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Ideals and symmetric reverse bi-derivations of prime and semiprime rings

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Abstract

Let R be a prime ring of char $R \neq 2$ and I a nonzero ideal of R. Suppose that there exist symmetric reverse bi-derivations $D_1(.,.) : RXR \to R$ and $D_2(.,.) : RXR \to R$ such that $D_1(d_2(x),x) = 0$ for all $x \in I$, where d_2 denotes the trace of D_2. Then either $D_1 = 0$ or $D_2 = 0$.

Keywords

Derivation, Reverse derivation, Symmetric bi-derivation, Symmetric reverse bi-derivation, Prime rings, Semiprime rings, Trace.

AMS Subject Classification

16W25, 16N60, 16U80.

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Contents

1. Introduction

The concept of a symmetric bi-derivation has been introduced by Maksa. Gy in [6]. In [8], J. Vukman has proved some results concerning symmetric bi-derivation on prime and semiprime rings. Jaya Subba Reddy. C et al. [3 and 4] has studied reverse derivations and Symmetric reverse bi-derivations on prime rings. M. S. Yenigul and N. Argac [7] studied few results ideals and symmetric bi-derivations of prime and semiprime rings. In this paper, we extended some results on ideals and symmetric reverse bi-derivations of prime and semiprime rings.

2. Preliminaries

Throughout this paper, R considered as an associative ring with the center $Z(R)$. Recall that a ring R is called prime if for any $a,b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$, for any $x,y \in R$. A ring R is said to be semiprime if $aRa = 0$ with $a \in R$ implies $a = 0$. We shall write $[x,y]$ for $xy - yx$. A mapping $D(.,.) : RXR \to R$ is called symmetric if $D(x,y) = D(y,x)$, for all $x,y \in R$. A mapping $d : R \to R$ defined by $d(x) = D(x,x)$ is called the trace of D, where $D(.,.) : RXR \to R$ is a symmetric mapping. It is obvious that, if $D(.,.) : RXR \to R$ is a symmetric mapping which is also bi-additive (i.e. additive in both arguments), then the trace d of D satisfies the relation $d(x+y) = d(x) + d(y) + 2D(x,y)$, for all $x,y \in R$. Let R be a ring and I be a nonzero right (resp. left) ideal of R. We shall say that a mapping $D(.,.) : RXR \to R$ acts as a right (resp. left) R homomorphism on I if $D(rx,y) = D(x,y)r$ and $D(x,ry) = D(x,y)r$ (resp. $D(xr,y) = rD(x,y)$ and $D(x,yr) = rD(x,y)$) for all $x,y,z \in R$. Let S be a set. $r_R(S)$ (resp. $l_R(S)$) will be denote the right (resp. left) annihilator of S. An additive mapping $d : R \to R$ is said to be a derivation if $d(xy) = d(x)y + xd(y)$, for all $x,y \in R$. An additive mapping $d : R \to R$ is said to be a reverse derivation if $d(xy) = d(y)x + yd(x)$, for all $x,y \in R$. If $D(.,.)$ is bi-additive and satisfies the identities $D(xy,z) = D(x,z)y + xD(y,z)$ and $D(x,yz) = D(x,y)z + yD(x,z)$, for all $x,y,z \in R$, then $D(.,.)$ is called a symmetric bi-derivation. If $D(.,.)$ is reverse bi-additive and satisfies the identity $D(xy,z) = D(y,z)x + yD(x,z)$ and $D(x,yz) = D(x,z)y + zD(x,y)$, for all $x,y,z \in R$, then $D(.,.)$ is called a symmetric reverse bi-derivation. We shall make use of commutator identities; $[x,yz] = [x,y]z + y[x,z]$ and $[xy,z] = [x,z]y + x[y,z]$, for all $x,y,z \in R$.

Lemma 2.1. *Let $D : R \to R$ be a reverse derivation of a prime* *ring R and I a nonzero ideal of R. Suppose that either (i) $aD(x) = 0$, for all $x \in I$ or (ii) $D(x)a = 0$, for all $x \in I$ holds. Then $a = 0$ or $D = 0$.*

Proof. (i) for any $x \in I$, we have

$$D(x)a = 0 \tag{2.1}$$

Replacing x by yx in above equation then, we get

$$aD(yx) = 0$$

$$a(D(x)y + xD(y)) = 0$$

$$aD(x)y + axD(y) = 0$$

Using equation (2.1) then, we get $axD(y) = 0$, for all $y \in I$.

$$aRD(y) = 0$$

Since R is a prime which implies that either $a = 0$ or $D = 0$.
(ii) for any $x \in I$, we have

$$aD(x) = 0 \tag{2.2}$$

Replacing x by yx in above equation then, we get

$$D(yx)a = 0$$

$$(D(x)y + xD(y))a = 0$$

$$D(x)ya + xD(y)a = 0$$

Using equation (2.2) then, we get $D(x)ya = 0$, for all $x, y \in I$.

$$D(x)Ra = 0$$

. Since R is a prime which implies that either $a = 0$ or $D = 0$. □

Lemma 2.2 (2, Lemma 1)**.** *Let R be a prime ring and I a nonzero right ideal of R. If I is a commutative, then R is a commutative.*

Lemma 2.3 (7, Lemma 3)**.** *Let R be a prime ring of char $R \neq 2$ and I a nonzero ideal of R. Let a, b be a fixed elements of R. If $axb + bxa = 0$ is fulfilled for all $x \in I$, either $a = 0$ or $b = 0$.*

Lemma 2.4. *Let R be a prime ring of char $R \neq 2$ and I a nonzero left (or right) ideal of R. Let $D(.,.) : RXR \to R$ be a symmetric reverse bi-derivation and d the trace of D. Suppose that $d(x) = 0$ for all $x \in I$. Then $d = 0$, that is, $D = 0$.*

Proof. for any $x \in I$, we have

$$d(x) = 0 \tag{2.3}$$

The linearization of equation (2.3) then, we get

$$d(x+y) = 0$$

$$d(x) + d(y) + 2D(x,y) = 0$$

, for all $x, y \in I$. Since $d(x) = d(y) = 0$ and char $R \neq 2$, then

$$D(x,y) = 0 \tag{2.4}$$

Replacing y by yr ($r \in R$) in equation (2.4), and using equation (2.4), we get

$$D(x,yr) = 0$$

$$D(x,r)y + rD(x,y) = 0$$

$D(x,r)y = 0$, for all $x, y \in I, r \in R$. Since the left annihilator of a nonzero left ideal is zero, we have

$$D(x,r) = 0 \tag{2.5}$$

Replacing x by xr in equation (2.5), and using equation (2.5), we get

$$D(xr,r) = 0$$

$$D(r,r)x + rD(x,r) = 0$$

$d(r)x = 0$, for all $x \in I, r \in R$. Hence $d(r)$ is an element of the left annihilator of I then, $d(r) = 0$, for all $r \in R$. □

3. Main Results

Theorem 3.1. *Let R be a non commutative prime ring and I a nonzero ideal of R. Let $D(.,.) : RXR \to R$ be a symmetric reverse bi-derivation such that $D(I,I) \subset I$ and d the trace of D.*
(i) If char $R \neq 2$ and $[x, d(x)] = 0$, for all $x \in I$, then $D = 0$.
(ii) If char $R \neq 2,3$ and $[x, d(x)] \in Z(R)$, for all $x \in I$, then $D = 0$.

Proof. (i) I is not a commutative ideal of R by Lemma 2.2. Since I is a nonzero ideal of a prime ring R of char $R \neq 2$, I itself is a non commutative prime ring of char $I \neq 2$. Therefore, $d(x) = 0$, for all $x \in I$ by the proof of [4,Theorem 1] and $d(r) = 0$, for all $r \in R$ by Lemma 2.4. Hence $D = 0$.
(ii) Since char $I \neq 2,3$, we have $[x, d(x)] = 0$, for all $x \in I$ by the proof of [4,Theorem 2]. Hence $d(r) = 0$, for all $r \in R$ by (i). Hence $D = 0$. □

Theorem 3.2. *Let R be a prime ring of char $R \neq 2$ and I a nonzero ideal of R. Suppose that there exist symmetric reverse bi-derivations $D_1(.,.) : RXR \to R$ and $D_2(.,.) : RXR \to R$ such that $D_1(d_2(x), x) = 0$ for all $x \in I$, where d_2 denotes the trace of D_2. Then either $D_1 = 0$ or $D_2 = 0$.*

Proof. It is enough to show that $d_1(I) = 0$ or $d_2(I) = 0$ by Lemma 2.4 and by the proof of [4,Theorem 3], for any $x, y \in I$

$$d_1(x)yd_2(x) + d_2(x)yd_1(x) = 0 \tag{3.1}$$

By using Lemma 2.3, we get $d_1(I) = 0$ or $d_2(I) = 0$. Hence $D_1 = 0$ or $D_2 = 0$. □

Theorem 3.3. *Let R be a ring and I a nonzero right (resp. left) ideal of R such that $r_R(I) = 0$ (resp. $l_R(I) = 0$). Let $D(.,.) : RXR \to R$ be a symmetric reverse bi-derivation. If D acts as a right (resp. left) R homomorphism on I, then $D = 0$.*

Proof. Suppose that I is a right ideal such that $r_R(I) = 0$ and D acts as a right R homomorphism on I. Then $D(x,y)r = D(x,ry) = D(x,y)r + yD(x,r)$, for all $x,y \in I$, $r \in R$ and $yD(x,r) = 0$, for all $x,y \in I$, $r \in R$. Hence $D(x,r) \in r_R(I) = 0$. Then we have $0 = D(sx,r) = D(x,r)s + xD(s,r) = xD(s,r)$, for all $x \in I$, $r,s \in R$. As the above, $D(r,s) = 0$, for all $r,s \in R$. □

Corollary 3.4. *Let R be a prime ring and I a nonzero right (resp. left) ideal of R. Let $D(.,.) : RXR \to R$ be a symmetric reverse bi-derivation. If D acts as a right (resp. left) R homomorphism on R, then $D = 0$.*

Corollary 3.5. *Let R be a semiprime ring and $D(.,.) : RXR \to R$ be a symmetric reverse bi- derivation. If D acts as a right (resp. left) R homomorphism on R, then $D = 0$.*

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