A)})\} \\ = \max\{\mu_{[F,V_i]}(x_{(F,A)}), \mu_{[F,V_i]}(y_{(F,A)})\}.$$

Therefore,

$$\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \\ \leq \max\{\mu_{(F,A)}x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in $R$. And,

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \\ = \mu_{[F,V_i]}(x_{(F,A)}y_{(F,A)}) \\ \leq \max\{\mu_{[F,V_i]}(x_{(F,A)}), \mu_{[F,V_i]}(y_{(F,A)})\} \\ = \max\{\mu_{[F,V_i]}(x_{(F,A)}), \mu_{[F,V_i]}(y_{(F,A)}])\} \\ = \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}.$$

Therefore,

$$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \\ \leq \max\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\},$$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in $R$. That is, $[F,A]$ is an interval valued anti-fuzzy softsubhemiring of a hemiring $R$. Hence, the union of a family of interval valued anti-fuzzy soft subhemirings of $R$ is an anti-fuzzy soft subhemiring of $R$.

□

**Theorem 3.3.** *If $[F,A]$ and $[G,B]$ are any two interval valued anti-fuzzy soft subhemirings of the hemirings $R_1$ and $R_2$ respectively, then anti-product $[F,A] \times [G,B]$ is an interval valued anti-fuzzy soft subhemiring of $R_1 \times R_2$.*

*Proof.* Let $[F,A]$ and $[G,B]$ be two interval valued anti-fuzzy soft subhemirings of the hemirings $R_1$ and $R_2$ respectively.Let $x_{[F,A]1}$ and $x_{[F,A]2}$ be in $R_1, y_{[G,B]1}$ and $y_{[G,B]2}$ be in $R_2$. Then $(x_{[F,A]1}, y_{[G,B]1})$ and $(x_{[F,A]2}, y_{[G,B]2})$ are in $R_1 \times R_2$. Now,

$$\mu_{[F,A]\times[G,B]}[(x_{[F,A]1}, y_{[G,B]1}) \\ + (x_{[F,A]2}, y_{[G,B]2})] \\ = \mu_{[F,A]\times[G,B]}(x_{[F,A]1} + x_{[F,A]2}, \\ y_{[G,B]1} + y_{[G,B]2}) \\ = \max\{\mu_{[F,A]}(x_{[F,A]1} + x_{[F,A]2}), \\ \mu_{[G,B]}(y_{[G,B]1} + y_{[G,B]2})\} \\ \leq \max\{\max\{\mu_{[F,A]}(x_{[F,A]1}), \mu_{[F,A]}(x_{[F,A]2})\}, \\ \max\{\mu_{[G,B]}(y_{[G,B]1}), \mu_{[G,B]}(y_{[G,B]2})\}\} \\ = \max\{\max\{\mu_{[F,A]}(x_{[F,A]1}), \mu_{[G,B]}(y_{[G,B]1})\}, \\ \max\{\mu_{[F,A]}(x_{[F,A]2}), \mu_{[G,B]}(y_{[G,B]2})\} \\ = \max\{\mu_{[F,A]\times[G,B]}(x_{[F,A]1}, y_{[G,B]1}), \\ \mu_{[F,A]\times[G,B]}(x_{[F,A]2}, y_{[G,B]2})\}.$$

Therefore,

$$\mu_{[F,A]\times[G,B]}[(x_{[F,A]1}, y_{[G,B]1}) \\ + (x_{[F,A]2}, y_{[G,B]2})] \\ \leq \max\{\mu_{[F,A]\times[G,B]}\}(x_{[F,A]1}, y_{[G,B]1}), \\ \{\mu_{[F,A]\times[G,B]}\}(x_{[F,A]2}, y_{[G,B]2})\}.$$

Also,

$$\mu_{[F,A]\times[G,B]}[(x_{[F,A]1},$$
$$y_{[G,B]1})(x_{[F,A]2},y_{[G,B]2})]$$
$$=\mu_{[F,A]\times[G,B]}(x_{[F,A]1\times[F,A]2},$$
$$y_{[G,B]1\times[G,B]2})$$
$$=\max\{\mu_{[F,A]}(x_{[F,A]1\times[F,A]2}),$$
$$\mu_{(GxB)}(y_{[G,B]1y[G,B]2})\}$$
$$\leq\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),$$
$$\mu_{[F,A]}(x_{[F,A]2})\},\max\{\mu_{(G\times B)}(y_{[G,B]1}),$$
$$\mu_{(G\times B)}(y_{[G,B]2})\}\}$$
$$=\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{(G\times B)}(y_{[G,B]1})\},$$
$$\max\{\mu_{[F,A]}(x_{[F,A]2}),\mu_{(GxB)}(y_{[G,B]2})\}\}$$
$$=\max\{\mu_{[F,A]\times[G,B]}(x_{[F,A]1},y_{[G,B]1}),$$
$$\mu_{[F,A]\times[G,B]}(x_{[F,A]2},y_{[G,B]2})\}.$$

Therefore,

$$\mu_{[F,A]\times[G,B]}[(x_{[F,A]1},y_{[G,B]1})(x_{[F,A]2},y_{[G,B]2})]$$
$$\leq\max\{\mu_{[F,A]\times[G,B]}(x_{[F,A]1},y_{[G,B]1}),$$
$$\mu_{[F,A]\times[G,B]}(x_{[F,A]2},y_{[G,B]2})\}.$$

Hence $[F,A]\times[G,B]$ is an interval valued anti-fuzzy soft subhemiring of hemiring of $R_1\times R_2$.

□

**Theorem 3.4.** *Let $[F,A]$ be a interval valued fuzzy interval valued soft subset of a hemiring $R$ and $[G,V]$ be the strongest interval valued anti-fuzzy interval valued soft relation of $R$. Then $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of $R$ if and only if $[G,V]$ is an interval valued anti-fuzzy interval valued soft subhemiring of $R\times R$.*

*Proof.* Suppose that $[F,A]$ is an interval valued anti-fuzzy subhemiring of a hemiring $R$. Then for any $x=(x_{[F,A]1},x_{[F,A]2})$ and $y=(y_{[F,A]1},y_{[F,A]2})$ are in $R\times R$. We have,

$$\mu_{[G,V]}(x_{[G,V]}+y_{[G,V]})$$
$$=\mu_{[G,V]}[(x_{[G,V]1},x_{[G,V]2})+(y_{[G,V]1},y_{[G,V]2})]$$
$$=\mu_{[G,V]}(x_{[G,V]1}+y_{[G,V]1},x_{[G,V]2}+y_{[G,V]2})$$
$$=\max\{\mu_{[F,A]}(x_{[F,A]1}+y_{[F,A]1}),$$
$$\mu_{[F,A]}(x_{[F,A]2}+y_{[F,A]2})\}$$
$$\leq\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]1})\},$$
$$\max\{\mu_{[F,A]}(x_{[F,A]2}),\mu_{[F,A]}(y_{[F,A]2})\}\}$$
$$=\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2})\},$$
$$\max\{\mu_{[F,A]}(y_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]2})\}\}$$
$$=\max\{\mu_{[G,V]}(x_{[G,V]1},x_{[G,V]2}),\mu_{[G,V]}(y_{[G,V]1},y_{[G,V]2})\}$$
$$=\max\{\mu_{[G,V]}(x_{[G,V]}),\mu_{[[G,V]}(y_{[G,V]})\}.$$

Therefore,

$$\mu_{[G,V]}(x_{[G,V]}+y_{[G,V]})$$
$$\leq\max\{\mu_{[G,V]}(x_{[G,V]}),$$
$$\mu_{[G,V]}(y_{[G,V]})\},$$

for all $x_{[G,V]}$ and $y_{[G,V]}$ in $R\times R$. And,

$$\mu_{[G,V]}(x_{[G,V]}y_{[G,V]})$$
$$=\mu_{[G,V]}[(x_{[G,V]1},x_{[G,V]2})(y_{[G,V]1},y_{[G,V]2})]$$
$$=\mu_{[G,V]}(x_{[G,V]1}y_{[G,V]1},x_{[G,V]2}y_{[G,V]2})$$
$$=\max\{\mu_{[F,A]}(x_{[F,A]1}y_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2}y_{[F,A]2})\}$$
$$\leq\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]1})\},$$
$$\max\{\mu_{[F,A]}(x_{[F,A]2}),\mu_{[F,A]}(y_{[F,A]2})\}$$
$$=\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2})\},$$
$$\max\{\mu_{[F,A]}(y_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]2})\}\}$$
$$=\max\{\mu_{[G,V]}(x_{[G,V]1},x_{[G,V]2}),\mu_{[G,V]}(y_{[G,V]1},y_{[G,V]2})\}$$
$$=\max\{\mu_{[G,V]}(x_{[G,V]}),\mu_{[G,V]}(y_{[G,V]})\}.$$

Therefore,

$$\mu_{[G,V]}(x_{[G,V]}y_{[G,V]})$$
$$\leq\max\{\mu_{[G,V]}(x_{[G,V]}),\mu_{[G,V]}(y_{[G,V]})\},$$

for all $x_{[G,V]}$ and $y_{[G,V]}$ in $R\times R$. This proves that $[G,V]$ is an interval valued anti-fuzzy soft subhemiring of $R\times R$. Conversely assume that $[G,V]$ is an interval valued anti-fuzzy soft subhemiring of $R\times R$, then for any $x=(x_{[G,V]1},x_{[G,V]2})$ and $y=(y_{[G,V]1},y_{[G,V]2})$ are in $R\times R$, we have

$$\max\{\mu_{[F,A]}(x_{[F,A]1}+y_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2}+y_{[F,A]2})\}$$
$$=\mu_{[G,V]}(x_{[G,V]1}+y_{[G,V]1},x_{[G,V]2}+y_{[G,V]2})$$
$$=\mu_{[G,V]}[(x_{[G,V]1},x_{[G,V]2})+(y_{[G,V]1},y_{[G,V]2)}]$$
$$=\mu_{[G,V]}(x_{[G,V]}+y_{[G,V]})$$
$$\leq\max\{\mu_{[G,V]}(x_{[G,V]}),\mu_{[G,V]}(y_{[G,V]})\}$$
$$=\max\{\mu_{[G,V]}(x_{[G,V]1},x_{[G,V]2}),\mu_{[G,V]}(y_{[G,V]1},y_{[G,V]2})\}$$
$$=\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2})\},$$
$$\max\{\mu_{[F,A]}(y_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]2})\}\}.$$

If

$$\mu_{[F,A]}(x_{[F,A]1}+y_{[F,A]1})$$
$$\geq\mu_{[F,A]}(x_{[F,A]2}+y_{[F,A]2}),\mu_{[F,A]}(x_{[F,A]1})$$
$$\geq\mu_{[F,A]}(x_{[F,A]2}),\mu_{[F,A]}(y_{[F,A]1})$$
$$\geq\mu_{[F,A]}(y_{[F,A]2}),$$

we get,

$$\mu_{[F,A]}(x_{[F,A]1}+y_{[F,A]1})$$
$$\leq\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]1})\},$$

for all $x_{[F,A]1}$ and $y_{[F,A]1}$ in $R$. And,

$$\begin{aligned}&\max\{\mu_{[F,A]}(x_{[F,A]1}y_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2}y_{[F,A]2})\}\\&=\mu_{[G,V]}(x_{[G,V]1}y_{[G,V]1},x_{[G,V]2}y_{[G,V]2})\\&=\mu_{[G,V]}[(x_{[G,V]1},x_{[G,V]2})(y_{[G,V]1},y_{[G,V]2})]\\&=\mu_{[G,V]}(x_{[G,V]}y_{[G,V]})\\&\le\max\{\mu_{[G,V]}(x_{[G,V]}),\mu_{[G,V]}(y_{[G,V]})\}\\&=\max\{\mu_{[G,V]}(x_{[G,V]1},x_{[G,V]2}),\mu_{[G,V]}(y_{[G,V]1},y_{[G,V]2})\}\\&=\max\{\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(x_{[F,A]2})\},\\&\quad\max\{\mu_{[F,A]}(y_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]2})\}.\end{aligned}$$

If

$$\begin{aligned}&\mu_{[F,A]}(x_{[F,A]1}y_{[F,A]1})\\&\ge\mu_{[F,A]}(x_{[F,A]2}y_{[F,A]2}),\mu_{[F,A]}(x_{[F,A]1})\\&\ge\mu_{[F,A]}(x_{[F,A]2}),\mu_{[F,A]}(y_{[F,A]1})\\&\ge\mu_{[F,A]}(y_{[F,A]2}),\end{aligned}$$

we get

$$\begin{aligned}&\mu_{[F,A]}(x_{[F,A]1}y_{[F,A]1})\\&\le\max\{\mu_{[F,A]}(x_{[F,A]1}),\mu_{[F,A]}(y_{[F,A]1})\},\end{aligned}$$

for all $x_{[F,A]1}$ and $y_{[F,A]1}$ in $R$. Therefore $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of $R$. □

**Theorem 3.5.** *$[F,A]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring $(R,+,\cdot)$ if and only if $\mu_{[F,A]}(x_{[F,A]}+y_{[F,A]})\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\}$, $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]})\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\}$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in $R$.*

*Proof.* It is trivial. □

**Theorem 3.6.** *If $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring $(R,+,\cdot)$, then $H=\{x_{[F,A]}/x_{[F,A]}\in R:\mu_{[F,A]}(x_{[F,A]})=0\}$ is either empty or is a subhemiring of $R$.*

*Proof.* If no element satisfies this condition, then $H$ is empty. If $x_{[F,A]}$ and $y_{[F,A]}$ in $H$, then

$$\begin{aligned}&\mu_{[F,A]}(x_{[F,A]}+y_{[F,A]})\\&\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\}\\&=\max\{0,0\}\\&=0.\end{aligned}$$

Therefore,

$$\mu_{[F,A]}(x_{[F,A]}+y_{[F,A]})=0.$$

And

$$\begin{aligned}&\mu_{[F,A]}(x_{[F,A]}y_{[F,A]})\\&\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\}\\&=\max\{0,0\}\\&=0.\end{aligned}$$

Therefore,

$$\mu_{[F,A]}(x_{[F,A]}y_{[F,A]})=0.$$

We get

$$x_{[F,A]}+y_{[F,A]},x_{[F,A]}y_{[F,A]}$$

in $H$. Therefore, $H$ is a subhemiring of $R$. Hence $H$ is either empty or is a subhemiring of $R$. □

**Theorem 3.7.** *Let $[F,A]$ be an interval valued anti-fuzzy soft subhemiring of a hemiring $(R,+,\cdot)$. If $\mu_{[F,A]}(x_{[F,A]}+y_{[F,A]})=1$, then either $\mu_{[F,A]}(x_{[F,A]})=1$ or $\mu_{[F,A]}(y_{[F,A]})=1$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in $R$.*

*Proof.* Let $x_{[F,A]}$ and $y_{[F,A]}$ in $R$. By the definition

$$\begin{aligned}&\mu_{[F,A]}(x_{[F,A]}+y_{[F,A]})\\&\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\},\end{aligned}$$

which implies that

$$1\le\max\{\mu_{[F,A]}(x_{[F,A]}),\mu_{[F,A]}(y_{[F,A]})\}.$$

Therefore, either

$$\mu_{[F,A]}(x_{[F,A]})=1$$

or

$$\mu_{[F,A]}(y_{[F,A]})=1.$$

□

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